1. BY LONG DIVISION

$$\begin{array}{r|l} & 2x \\ x^2+1 & 2x^3+0x^2+x-2 \\ & -2x^3 \qquad -2x \\ \hline & \qquad -x-2 \end{array}$$

$$\therefore \frac{2x^3+x-2}{x^2+1} = 2x + \frac{-x-2}{x^2+1} = 2x - \frac{x+2}{x^2+1}$$

$A=2,\ B=0,\ C=-1,\ D=-2$

2. (a) LET $y = \frac{x}{x+3}$

$\Rightarrow yx+3y = x$

$\Rightarrow 3y = x - yx$

$\Rightarrow 3y = x(1-y)$

$\Rightarrow x = \frac{3y}{1-y}$

$\therefore f^{-1}(x) = \frac{3x}{1-x}$

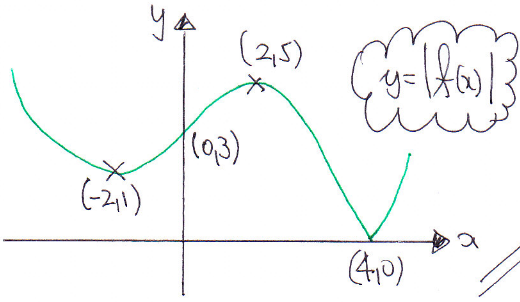
(b) $f(g(\tfrac{2}{3})) = f\left(\frac{2}{\frac{2}{3}}\right)$

$= f(3)$

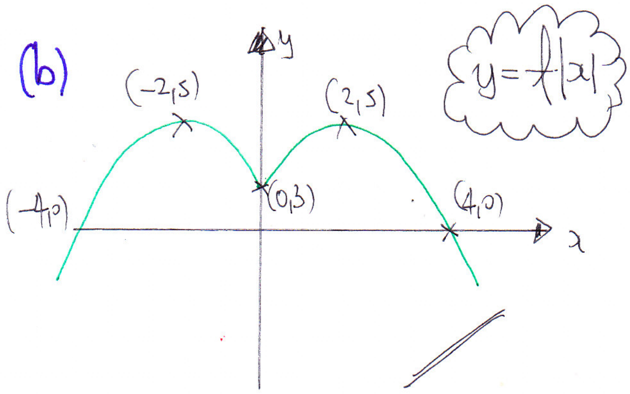
$= \frac{3}{3+3}$

$= \frac{1}{2}$

3. (a)

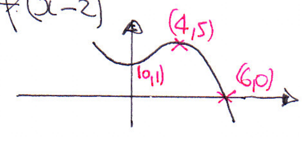


(b)

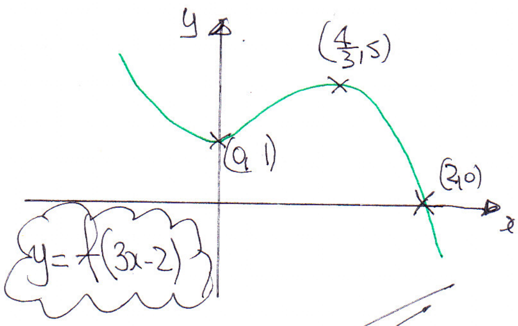


(c) • THIS IS TRANSLATION, RIGHT, BY 2 FOLLOWED BY HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{3}$

• THUS $f(x-2)$



THUS



4. (a) $\sqrt{2}\cos\theta - \sqrt{6}\sin\theta \equiv R\cos(\theta+\alpha)$
$\equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$
$\equiv (R\cos\alpha)\cos\theta - (R\sin\alpha)\sin\theta$

$R\cos\alpha = \sqrt{2}$
$R\sin\alpha = \sqrt{6}$

• SQUARE & ADD
$R = \sqrt{(\sqrt{2})^2 + (\sqrt{6})^2}$
$R = \sqrt{2+6}$
$R = \sqrt{8}$

• DIVIDE EQUATIONS
$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{6}}{\sqrt{2}}$
$\tan\alpha = \sqrt{3}$
$\alpha = \frac{\pi}{3} = 60°$

$\therefore\ y = \sqrt{8}\cos(\theta + 60°)$

(b) $y = 2$
$\Rightarrow \sqrt{8}\cos(\theta+60°) = 2$
$\Rightarrow \cos(\theta+60°) = \frac{2}{\sqrt{8}}$
$\arccos\left(\frac{2}{\sqrt{8}}\right) = 45°$

$\begin{cases} \theta + 60° = 45 \pm 360n \\ \theta + 60° = 315 \pm 360n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\begin{cases} \theta = -15 \pm 360n \\ \theta = 255 \pm 360n \end{cases}$

BUT $0 < \theta < 360$

$\therefore\ \theta = 255°, 345°$

(c) (I) LOOKING AT
$y = \sqrt{8}\cos(\theta+60°)$
$y^2 = 8\cos^2(\theta+60°)$
MIN OCCURS WHEN $\cos(\theta+60) = 0$
$\therefore$ MINIMUM IS 0

(II) $\frac{1}{y^2} = \frac{1}{8\cos^2(\theta+60)}$

MIN OCCURS WHEN THE DENOMINATOR IS LARGEST

$\therefore$ MINIMUM VALUE IS $\frac{1}{8}$

(P.T.O)

