1. a) $e^{2x} = 9$

$2x = \ln 9$

$x = \frac{1}{2}\ln 9$

(OR $x = \ln 3$)

b) $\ln(4-y) = 2$

$4 - y = e^2$

$4 - e^2 = y$

$y = 4 - e^2$

c) $\ln t + \ln 3 = \ln 12$

$\ln 3t = \ln 12$

$3t = 12$

$t = 4$

2. a) $e^{3x} = x + 20$

$e^{3x} - x - 20 = 0$

$f(x) = e^{3x} - x - 20$

$f(1) = -0.914\ldots$

$f(2) = 381.42\ldots$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN, THERE MUST BE A ROOT α BETWEEN 1 & 2

b) $x_{n+1} = \frac{1}{3}\ln(x_n + 20)$

$x_0 = 1.5$

$x_1 \simeq 1.0227$

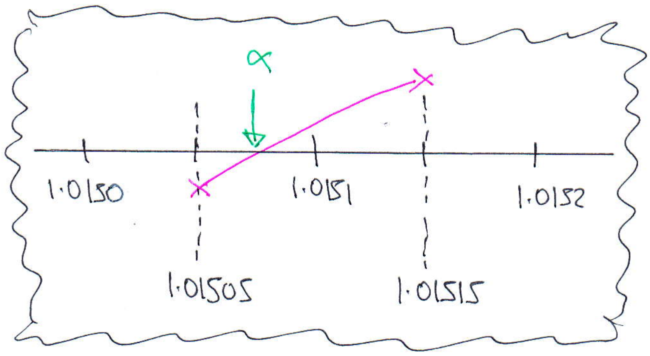
$x_2 \simeq 1.0152$

$x_3 \simeq 1.0151$

c) $f(x) = e^{3x} - x - 20$

$f(1.01505) = -0.0019$

$f(1.01515) = 0.0043$



CHANGE OF SIGN $\Rightarrow$

$1.01505 < \alpha < 1.01515$

$\alpha = 0.0151$

CORRECT TO 4 d.p.

C3, IYGB, PAPER E

3. (a)

$$\sin x + \sqrt{3}\cos x \equiv R\cos(x-\alpha)$$

$$\sin x + \sqrt{3}\cos x \equiv R\cos x\cos\alpha + R\sin x\sin\alpha$$

$$\sin x + \sqrt{3}\cos x \equiv (R\cos\alpha)\cos x + (R\sin\alpha)\sin x$$

$$\left.\begin{array}{l} R\cos\alpha = \sqrt{3} \\ R\sin\alpha = 1 \end{array}\right\} \Rightarrow \text{SQUARE AND ADD } R = \sqrt{(\sqrt{3})^2 + 1^2}$$

$$\boxed{R = 2}$$

$$\Rightarrow \text{DIVIDE EQUATIONS } \frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{1}{\sqrt{3}}$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\boxed{\alpha = \frac{\pi}{6}}$$

$$\therefore f(x) = 2\cos\left(x - \frac{\pi}{6}\right)$$

b)

	MIN	MAX
$f(x)$	-2	2
$[f(x)]^2 = 4\cos^2\left(x - \frac{\pi}{6}\right)$	0	4
$\frac{1}{5+f(x)} = \frac{1}{5 + 2\cos\left(x-\frac{\pi}{6}\right)}$	$\frac{1}{5+2} = \frac{1}{7}$	$\frac{1}{5-2} = \frac{1}{3}$

4. a) $y = \frac{2x^2+3x}{2x^2-x-6} - \frac{6}{x^2-x-2} = \frac{x(\cancel{2x+3})}{(\cancel{2x+3})(x-2)} - \frac{6}{(x-2)(x+1)}$

$= \frac{x}{x-2} - \frac{6}{(x-2)(x+1)} = \frac{x(x+1)-6}{(x-2)(x+1)} = \frac{x^2+x-6}{(x-2)(x+1)}$

$= \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \frac{x+3}{x+1}$

b) $\frac{dy}{dx} = \frac{(x+1)\times 1-(x+3)\times 1}{(x+1)^2} = \frac{x+1-x-3}{(x+1)^2} = -\frac{2}{(x+1)^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{1}{2}$

