C3, IYGB, PAPER C

1. $y = (x^2+1)^{\frac{1}{2}} = \sqrt{x^2+1}$

- $\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x = x(x^2+1)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2+1}}$
- WHEN $x=1$ $\quad y = \sqrt{1^2+1} = \sqrt{2} \quad \therefore (1, \sqrt{2})$
- $\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{\sqrt{1^2+1}} = \frac{1}{\sqrt{2}}$
- NORMAL GRADIENT IS $-\sqrt{2}$
- $y - y_0 = m(x - x_0) \implies y - \sqrt{2} = -\sqrt{2}(x-1)$

$$y - \sqrt{2} = -\sqrt{2}x + \sqrt{2}$$
$$y = 2\sqrt{2} - \sqrt{2}x$$
$$y = \sqrt{2}(2-x)$$

AS REQUIRED

2. a) $e^{-x} + \sqrt{x} = 2$

$$e^{-x} + \sqrt{x} - 2 = 0$$
$$f(x) = e^{-x} + \sqrt{x} - 2$$

$$\begin{cases} f(3) = e^{-3} + \sqrt{3} - 2 = -0.218\ldots \\ f(4) = e^{-4} + \sqrt{4} - 2 = 0.018\ldots \end{cases}$$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN BETWEEN 3 & 4, THERE MUST BE A ROOT BETWEEN 3 & 4

b) $x_{n+1} = (2 - e^{-x_n})^2$

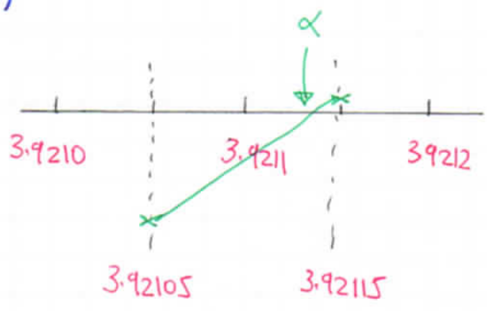
$x_0 = 4$

$x_1 = 3.92707\ldots \simeq 3.927$

$x_2 = 3.92158\ldots \simeq 3.922$

$x_3 = 3.92115\ldots \simeq 3.921$

c)



$f(3.92105) = -0.000016$

$f(3.92115) = 0.0000077$

THE CHANGE OF SIGN IMPLIES THAT

$$3.92105 < \alpha < 3.92115$$

$$\therefore \alpha = 3.9211$$

4 d.p.

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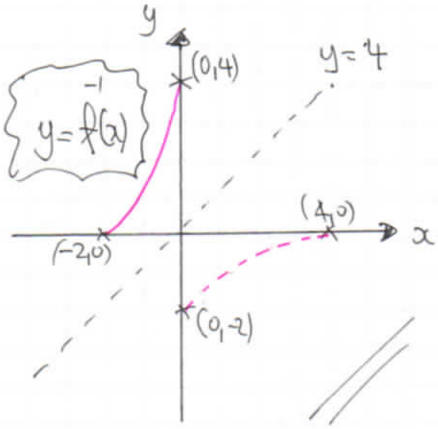
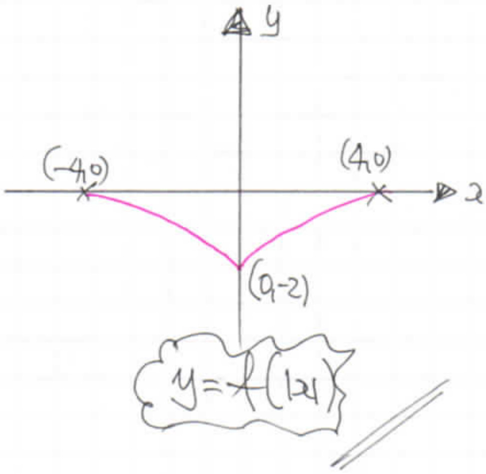
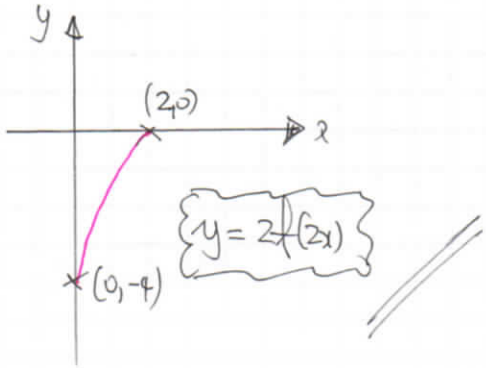
3. a)



b)



c)



