C2, IYGB, PAPER Y

1. a) $\boxed{C = \frac{192}{v} + \frac{v^2}{144}}$

$\Rightarrow C = 192v^{-1} + \frac{1}{144}v^2$

$\Rightarrow \frac{dC}{dv} = -192v^{-2} + \frac{1}{72}v$

- SOLVE FOR ZERO

$\Rightarrow -\frac{192}{v^2} + \frac{1}{72}v = 0$

$\Rightarrow \frac{1}{72}v = \frac{192}{v^2}$

$\Rightarrow v^3 = 13824$

$\Rightarrow v = 24$

b) $\frac{d^2C}{dv^2} = 384v^{-3} + \frac{1}{72}$

$\left.\frac{d^2C}{dv^2}\right|_{v=24} = \frac{384}{24^3} + \frac{1}{72} = \frac{1}{24} > 0$

INDEED IT MINIMIZES COST

c) WHEN $v = 24$

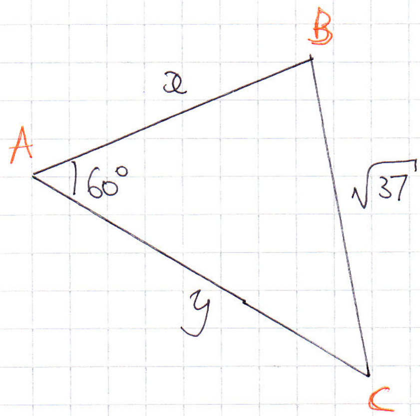
$C = \frac{192}{24} + \frac{24^2}{144}$

$C = 12 \leftarrow$ pence per mile

$\therefore$ REQUIRED COST IS

$600 \times 0.12 = £72$

2.



- AREA $= 7\sqrt{3}$

$\frac{1}{2}xy\sin 60 = 7\sqrt{3}$

$\frac{1}{4}\cancel{\sqrt{3}}\,xy = 7\cancel{\sqrt{3}}$

$\boxed{xy = 28}$

- BY THE COSINE RULE

$(\sqrt{37})^2 = x^2 + y^2 - 2xy\cos 60$

$\boxed{37 = x^2 + y^2 - xy}$ $\left(\cos 60° = \frac{1}{2}\right)$

THUS $\left.\begin{array}{l} xy = 28 \\ x^2 + y^2 - xy = 37 \end{array}\right)$

$x^2 + y^2 - 28 = 37$

$\boxed{x^2 + y^2 = 65}$

BUT $y = \frac{28}{x}$

$x^2 + \left(\frac{28}{x}\right)^2 = 65$

$x^2 + \frac{784}{x^2} = 65$

$x^4 + 784 = 65x^2$

$x^4 - 65x^2 + 784 = 0$

QUADRATIC FORMULA OR FACTORIZATION

$(x^2 - 49)(x^2 - 16) = 0$

C2, IYGB, PAPER Y

$$x^2 = \begin{cases} 49 \\ 16 \end{cases} \Rightarrow x = \begin{cases} 7 \\ 4 \end{cases} \quad (x>0) \qquad y = \begin{cases} 4 \\ 7 \end{cases}$$

$\therefore \ x = 4 \ \& \ y = 7$ (EITHER ORDER)

3. $4\tan\psi\sin\psi\cos\psi + 4\tan\psi\cos\psi + 1 = 0$

$$\Rightarrow 4\left(\frac{\sin\psi}{\cancel{\cos\psi}}\right)\sin\psi\cancel{\cos\psi} + 4\left(\frac{\sin\psi}{\cancel{\cos\psi}}\right)\cancel{\cos\psi} + 1 = 0$$

$$\Rightarrow 4\sin^2\psi + 4\sin\psi + 1 = 0$$

$$\Rightarrow (2\sin\psi + 1)^2 = 0$$

$$\Rightarrow \sin\psi = -\frac{1}{2}$$

$$\arcsin\left(-\frac{1}{2}\right) = -30^\circ$$

$$\Rightarrow \begin{cases} \psi = -30 \pm 360n \\ \psi = 210 \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots \qquad \psi_1 = 330^\circ \quad \psi_2 = 210^\circ$$