NORMAL GRADIENT = 2

WHEN $x=1$ $\quad y = \frac{1+3}{1+1} = 2 \quad$ i.e. $(1,2)$

$y-y_0 = m(x-x_0)$

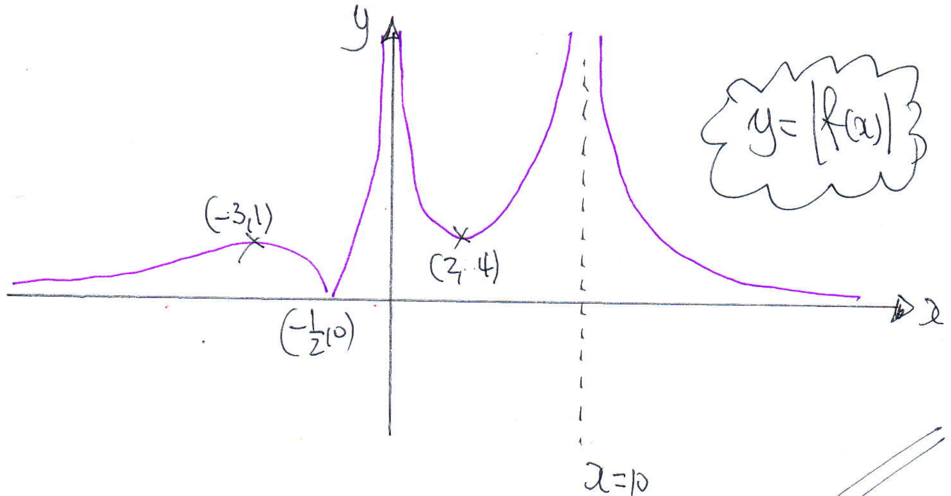
$y-2 = 2(x-1)$

$y-\cancel{2} = 2x-\cancel{2}$

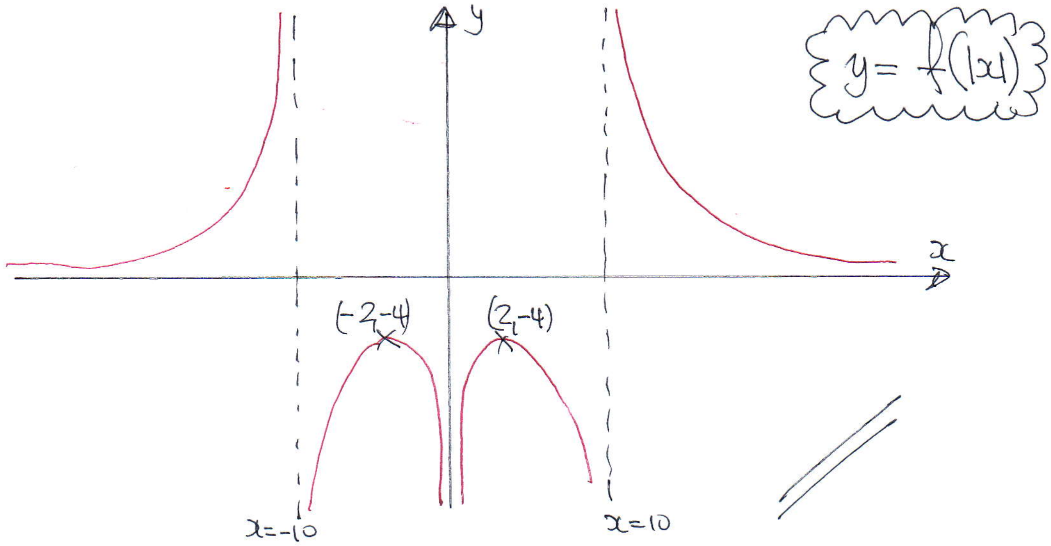
$y = 2x$

$\therefore$ THROUGH THE ORIGIN.

5. (a)



(b)



6.