4. a) $2\sqrt{2}\cos x + 2\sqrt{2}\sin x \equiv R\sin(x+\alpha)$

$$\equiv R\sin x\cos\alpha + R\cos x\sin\alpha$$

$$\equiv (R\cos\alpha)\sin x + (R\sin\alpha)\cos x$$

$$\left\{\begin{array}{l} R\cos\alpha = 2\sqrt{2} \\ R\sin\alpha = 2\sqrt{2} \end{array}\right\}$$

- SQUARE & ADD

$R=\sqrt{(2\sqrt{2})^2+(2\sqrt{2})^2}$

$R=\sqrt{8+8}$

$R=4$

- DIVIDE EQUATIONS

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \frac{2\sqrt{2}}{2\sqrt{2}}$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$\therefore y = 4\sin\left(x+\frac{\pi}{4}\right)$

b) $y=2$

$\Rightarrow 4\sin\left(x+\frac{\pi}{4}\right)=2$

$\Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{2}$

$\arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$$\begin{cases} x+\frac{\pi}{4} = \frac{\pi}{6} \pm 2n\pi \\ x+\frac{\pi}{4} = \frac{5\pi}{6} \pm 2n\pi \end{cases} \quad n=0,1,2,3\ldots$$

$$\begin{cases} x = -\frac{\pi}{12} \pm 2n\pi \\ x = \frac{7\pi}{12} \pm 2n\pi \end{cases}$$

$$\therefore x_1 = \frac{23\pi}{12}$$
$$x_2 = \frac{7\pi}{12}$$

c) $y_{MAX} = 4$

d) FOR $y_{MAX} \Longrightarrow \sin\left(x+\frac{\pi}{4}\right)=1$

$$x+\frac{\pi}{4} = \frac{\pi}{2}$$
$$x = \frac{\pi}{4}$$

5. a) $y = xe^{-\frac{1}{2}x^2}$

$$\frac{dy}{dx} = 1\times e^{-\frac{1}{2}x^2} + xe^{-\frac{1}{2}x^2}\times(-x)$$

$$\frac{dy}{dx} = e^{-\frac{1}{2}x^2} - x^2e^{-\frac{1}{2}x^2}$$

b) SOLVE FOR ZERO

$$e^{-\frac{1}{2}x^2} - x^2e^{-\frac{1}{2}x^2} = 0$$
$$e^{-\frac{1}{2}x^2}(1-x^2) = 0$$
$$1-x^2 = 0 \qquad e^{-\frac{1}{2}x^2} \neq 0$$
$$x^2 = 1$$
$$x = \pm 1$$

$$x = \begin{cases} 1 \\ -1 \end{cases} \qquad y = \begin{cases} e^{-\frac{1}{2}} \\ -e^{-\frac{1}{2}} \end{cases} \qquad \therefore \begin{matrix} (1, e^{-\frac{1}{2}}) \\ (-1, -e^{-\frac{1}{2}}) \end{matrix}$$

6. a) $\frac{1}{x+2}+\frac{2x+11}{2x^2+x-6} = \frac{1}{x+2}+\frac{2x+11}{(2x-3)(x+2)}$

