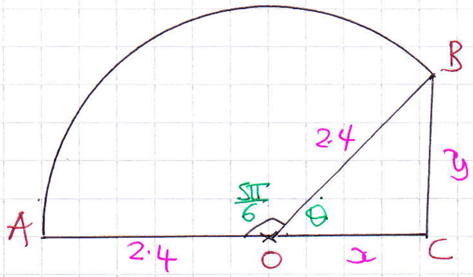
C2, IYGB, PAPER W

1.



- $B\hat{O}C = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$
- LOOKING AT $\overset{\Delta}{OBC}$

$\cos\theta = \frac{x}{2.4}$ $\quad$ $\sin\theta = \frac{y}{2.4}$

$x = 2.4\cos\theta$ $\quad$ $y = 2.4\sin\theta$

$x = 2.4\cos\frac{\pi}{6}$ $\quad$ $y = 2.4\sin\frac{\pi}{6}$

$x = \frac{6}{5}\sqrt{3}$ $\quad$ $y = \frac{6}{5}$

THUS AREA OF $\overset{\Delta}{OBC} = \frac{1}{2}xy = \frac{1}{2} \times \frac{6}{5}\sqrt{3} \times \frac{6}{5} = \frac{18}{25}\sqrt{3}$

AREA OF SECTOR $= \frac{1}{2}r^2\theta^c = \frac{1}{2} \times 2.4^2 \times \frac{5\pi}{6} = \frac{12\pi}{5}$

$\therefore$ TOTAL AREA $= \frac{18}{25}\sqrt{3} + \frac{12}{5}\pi \simeq 8.79$

2.

$y = \frac{x^2+4}{4x} = \frac{x^2}{4x} + \frac{4}{4x} = \frac{x}{4} + \frac{1}{x} = \frac{1}{4}x + x^{-1}$

$\frac{dy}{dx} = \frac{1}{4} - x^{-2}$

INCREASING $\Rightarrow \frac{1}{4} - \frac{1}{x^2} > 0$

$\Rightarrow -\frac{1}{x^2} > \frac{1}{4}$

$\Rightarrow \frac{1}{x^2} < \frac{1}{4}$

$\Rightarrow 4 < x^2$

$\Rightarrow x^2 > 4$

$\Rightarrow x^2 - 4 > 0$

$\Rightarrow (x-2)(x+2) > 0$

$x < -2$ OR $x > 2$

3.

$\boxed{a_n = ar^{n-1}}$

$\left.\begin{array}{l} a + ar = 240 \\ a + ar^2 = 200 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a(1+r) = 240 \\ a(1+r^2) = 200 \end{array}\right\} \Rightarrow$ DIVIDE EQUATIONS

$\Rightarrow \frac{\cancel{a}(1+r^2)}{\cancel{a}(1+r)} = \frac{200}{240}$

$\Rightarrow \frac{1+r^2}{1+r} = \frac{5}{6}$

$\Rightarrow 6r^2+6 = 5r+5$

$\Rightarrow 6r^2-5r+1=0$

$\Rightarrow (3r-1)(2r-1)=0$

$\Rightarrow r = \begin{cases} 1/2 \\ 1/3 \end{cases}$

BUT $\boxed{a = \frac{240}{1+r}}$ $a \begin{cases} \frac{240}{1+\frac{1}{2}} = \frac{240}{3/2} = 160 \\ \frac{240}{1+\frac{1}{3}} = \frac{240}{4/3} = 180 \end{cases}$

$\therefore$ EITHER $S_\infty = \frac{a_1}{1-r_1} = \frac{160}{1-\frac{1}{2}} = 320$

