IYGB GCE

Core Mathematics C3

Advanced

Practice Paper E

Difficulty Rating: 2.8600/1.2739

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Determine, in exact form where appropriate, the solution of each of the following equations.

a) $e^{2x} = 9$ **(2)**

b) $\ln(4 - y) = 2$ **(2)**

c) $\ln t + \ln 3 = \ln 12$ **(2)**

Question 2

$$e^{3x} = x + 20$$

a) Show that the above equation has a root α between 1 and 2. **(2)**

The recurrence relation

$$x_{n+1} = \tfrac{1}{3}\ln(x_n + 20)$$

starting with $x_0 = 1.5$ is to be used to find α.

b) Find to 4 decimal places, the value of x_1, x_2 and x_3. **(3)**

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha = 1.0151$, correct to 4 decimal places. **(3)**

Question 3

$$f(x) \equiv \sin x + \sqrt{3}\cos x, \ 0 \le x < 2\pi .$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$. **(4)**

b) State the minimum and maximum value of …

i. … $f(x)$. **(2)**

ii. … $\left[f(x)\right]^2$. **(2)**

iii. … $\dfrac{1}{5+f(x)}$. **(2)**

Question 4

A curve has equation

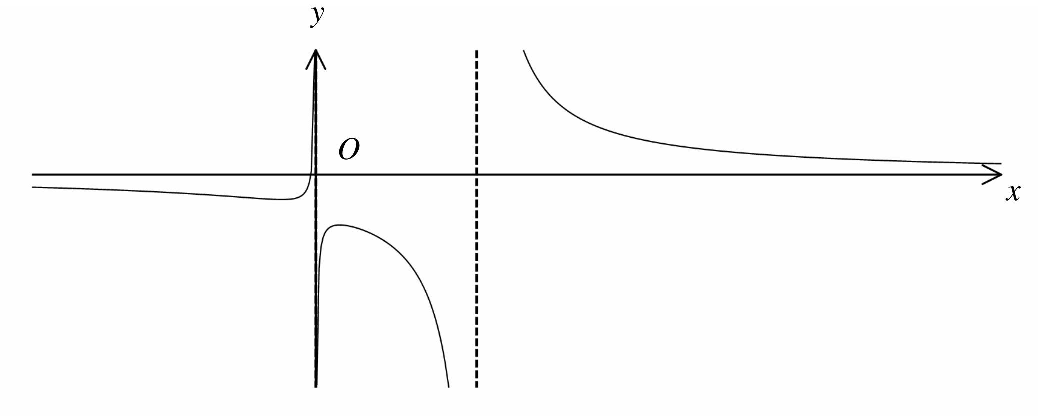
$$y = \frac{2x^2+3x}{2x^2-x-6} - \frac{6}{x^2-x-2}, \ x \in \mathbb{R}, \ 0<x<2 .$$

a) Show clearly that

$$y = \frac{x+3}{x+1}, \ x \in \mathbb{R}, \ 0<x<2 .$$ **(5)**

b) Show further that the equation of the normal to the curve at the point where $x=1$ passes through the origin. **(8)**

Question 5



The figure above shows the graph of the curve with equation $y = f(x)$.

The graph of $y = f(x) \ldots$

… has as asymptotes the lines with equations $y = 0$, $x = 0$ and $x = 10$.

… crosses the x axis at the point $A\left(-\frac{1}{2}, 0\right)$.

… has local minimum and local maximum at $B(-3,-1)$ and $C(2,-4)$, respectively.

Sketch on separate diagrams the graph of …

a) … $y = |f(x)|$ **(4)**

b) … $y = f(|x|)$ **(3)**

Each of the two sketches must clearly indicate the coordinates of the new position of A, B and C, and the equations of any asymptotes

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Question 6

A curve has equation

$$y(y-1)=5x-3.$$

Find the gradient at each of the points on the curve where $x=3$. **(8)**

Question 7

The acute angles α and β satisfy the relationships

$$7\cot^2\alpha+6\cot\alpha=1 \quad \text{and} \quad 6\tan\beta=8+\sec^2\beta.$$

a) Determine the value of $\tan\alpha$ and the value of $\tan\beta$. **(6)**

b) Show clearly that

$$\tan(\alpha+\beta)=-\tfrac{1}{2}.$$ **(2)**

Question 8

The functions $f(x)$ and $g(x)$ are given by

$$f(x) = x^2, \;\; x \in \mathbb{R}.$$

$$g(x) = \frac{1}{x+2}, \quad x \in \mathbb{R}, x \neq -2.$$

a) State the range of $f(x)$. **(1)**

b) Solve the equation

$$fg(x) = \frac{4}{9}.$$ **(5)**

c) Find, in its simplest form, an expression for $g^{-1}(x)$. **(3)**

Question 9

Solve the trigonometric equation

$$4 - 4\cos 2\theta = \operatorname{cosec}\theta, \;\; 0 \leq \theta < 2\pi,$$

giving the answers in terms of π. **(6)**