$= \frac{(2x-3)+(2x+11)}{(2x-3)(x+2)}$

$= \frac{4x+8}{(2x-3)(x+2)} = \frac{4\cancel{(x+2)}}{(2x-3)\cancel{(x+2)}}$

$= \frac{4}{2x-3}$ // AS REQUIRED

b) LET $y=\frac{4}{2x-3}$

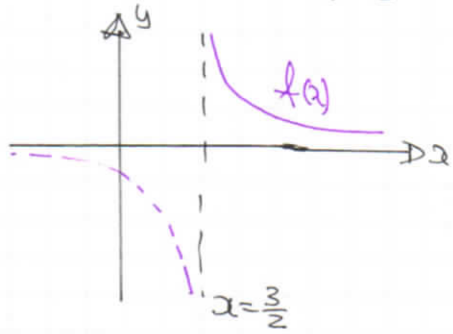
$2xy-3y=4$

$2xy=4+3y$

$x=\frac{4+3y}{2y}$

$\therefore f^{-1}(x)=\frac{3x+4}{2x}$ //

c) THE GRAPH OF $y=\frac{4}{2x-3}$



	f	f^{-1}
D	$x>\frac{3}{2}$	$x>0$
R	$f(x)>0$	$f^{-1}(x)>\frac{3}{2}$

$\therefore$ DOMAIN $x>0$ //

d) $f(g(x))=-2$

$\Rightarrow f(\ln(x-1))=-2$

$\Rightarrow \frac{4}{2\ln(x-1)-3}=-2$

$\Rightarrow 4=-4\ln(x-1)+6$

$\Rightarrow 4\ln(x-1)=2$

$\Rightarrow \ln(x-1)=\frac{1}{2}$

$\Rightarrow x-1=e^{\frac{1}{2}}$

$\Rightarrow x=1+e^{\frac{1}{2}}$

OR

$x=1+\sqrt{e}$ // AS REQUIRED

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7. $\left.\begin{array}{l} e^{2x+4} = ey \\ \ln y = 4x+6 \end{array}\right\} \Longrightarrow y = \dfrac{e^{2x+4}}{e}$

$\Longrightarrow y = e^{2x+3}$

$\Longrightarrow \ln y = 2x+3$

$\therefore \left.\begin{array}{l} \ln y = 4x+6 \\ \ln y = 2x+3 \end{array}\right\} \Longrightarrow 4x+6 = 2x+3$

$2x = -3$

$x = -\frac{3}{2}$ //

$\ln y = 2\times\left(-\frac{3}{2}\right)+3$

$\ln y = 0$

$y = e^0$

$y = 1$ //

8. a) $\text{LHS} = \dfrac{1+\cot^2 x}{\cot x \operatorname{cosec} x} = \dfrac{\operatorname{cosec}^2 x}{\cot x \operatorname{cosec} x} = \dfrac{\operatorname{cosec} x}{\cot x}$

$= \dfrac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} = \dfrac{\cancel{\sin x}}{\cancel{\sin x}\cos x} = \sec x = \text{RHS}$

ALTERNATIVE

$\text{LHS} = \dfrac{1+\cot^2 x}{\cot x \operatorname{cosec} x} = \dfrac{1+\frac{\cos^2 x}{\sin^2 x}}{\frac{\cos x}{\sin x}\times\frac{1}{\sin x}} = \dfrac{1+\frac{\cos^2 x}{\sin^2 x}}{\frac{\cos x}{\sin^2 x}}$

= MULTIPLY TOP/BOTTOM BY $\sin^2 x$ ...

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x} = \dfrac{1}{\cos x} = \sec x = \text{RHS}$

b) $\frac{4(1+\cot^2 x)}{\cot x \operatorname{cosec} x} = \tan^2 x + 5$

$4\sec x = \tan^2 x + 5$

$4\sec x = (\sec^2 x - 1) + 5$

$0 = \sec^2 x - 4\sec x + 4$

$(\sec x - 2)(\sec x - 2) = 0$

$1 + \tan^2 x \equiv \sec^2 x$
$\tan^2 x \equiv \sec^2 x - 1$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$\begin{cases} x = \frac{\pi}{3} \pm 2n\pi \\ x = \frac{5\pi}{3} \pm 2n\pi \end{cases}$ $n = 0, 1, 2, 3 \ldots$

$\therefore x_1 = \frac{\pi}{3}$
$x_2 = \frac{5\pi}{3}$

9. $\pi - 3\arccos(x+1) = 0$

$3\arccos(x+1) = \pi$

$\arccos(x+1) = \frac{\pi}{3}$

$\cos[\arccos(x+1)] = \cos\left(\frac{\pi}{3}\right)$

$x+1 = \frac{1}{2}$

$x = -\frac{1}{2}$