5. (a) $N = Ae^{-kt}$

- WHEN $t=0$, $N=12000$

$12000 = Ae^{0}$

$12000 = A$

$\therefore N = 12000e^{-kt}$

- WHEN $t=24$, $N=2000$

$2000 = 12000e^{-k\times 24}$

$\frac{1}{6} = e^{-24k}$

$e^{24k} = 6$

$24k = \ln 6$

$k = \frac{1}{24}\ln 6 \simeq 0.07466$

AS REQUIRED

(b) $N = 12000e^{-0.07466t}$

$1000 = 12000e^{-0.07466t}$

$\Rightarrow \frac{1}{12} = e^{-0.07466t}$

$\Rightarrow 12 = e^{0.07466t}$

$\Rightarrow 0.07466t = \ln 12$

$\Rightarrow t = \frac{\ln 12}{0.07466}$

$\Rightarrow t \simeq 33.2829...$

$\Rightarrow t \simeq 33.3$

6. $y = \frac{x}{y^2+\ln y}$

$\Rightarrow y^3 + y\ln y = x$

$\Rightarrow x = y^3 + y\ln y$

$\Rightarrow \frac{dx}{dy} = 3y^2 + 1\times\ln y + y\times\frac{1}{y}$

$\Rightarrow \frac{dx}{dy} = 3y^2 + \ln y + 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3y^2+\ln y+1}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{3+0+1} = \frac{1}{4}$

NORMAL GRADIENT IS -4

$y - y_0 = m(x-x_0)$

$y - 1 = -4(x-1)$

$y - 1 = -4x + 4$

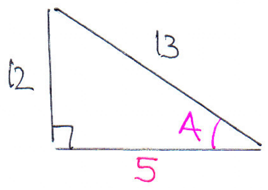
$y + 4x = 5$

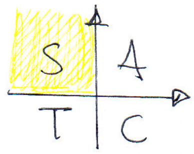
or

$4x + y = 5$

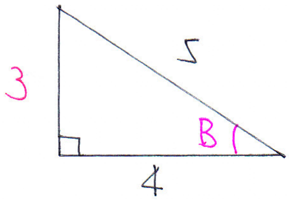
AS REQUIRED

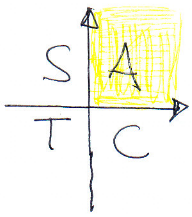
7.





$\cos A = -\frac{5}{13}$





$\sin B = \frac{3}{5}$

$$\sin(A+B) = \sin A\cos B + \cos A \sin B$$
$$= \frac{12}{13} \times \frac{4}{5} + \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right)$$
$$= \frac{48}{65} - \frac{15}{65}$$
$$= \frac{33}{65}$$

AS REQUIRED

8. (a) $y = e^{2x} - 4e^{x} - 16x$

$\frac{dy}{dx} = 2e^{2x} - 4e^{x} - 16$

SOLVE FOR ZERO

$2e^{2x} - 4e^{x} - 16 = 0$

$e^{2x} - 2e^{x} - 8 = 0$

(b) $e^{2x} - 2e^{x} - 8 = 0$

$(e^{x} + 2)(e^{x} - 4) = 0$

$e^{x} = \cancel{-2}$ or 4

$x = \ln 4$

$x = 2\ln 2$

NOW $y = e^{2(2\ln 2)} - 4e^{2\ln 2} - 16(2\ln 2)$

$y = \cancel{16} - \cancel{4\times 4} - 32\ln 2$

$y = -32\ln 2$

$\therefore (2\ln 2, -32\ln 2)$

9. a) $y = \dfrac{3x+1}{x^3-x^2+5}$

$$\frac{dy}{dx} = \frac{(x^3-x^2+5)(3)-(3x+1)(3x^2-2x)}{(x^3-x^2+5)^2} = \frac{3x^3-3x^2+15-(9x^3-6x^2+3x^2-2x)}{(x^3-x^2+5)^2}$$

$$= \frac{3x^3 \cancel{-3x^2} - 15 - 9x^3 \cancel{+3x^2} + 2x}{(x^3-x^2+5)^2} = \frac{-6x^3+2x+15}{(x^3-x^2+5)^2}$$

SOLVE FOR ZERO

$$\frac{-6x^3+2x+15}{(x^3-x^2+5)^2} = 0 \implies -6x^3+2x+15=0$$

$$\implies 6x^3-2x-15=0$$

$$\implies 6x^3 = 2x+15$$

$$\implies x^3 = \tfrac{1}{3}x + \tfrac{5}{2}$$

$$\implies x = \sqrt{\tfrac{1}{3}x + \tfrac{5}{2}}$$

AS REQUIRED

b) $x_{n+1} = \sqrt[3]{\tfrac{1}{3}x_n + \tfrac{5}{2}}$

START AT $x_1 = 1.4$

$x_2 = 1.43689\ldots$

$x_3 = 1.43887\ldots$

$x_4 = 1.43898\ldots$

$x_5 = 1.43898\ldots$

NOW $y = \dfrac{3(1.43898)+1}{(1.43898)^3-(1.43898)^2+5}$

$y = 0.899806\ldots$

$\therefore M(1.439, 0.900)$

3 d.p