$y(y-1) = 5x-3$

$\Rightarrow y^2 - y = 5x-3$

$\Rightarrow y^2 - y + 3 = 5x$

$\Rightarrow x = \frac{1}{5}y^2 - \frac{1}{5}y + \frac{3}{5}$

$\frac{dx}{dy} = \frac{2}{5}y - \frac{1}{5}$

$\frac{dy}{dx} = \frac{1}{\frac{2}{5}y - \frac{1}{5}}$

OR $\frac{dy}{dx} = \frac{5}{2y-1}$

NOW when $x=3$

$y^2 - y = 12$

$y^2 - y - 12 = 0$

$(y+3)(y-4) = 0$

$y = \begin{cases} 4 \\ -3 \end{cases}$

$\left.\frac{dy}{dx}\right|_{y=4} = \frac{5}{2\times4-1} = \frac{5}{7}$

$\left.\frac{dy}{dx}\right|_{y=-3} = \frac{5}{2(-3)-1} = -\frac{5}{7}$

7. a) $7\cot^2\alpha + 6\cot\alpha = 1$

$\Rightarrow 7\cot^2\alpha + 6\cot\alpha - 1 = 0$

$\Rightarrow (7\cot\alpha - 1)(\cot\alpha + 1) = 0$

$\Rightarrow \cot\alpha = \begin{cases} -1 \\ \frac{1}{7} \end{cases}$

$\Rightarrow \tan\alpha = \begin{cases} \cancel{-1} \quad (\alpha \text{ acute}) \\ 7 \end{cases}$

• $6\tan\theta = 8 + \sec^2\theta$

$\Rightarrow 6\tan\theta = 8 + (1 + \tan^2\theta)$

$\Rightarrow 0 = \tan^2\theta - 6\tan\theta + 9$

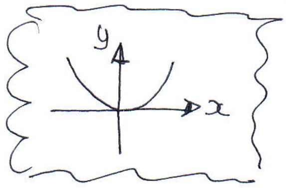
$\Rightarrow (\tan\theta - 3)^2 = 0$

$\tan\theta = 3$

b) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{7+3}{1-3\times7} = \dfrac{10}{-20} = -\dfrac{1}{2}$

AS REQUIRED

8. a) $f(x) = x^2$



$\therefore$ RANGE $f(x) \geqslant 0$

b) $f(g(x)) = \frac{4}{9}$

$f\left(\frac{1}{x+2}\right) = \frac{4}{9}$

$\frac{1}{(x+2)^2} = \frac{4}{9}$

$(x+2)^2 = \frac{9}{4}$

$x+2 = \begin{cases} 3/2 \\ -\frac{3}{2} \end{cases}$

$x = \begin{cases} -\frac{1}{2} \\ -\frac{7}{2} \end{cases}$

OR

$4(x+2)^2 = 9$

$4(x^2+4x+4) = 9$

$4x^2+16x+16 = 9$

$4x^2+16x+7 = 0$

$(2x-7)(2x-1) = 0$

$x = \begin{cases} -1/2 \\ -\frac{7}{2} \end{cases}$

c) LET $y = \frac{1}{x+2}$

$\Rightarrow yx + 2y = 1$

$\Rightarrow yx = 1-2y$

$\Rightarrow x = \frac{1-2y}{y}$

$\therefore f^{-1}(x) = \frac{1-2x}{x}$

$\Rightarrow x+2 = \frac{1}{y}$

$\Rightarrow x = \frac{1}{y} - 2$

$\therefore f^{-1}(x) = \frac{1}{x} - 2$

9. $4-4\cos 2\theta = \operatorname{cosec}\theta$

$\cos 2\theta \equiv 1-2\sin^2\theta$

$4-4(1-2\sin^2\theta) = \frac{1}{\sin\theta}$

$\cancel{4-4} + 8\sin^2\theta = \frac{1}{\sin\theta}$

$\Rightarrow 8\sin^2\theta = \frac{1}{\sin\theta}$

$\Rightarrow 8\sin^3\theta = 1$

$\Rightarrow \sin^3\theta = \frac{1}{8}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\theta = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\theta = \frac{\pi}{6} \pm 2n\pi$

$\theta = \frac{5\pi}{6} \pm 2n\pi$ $\quad n = 0, 1, 2, 3, \ldots$

$\therefore \theta = \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}$ //