IYGB GCE

Core Mathematics C3

Advanced

Practice Paper G

Difficulty Rating: 3.4667/1.5789

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Show that

$$1+\frac{x-8}{x^2+2x-8}-\frac{2}{x+4}\equiv\frac{x-p}{x-q},$$

stating the value of each of the integer constants, p and q. **(5)**

Question 2

The function $f(x)$ is given by

$$f(x)=4-\ln(2x-1),\ x\in\mathbb{R},\ x>\frac{1}{2}.$$

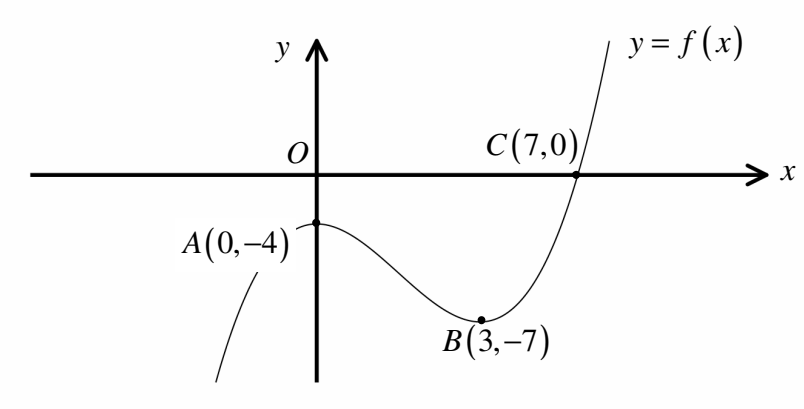
a) Find an expression for $f^{-1}(x)$, in its simplest form. **(4)**

b) Determine the exact value of $ff(1)$. **(2)**

c) Hence, or otherwise, solve the equation

$$f(x)=ff(1).$$ **(3)**

Question 3



The figure above shows part of the graph of the curve with equation $y = f(x)$.

The graph meets the coordinate axes at $A(0,-4)$ and $C(7,0)$ and has a stationary point at $B(3,-7)$.

Sketch on separate diagrams, indicating the new coordinates of the points A, B and C, the graph of …

a) … $y = |f(x)|$. **(2)**

b) … $y = f(|x|)$. **(2)**

c) … $y = 3f(3x)$. **(2)**

Question 4

The angle x is acute so that $\tan x = \frac{1}{2}$.

a) Find the exact value of $\operatorname{cosec} x$. **(2)**

It is further given that $\tan(x+y) = 2$, where y is another angle.

b) Determine the value of $\tan y$. **(3)**

Question 5

The volume of water in a tank V m^3, t hours after midnight, is given by the equation

$$V = 10 + 8\mathrm{e}^{-\frac{1}{12}t}, \ t \geq 0.$$

a) State the volume of water in the tank at midnight. **(1)**

b) Find the time, using 24 hour clock notation, when the volume of the water in the tank is 14 m^3. **(5)**

c) Determine the rate at which the volume of the water is changing at midday, explaining the significance of its sign. **(4)**

d) State the limiting value of V. **(1)**

Question 6

A curve C has equation

$$y = \mathrm{e}^{2x}\left(x^2 - 4x - 2\right), \ x \in \mathbb{R}.$$

a) Show clearly that

$$\frac{dy}{dx} = 2\mathrm{e}^{2x}\left(x^2 - 3x - 4\right). \quad \textbf{(5)}$$

b) Hence find the exact coordinates of the stationary points of C.
You need not determine their nature. **(4)**

Question 7

The curve C has equation

$$y = \sqrt{\mathrm{e}^{2x} - 2x}\,,\ \mathrm{e}^{2x} > 2x\,.$$

The tangent to C at the point P ,where $x = p$, passes through the origin.

a) Show that $x = p$ is a solution of the equation

$$(1-x)\mathrm{e}^{2x} = x\,. \qquad (\mathbf{8})$$

b) Show further that the equation of part **(a)** has a root between 0.8 and 1. $(\mathbf{3})$

The iterative formula

$$x_{n+1} = 1 - x_n\,\mathrm{e}^{-2x_n}$$

with $x_0 = 0.8$ is used to find this root.

c) Find, to 3 decimal places, the value of x_1, x_2, x_3 and x_4. $(\mathbf{2})$

d) Hence show that the value of p is 0.8439 , correct to 4 decimal places. $(\mathbf{2})$

Question 8

$$f(x)=\sqrt{3}\cos x-\sin x,\ x\in\mathbb{R}.$$

a) Express $f(x)$ in the form $R\cos(x+\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$. **(3)**

b) State the maximum value of $f(x)$ and find the smallest positive value of x for which this maximum occurs. **(3)**

The depth of the water, D metres, in a harbour is modelled by the equation

$$D=13+\sqrt{3}\cos\left(\frac{\pi t}{6}\right)-\sin\left(\frac{\pi t}{6}\right),\ 0\le t<24$$

where t is the time in hours measured since midnight.

c) State the maximum depth of the water in the harbour and a time when this maximum depth occurs. **(3)**

d) Find the times when the depth of the water in the harbour is 12 metres. **(6)**