IYGB GCE

Core Mathematics C3

Advanced

Practice Paper D

Difficulty Rating: 3.1733/1.4151

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Express

$$\frac{3x-4}{x^2-5x-6}-\frac{2}{x-6},$$

as a single fraction in its simplest form. **(4)**

Question 2

Differentiate each of the following expressions with respect to x.

a) $y=\dfrac{1}{\sqrt{1-2x}}$. **(2)**

b) $y=\mathrm{e}^{3x}\left(\sin x+\cos x\right)$. **(3)**

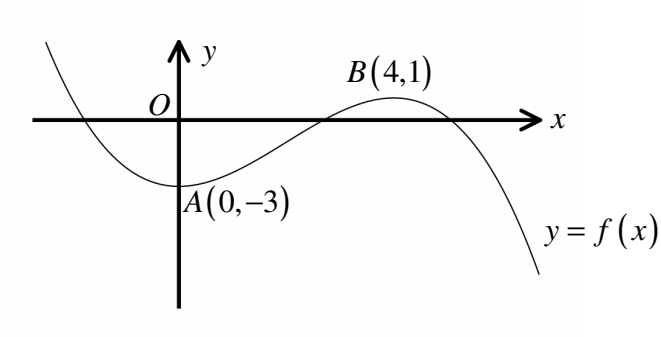
c) $y=\dfrac{\ln x}{x^2}$. **(3)**

Question 3

Prove the validity of the following trigonometric identity

$$\frac{1+\cot^2\theta}{2\cot\theta}\equiv \operatorname{cosec} 2\theta .$$ **(4)**

Question 4



The figure above shows part of the graph of the curve with equation $y = f(x)$.

The graph has stationary points at $A(0,-3)$ and $B(4,1)$.

Sketch on separate diagrams the graph of …

a) … $y = |f(x)|$. **(2)**

b) … $y = f(|x|)$. **(3)**

c) … $y = f(2-x)$. **(2)**

Indicate the new coordinates of the points A and B in these graphs.

Question 5

The curves C_1 and C_2 have respective equations

$$y = 9 - x^2,\ x \in \mathbb{R} \quad \text{and} \quad y = \mathrm{e}^x,\ x \in \mathbb{R}.$$

a) Sketch in the same diagram the graph of C_1 and the graph of C_2.

The sketch must include the coordinates of the points where each of the curves meet the coordinate axes. **(4)**

b) By considering the graphs sketched in part (a), show clearly that the equation

$$\left(9 - x^2\right)\mathrm{e}^{-x} = 1$$

has exactly one positive root and one negative root. **(2)**

To find the negative root of the equation the following iterative formula is used

$$x_{n+1} = -\sqrt{9 - \mathrm{e}^{x_n}},\ x_1 = -3.$$

c) Find, to 5 decimal places, the value of x_2, x_3 and x_4. **(2)**

To find the positive root of the equation the following iterative formula is used

$$x_{n+1} = \sqrt{9 - \mathrm{e}^{x_n}},\ x_1 = 2.$$

d) Explain, clearly but briefly, why do these iterations fail. **(1)**

Question 6

A car tyre develops a puncture.

The tyre pressure P, measured in suitable units known as p.s.i., t minutes after the tyre got punctured is given by the expression

$$P = 8 + 32\mathrm{e}^{-kt}, \ t \geq 0,$$

where k is a positive constant.

a) State the tyre pressure when the tyre got punctured. **(1)**

The tyre pressure halves 2 minutes after the puncture occurred.

b) Show that $k = 0.4904$, correct to 4 significant figures. **(4)**

c) Calculate the time it takes for the tyre pressure to drop to 12 p.s.i. **(3)**

d) Find the rate at which the pressure of the tyre is changing one minute after the puncture occurred. **(3)**

Question 7

The functions $f(x)$ and $g(x)$ are given by

$$f(x) = \frac{2x+3}{2x-3}, \ x \in \mathbb{R}, \ x \neq \frac{3}{2}.$$

$$g(x) = x^2 + 2, \ x \in \mathbb{R}.$$

a) State the range of $g(x)$. **(1)**

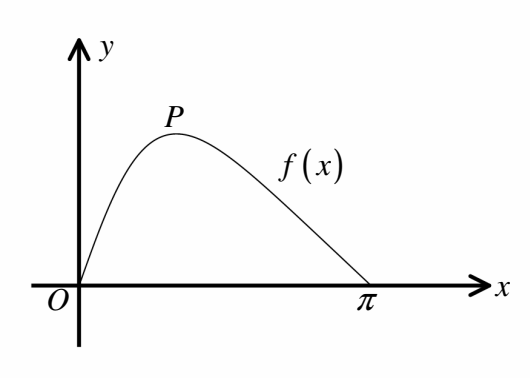
b) Find an expression, as a simplified algebraic fraction, for $fg(x)$. **(2)**

c) Determine an expression, as a simplified algebraic fraction, for $f^{-1}(x)$. **(4)**

d) Solve the equation

$$f^{-1}(x) = f(x). \quad \textbf{(4)}$$

Question 8



The figure above shows the graph of the curve with equation

$$f(x)=\frac{\sin x}{2-\cos x},\ 0\le x\le \pi .$$

The curve has a stationary point at P.

Determine the exact coordinates of P. **(8)**

Question 9

$$f(x)=\frac{6}{2\cos x+2\sin x} \text{ for } 0<x<\pi,\ x\neq \beta .$$

a) Express $2\cos x+2\sin x$ in the form $R\cos(x-\alpha)$, $R>0$, $0<a<\frac{\pi}{2}$. **(4)**

The curve with equation $y=f(x)$ has a vertical asymptote at $x=\beta$.

b) Determine the value of β. **(2)**

c) Solve the equation

$$f(3x)-\sqrt{6}=0,\ 0<x<\pi ,$$

giving the answers in terms of π. **(7)**