OR $S_\infty = \frac{a_2}{1-r_2} = \frac{180}{1-\frac{1}{3}} = 270$

4. $8\tan^2 x \sin x = \cos x$

$\Rightarrow 8\left(\frac{\sin^2 x}{\cos^2 x}\right)\sin x = \cos x$

$\Rightarrow \frac{8\sin^3 x}{\cos^2 x} = \cos x$

$\Rightarrow 8\sin^3 x = \cos^3 x$

$\Rightarrow \frac{8\sin^3 x}{\cos^3 x} = 1$

$\Rightarrow 8\tan^3 x = 1$

$\Rightarrow \tan^3 x = \frac{1}{8}$

$\Rightarrow \tan x = \frac{1}{2}$

$\arctan\left(\frac{1}{2}\right) = 0.464\ldots$

$x = 0.464 \pm n\pi \quad n = 0, 1, 2, 3.$

$x_1 = 0.46^c$

$x_2 = 3.61^c$

ALTERNATIVE

$8\tan^2 x \sin x = \cos x$

$\frac{8\tan^2 x \sin x}{\cos x} = \frac{\cos x}{\cos x}$

$8\tan^2 x\left(\frac{\sin x}{\cos x}\right) = 1$

$8\tan^3 x = 1$

$\tan^3 x = \frac{1}{8}$

ETC

5. a) $f(x) = x^3 - 2x^2 - x - 6$

$f(3) = 3^3 - 2\times3^2 - 3 - 6$

$f(3) = 27 - 18 - 3 - 6 = 0$

$\therefore (x-3)$ IS A FACTOR OF $f(x)$

b) $f(x) = (x-3)(x^2 + Ax + 2)$

$-3Ax$

$+2x$

BY LONG DIVISION OR INSPECTION

$$\boxed{\begin{aligned} -3Ax + 2x &= -x \\ -3A + 2 &= -1 \\ 3 &= 3A \\ A &= 1 \end{aligned}}$$

$\therefore f(x) = (x-3)(x^2 + x + 2)$

c) $y = 3x^4 - 8x^3 - 6x^2 - 72x + 240$

$\Longrightarrow \frac{dy}{dx} = 12x^3 - 24x^2 - 12x - 72$

- SOLVE FOR ZERO

$12x^3 - 24x^2 - 12x - 72 = 0$

$x^3 - 2x^2 - x - 6 = 0$

$(x-3)(x^2 + x + 2) = 0$

$b^2 - 4ac = 1^2 - 4\times1\times2 = 1 - 8 = -7 < 0$

NO MORE SOLUTIONS EXCEPT $x = 3$

- $y = 3\times3^4 - 8\times3^3 - 6\times3^2 - 72\times3 + 240$

$y = -3$

$\therefore$ ONLY STATIONARY POINT IS AT $(3, -3)$

- $\frac{d^2y}{dx^2} = 36x^2 - 48x - 12$

$\left.\frac{d^2y}{dx^2}\right|_{x=3} = 36\times3^2 - 48\times3 - 12 = 168 > 0$

$\therefore (3, -3)$ IS A MIN

6. $$\left[\frac{9}{2x} - \frac{2x^2}{3}\right]^{13} = \dots + \binom{13}{5}\left(\frac{9}{2x}\right)^5\left(-\frac{2x^2}{3}\right)^8 + \dots$$

(BY INSPECTION)

$$\dots + 1287 \times \frac{59049}{32x^5} \times \frac{256}{6561}x^{16} + \dots$$

$$+ 92664x^{11}$$

$$\therefore 92664$$

7. $$\boxed{P = A \times 10^{kt}}$$

- $t=3,\ P=19000 \implies 19000 = A \times 10^{3k}$
- $t=6,\ P=38000 \implies 38000 = A \times 10^{6k}$

} DIVIDE EQUATIONS

$$\implies \frac{\cancel{A} \times 10^{6k}}{\cancel{A} \times 10^{3k}} = \frac{38000}{19000}$$

$$\implies 10^{3k} = 2$$

$$\implies \log(10^{3k}) = \log 2$$

$$\implies 3k\cancel{\log 10} = \log 2$$

$$\implies k = \frac{1}{3}\log 2$$

$$\implies k \simeq 0.10$$

USING $19000 = A \times 10^{3k}$

$$19000 = A \times 2$$

$$A = 9500$$

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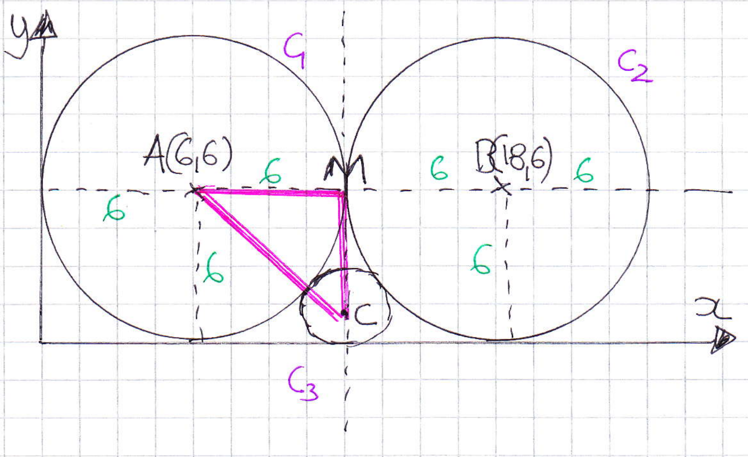
8.