4. $C = \frac{36}{T} + \frac{2T^2}{3}$

$$\boxed{C = 36T^{-1} + \frac{2}{3}T^2}$$

$$\Rightarrow \frac{dC}{dT} = -36T^{-2} + \frac{4}{3}T$$

INCREASING ...

$$\Rightarrow -\frac{36}{T^2} + \frac{4}{3}T > 0$$

$$\Rightarrow \frac{4}{3}T > \frac{36}{T^2}$$

$$\Rightarrow 4T^3 > 108$$

$$\Rightarrow T^3 > 27 \qquad \therefore T > 3$$

C2, IYGB, PAPER Y

5. $f(x) = \left(4x + \frac{1}{kx}\right)^7 = \dots + \binom{7}{5}(4x)^5\left(\frac{1}{kx}\right)^2 + \dots$

OR $\binom{7}{2}$

THUS $21 \times 4^5 \times \frac{1}{k^2} = 21$

$21504 = 21k^2$

$1024 = k^2$

$k = 32$ $\quad k > 0$

6. a) $x^2 + y^2 - 4x = 12$

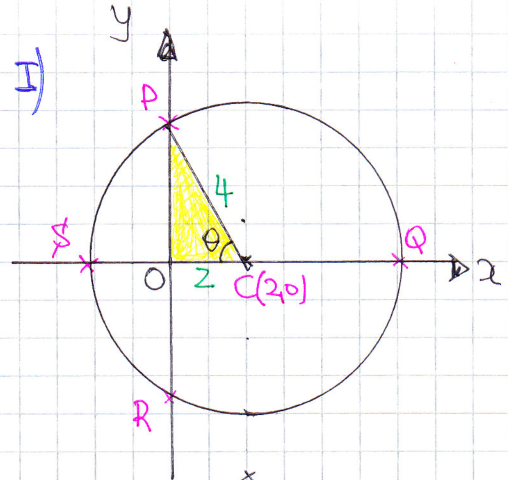
$x^2 - 4x + y^2 = 12$

$(x-2)^2 - 4 + y^2 = 12$

$(x-2)^2 + y^2 = 16$

CENTRE AT $(2,0)$ RADIUS $= \sqrt{16} = 4$

b) I)



LOOKING AT $\overset{\triangle}{OPC}$

$\cos\theta = \frac{2}{4}$

$\theta = \frac{\pi}{3}$ $(60°)$

$\therefore \ P\hat{C}Q = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

II) AREA OF $\overset{\triangle}{OPC} = \frac{1}{2} \times 2 \times 4 \times \sin\frac{\pi}{3}$

$= 2\sqrt{3}$

AREA OF SECTOR $PCQ = \frac{1}{2}r^2\theta^c$

$= \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$

$= \frac{16}{3}\pi$

$\therefore$ REQUIRED AREA

$= 2\sqrt{3} + \frac{16}{3}\pi$

$= \frac{2}{3}\left[3\sqrt{3} + 8\pi\right]$

AS REQUIRED

7.

$$\frac{\log_2 128 - \log_2 8}{\log_2 x} = \log_2 x$$

$$\Rightarrow \frac{\log_2\left(\frac{128}{8}\right)}{\log_2 x} = \log_2 x$$

$$\Rightarrow \frac{\log_2 16}{\log_2 x} = \log_2 x$$

$$\Rightarrow \frac{\log_2 2^4}{\log_2 x} = \log_2 x$$

$$\Rightarrow \frac{4\cancel{\log_2 2}}{\log_2 x} = \log_2 x$$

$$\Rightarrow 4 = (\log_2 x)^2$$

$$\Rightarrow \log_2 x = \begin{cases} 2 \\ -2 \end{cases}$$

$$\Rightarrow x = \begin{cases} 4 \\ \frac{1}{4} \end{cases}$$

NOTE

$\log_2 x = 2$

$\log_2 x = 2\log_2 2$

$\log_2 x = \log_2 2^2$

$\log_2 x = \log_2 4$

$x = 4$

AND SIMILARLY THE OTHER

8.

