C2, PAPER Q, IYGB

1. a)

$$x^2+y^2-2x-2y=8$$
$$x^2-2x+y^2-2y=8$$
$$(x-1)^2-1+(y-1)^2-1=8$$
$$(x-1)^2+(y-1)^2=10$$
$$\therefore\ C(1,1)\quad r=\sqrt{10}$$

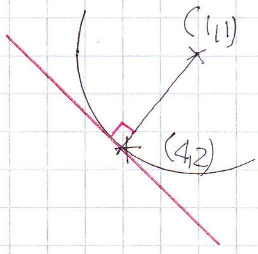
b) $C(1,1)\quad P(4,2)$

$$\text{GRAD } CP=\frac{y_2-y_1}{x_2-x_1}=\frac{2-1}{4-1}=\frac{1}{3}$$

GRADIENT OF TANGENT MUST BE -3

$$y-y_0=m(x-x_0)$$
$$y-2=-3(x-4)$$
$$y-2=-3x+12$$
$$y=14-3x$$

AS REQUIRED



2.

$$\int_2^a a-2x\,dx=\left[ax-x^2\right]_2^a=\cancel{(a^2-a^2)}-(2a-4)=4-2a$$

$$\therefore\ 4-2a=-5$$
$$9=2a$$
$$a=\frac{9}{2}$$

3.

$$\boxed{\begin{aligned}y&=x^4-2x^3+1\\ \frac{dy}{dx}&=4x^3-6x^2\\ \frac{d^2y}{dx^2}&=12x^2-12x\end{aligned}}$$

SOLVE $\frac{dy}{dx}=0 \implies 4x^3-6x^2=0$

$$2x^2(2x-3)=0$$

$$x=\begin{cases}0\\ \frac{3}{2}\end{cases}\qquad y=\begin{cases}1\\ -\frac{11}{16}\end{cases}$$

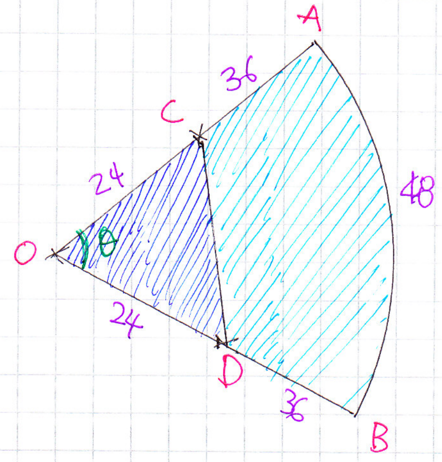
$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{3}{2}}=9>0\quad\therefore\ \left(\frac{3}{2},-\frac{11}{16}\right)\text{ IS A LOCAL MIN}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=0}=0$$ SO IT IS POSSIBLE POINT OF INFLEXION

$$\frac{d^3y}{dx^3}=24x-12$$

$$\left.\frac{d^3y}{dx^3}\right|_{x=0}=-12\neq 0\quad\therefore\ (0,1)\text{ IS A POINT OF INFLEXION}$$

4.



- USING $L = r\theta$

$48 = 60\theta$

$\theta = \frac{4}{5} = 0.8^c$

- AREA OF TRIANGLE

$= \frac{1}{2} \times 24 \times 24 \times \sin(0.8)$

$= 206.60\ldots$

- AREA OF SECTOR $\left(\frac{1}{2}r^2\theta^c\right)$

$= \frac{1}{2} \times 60^2 \times 0.8$

$= 1440$

THUS REQUIRED AREA $= 1440 - 206.60\ldots \approx 1233 \text{ cm}^2$

5. a) $a = 5000$, $r = 0.8$

$u_n = ar^{n-1}$

$u_5 = 5000 \times 0.8^4$

$u_5 = 2048$

AS REQUIRED

b) $S_n = \dfrac{a(1-r^n)}{1-r}$

$S_{24} = \dfrac{5000(1-0.8^{24})}{1-0.8}$

$S_{24} = 24882$

(OR 24881)

c) $a = 1000$, $r = 1.05$

$u_n = ar^{n-1}$

$u_{24} = 1000 \times 1.05^{23}$

$u_{24} = 3072$

(OR 3071)

d) $1000 \times 1.05^{k-1} > 5000 \times 0.8^{k-1}$

$1.05^{k-1} > 5 \times 0.8^{k-1}$

$\dfrac{1.05^{k-1}}{0.8^{k-1}} > 5$

$\left(\dfrac{1.05}{0.8}\right)^{k-1} > 5$

$\left(\dfrac{21}{16}\right)^{k-1} > 5$

e) $\log\left(\frac{21}{16}\right)^{k-1} > \log 5$

$(k-1)\log\left(\frac{21}{16}\right) > \log 5$

$k-1 > 5.918\ldots$

$k > 6.918$

$\therefore k = 7$

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6.

$$\frac{1}{2}\tan x - \sin x = 0$$

$$\Rightarrow \tan x - 2\sin x = 0$$

$$\Rightarrow \frac{\sin x}{\cos x} - 2\sin x = 0$$

$$\Rightarrow \sin x - 2\sin x\cos x = 0$$

$$\Rightarrow \sin x(1-2\cos x) = 0$$

$$\Rightarrow \sin x = 0 \quad \text{OR} \quad \cos x = \frac{1}{2}$$

- $\arcsin(0) = 0$

$$\begin{cases} x = 0 \pm 360n \\ x = 180 \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