- BY SYMMETRY THE CENTRE OF C_3 LIES ON $x = 12$
- LET r BE THE RADIUS OF C_3

$|AM| = 6$

$|AC| = 6 + r$

$|MC| = 6 - r$

BY PYTHAGORAS $\Rightarrow$ $6^2 + (6-r)^2 = (6+r)^2$

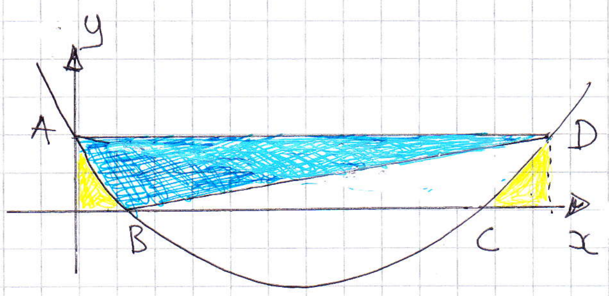
$$36 + \cancel{36} - \cancel{12r} + \cancel{r^2} = \cancel{36} + 12r + \cancel{r^2}$$

$$36 = 24r$$

$$r = \frac{3}{2}$$

$$\therefore (x-12)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

9.



$y = 2x^2 - 13x + 6$

- FIRSTLY $A(0,6)$
- $y = 0 \Rightarrow (2x-1)(x-6) = 0$

$$x = \begin{cases} 1/2 \\ 6 \end{cases} \qquad B\left(\tfrac{1}{2}, 0\right) \quad C(6,0)$$

- $D\left(\frac{13}{2}, 6\right)$ BY SYMMETRY
- YELLOW AREAS ARE CONGRUENT AND EQUAL TO

$$\int_0^{\frac{1}{2}} 2x^2 - 13x + 6 \, dx = \left[\frac{2}{3}x^3 - \frac{13}{2}x^2 + 6x\right]_0^{\frac{1}{2}}$$

$$= \left(\frac{1}{12} - \frac{13}{8} + 3\right) - (0)$$

$$= \frac{35}{24}$$

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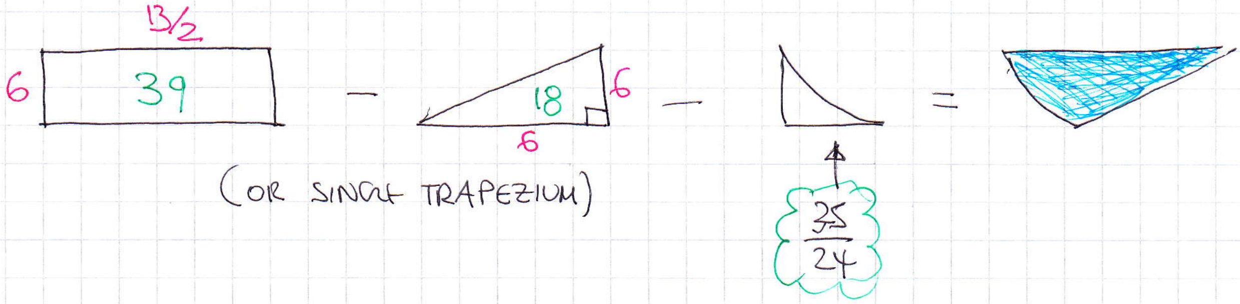


(OR SINGLE TRAPEZIUM)

$\therefore$ REQUIRED AREA $= \dfrac{469}{24}$

10. a)

x	1	1.4	1.8	2.2	2.6	3
y	$\frac{a}{2}$	$\frac{5}{12}a$	$\frac{5}{14}a$	$\frac{5}{16}a$	$\frac{5}{18}a$	$\frac{a}{4}$

$\dfrac{3-1}{5} = 0.4$

$$\text{AREA} \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times \text{REST}\right]$$

$$701.2 \simeq \frac{0.4}{2}\left[\frac{a}{2} + \frac{a}{4} + 2\left[\frac{5}{12}a + \frac{5}{14}a + \frac{5}{16}a + \frac{5}{18}a\right]\right]$$

$$701.2 \simeq \frac{1}{5}\left[\frac{3}{4}a + \frac{1375}{504}a\right]$$

$$701.2 \simeq \frac{1753}{2520}a$$

$$a = 1008$$

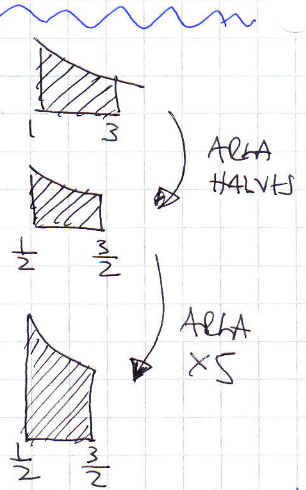
b)

LOOKING AT TRANSFORMATIONS

$$f(x) = \frac{a}{x+1}$$

$$f(2x) = \frac{a}{2x+1}$$

$$5f(2x) = \frac{5a}{2x+1}$$



$$\therefore \int_{0.5}^{1.5} \frac{5a}{2x+1}\,dx \approx \frac{701.2}{2} \times 5 \simeq 1753$$