$$\text{AREA} = \int_k^{2k} \frac{x^2+6}{x^4}\,dx = \int_k^{2k} \frac{x^2}{x^4} + \frac{6}{x^4}\,dx = \int_k^{2k} x^{-2} + 6x^{-4}\,dx$$

$$= \left[-x^{-1} - 2x^{-3}\right]_k^{2k} = \left[-\frac{1}{x} - \frac{2}{x^3}\right]_k^{2k}$$

$$= \left[\frac{1}{x} + \frac{2}{x^3}\right]_{2k}^{k} = \left(\frac{1}{k} + \frac{2}{k^3}\right) - \left(\frac{1}{2k} + \frac{2}{8k^3}\right)$$

$$= \frac{1}{k} + \frac{2}{k^3} - \frac{1}{2k} - \frac{1}{4k^3} = \frac{1}{2k} + \frac{7}{4k^3}$$

NOW $\frac{1}{2k} + \frac{7}{4k^3} = \frac{9}{4}$ MULTIPLY BY 4

$\frac{2}{k} + \frac{7}{k^3} = 9$ MULTIPLY BY k^3

$2k^2 + 7 = 9k^3$

$0 = 9k^3 - 2k^2 - 7$

AS REQUIRED

b) $9k^3 - 2k^2 - 7 = 0$

- BY INSPECTION $k=1$ IS A SOLUTION SINCE $9\times1^3 - 2\times1^2 - 7$
$= 9-2-7$
$= 0$

- THUS

$(k-1)(9k^2+4k+7)$
$(k-1)(9k^2+7k+7)$ BY INSPECTION OR LONG DIVISION

$b^2-4ac = 7^2-4\times9\times7 = 49-252 = -203 < 0$

IE DOES NOT REDUCE OVER THE REALS

ONLY SOLUTION IS $k=1$

9. a)

YEAR	START OF YEAR ...	END OF YEAR ...
1	1250	1250×1.06
2	$1250+1250\times1.06$	$[1250+1250\times1.06]\times1.06$ IE $1250\times1.06 + 1250\times1.06^2$
3	$1250+1250\times1.06+1250\times1.06^2$ $= £3979.50$	$[1250+1250\times1.06+1250\times1.06^2]\times1.06$ I.E $1250\times1.06+1250\times1.06^2+1250\times1.06^3$
⋮		

b) IN OTHER WORDS AT THE START OF YEAR ...

1 $\quad 1250\times1.06^0$

2 $\quad 1250+1250\times1.06^1$

3 $\quad 1250+1250\times1.06^1+1250\times1.06^2$

4 $\quad 1250+1250\times1.06^1+1250\times1.06^2+1250\times1.06^3$

⋮

n $\quad 1250+1250\times1.06^1+1250\times1.06^2+\dots+1250\times1.06^{39}$

C2, IYGB, PAPER Y

$\therefore$ TOTAL $= 1250 + 1250 \times 1.06^1 + 1250 \times 1.06^2 + \dots + 1250 \times 1.06^{39}$

$$= 1250\left[\underbrace{1 + 1.06 + 1.06^2 + 1.06^3 + \dots + 1.06^{39}}_{\text{THIS IS A G.P WITH } a=1,\ r=1.06,\ n=40}\right]$$

$$= 1250 \times \frac{1(1.06^{40}-1)}{1.06-1}$$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$= 193452.457$$

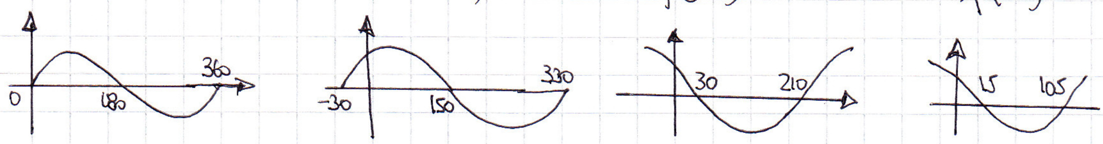
APPROX £193,000

10.

$y = \sin(30-2x)$

$\sin x \longmapsto \sin(x+30) \longmapsto \sin(-x+30) \longmapsto \sin(-2x+30)$

"$f(x+30)$" "$f(-x)$" "$f(2x)$"



THUS