- $\arccos\left(\frac{1}{2}\right) = 60°$

$$\begin{cases} x = 60 \pm 360n \\ x = 300 \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

$$x = 0°, 180°, 60°, 300°$$

7. a) $\boxed{P = A \times b^t}$

$$740 = 100 \times b^{21}$$

$$7.4 = b^{21}$$

$$b = \sqrt[21]{7.4}$$

$$b \simeq 1.099998231\ldots$$

$$b \simeq 1.10$$

$$A = 100$$

b)

$$P = 100 \times 1.10^t$$

$$100 \times 1.1^t > 10000$$

$$1.1^t > 100$$

$$\log 1.1^t > \log 100$$

$$t\log 1.1 > 2$$

$$t > 48.32$$

$$\therefore t = 49$$

$$\therefore 1970 + 49 = 2019$$

8. $\boxed{f(x) = x^3 - 9x^2 + 24x - 20}$

- $f(k) = -4$

$\Rightarrow k^3 - 9k^2 + 24k - 20 = -4$

$\Rightarrow k^3 - 9k^2 + 24k - 16 = 0$

LOOK FOR FACTORS OF ABOVE

- $k=1$, $1 - 9 + 24 - 16 = 0$

$\therefore (k-1)$ IS A FACTOR.

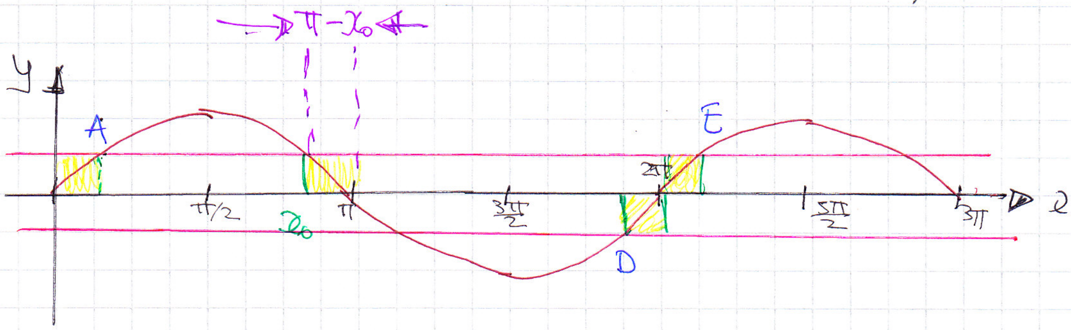
$$\begin{array}{r|l} & k^2 - 8k + 16 \\ k-1 & k^3 - 9k^2 + 24k - 16 \\ & -k^3 + k^2 \\ \hline & -8k^2 + 24k - 16 \\ & +8k^2 - 8k \\ \hline & 16k - 16 \\ & -16k + 16 \\ \hline & 0 \end{array}$$

$\therefore (k-1)(k^2 - 8k + 16) = 0$

$(k-1)(k-4)^2 = 0$

$\therefore k = \begin{cases} 1 \\ 4 \end{cases}$ //

9.



- A EVIDENTLY HAS x CO-ORDINATE $\pi - x_0$ //
- D HAS x CO-ORDINATE $2\pi - (\pi - x_0) = \pi + x_0$ //
- E HAS x CO-ORDINATE $2\pi + (\pi - x_0) = 3\pi - x_0$ //

10. $(2+ax)(1+bx)^7 = (2+ax)\left(1 + \frac{7}{1}(bx)^1 + \frac{7\times6}{1\times2}(bx)^2 + \dots\right)$

$= (2+ax)(1 + 7bx + 21b^2x^2 + \dots)$

$$\begin{aligned} = 2 + 14bx &+ 42b^2x^2 + \dots \\ + ax &+ 7abx^2 + \dots \end{aligned}$$

$= 2 + \underbrace{(14b+a)}_{-41}x + \underbrace{(42b^2 + 7ab)}_{357}x^2 + \dots$

$\left.\begin{array}{l} a + 14b = -41 \quad \times b \\ 42b^2 + 7ab = 357 \end{array}\right\} \Rightarrow \left.\begin{array}{l} ab + 14b^2 = -41b \\ 6b^2 + ab = 51 \end{array}\right\} \Rightarrow$ SOLVE FOR ab

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$$\left.\begin{array}{l} ab = -14b^2 - 41b \\ ab = 51 - 6b^2 \end{array}\right\} \Rightarrow 51 - 6b^2 = -14b^2 - 41b$$

$$8b^2 + 41b + 51 = 0$$

$$(b+3)(8b+17) = 0$$

$$b = \begin{cases} -3 \\ \cancel{-\frac{17}{8}} \end{cases} \quad (b \text{ IS AN INTEGER}$$

$$\therefore \quad a + 14b = -41$$

$$a + 14(-3) = -41$$

$$a - 42 = -41$$

$$a = 1$$