1.

x	0	0.25	0.5	0.75	1
y	1	$\frac{2}{3}$	$2-\sqrt{2}$	$4-2\sqrt{3}$	$\frac{1}{2}$

$$\text{AREA} \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$$

$$\text{AREA} \approx \frac{0.25}{2}\left[1 + \frac{1}{2} + 2\left(\frac{2}{3} + 2-\sqrt{2} + 4 - 2\sqrt{3}\right)\right]$$

$$\text{AREA} \simeq 0.635 \quad \text{3 d.p.}$$

2.

$a = 32$

$u_5 = 162$

USING $\quad u_n = ar^{n-1}$

$\Rightarrow 162 = 32 \times r^4$

$\Rightarrow \frac{81}{16} = r^4$

$\Rightarrow r = \sqrt[4]{\frac{81}{16}}$

$\Rightarrow \boxed{r = \frac{3}{2}}$

THUS

$u_3 = ar^2$

$u_3 = 32 \times \left(\frac{3}{2}\right)^2$

$u_3 = 72$

I.E 72 mph

3.

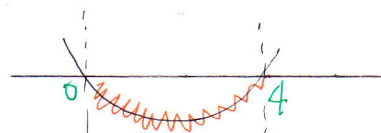$$y = x^3 - 6x^2 + 12$$

$$\frac{dy}{dx} < 0 \quad (\text{IF DECREASING}) \rightarrow$$

$$3x^2 - 12x < 0$$

$$3x(x-4) < 0$$
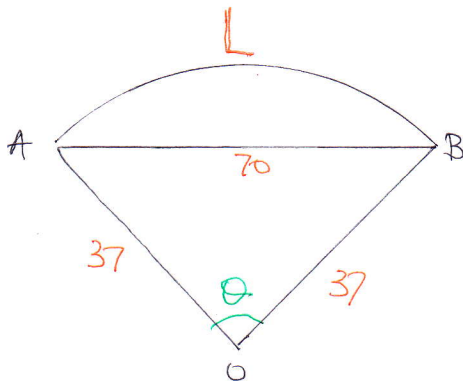
$$\text{C.V} = \begin{cases} 0 \\ 4 \end{cases}$$

$$0 < x < 4$$

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4. a)

● BY THE COSINE RULE

$$|AB|^2 = |OA|^2 + |OB|^2 - 2|OA||OB|\cos\theta$$

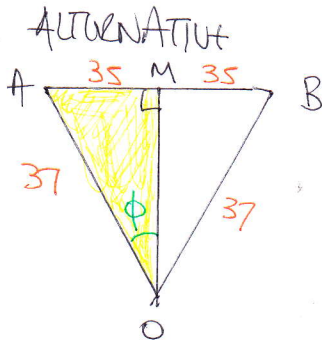$$70^2 = 37^2 + 37^2 - 2 \times 37 \times 37 \cos\theta$$

$$4900 = 1369 + 1369 - 2738\cos\theta$$

$$2738\cos\theta = -2162$$

$$\cos\theta = -\frac{2162}{2738}$$

$$\theta = 2.480997944\ldots$$

$$\theta \simeq 2.481^c$$

ALTERNATIVE

$$\sin\phi = \frac{35}{37}$$

$$\phi \simeq 1.24049\ldots$$

$$\therefore \widehat{AOB} = 2\phi \simeq 2.481^c \quad \text{AS BEFORE}$$

b) (I) $L = r\theta^c$.

$$L = 37 \times 2.481$$

$$L \simeq 91.80 \text{ m}$$

(II)

BY PYTHAGORAS

$$x^2 + 35^2 = 37^2$$

$$x^2 + 1225 = 1369$$

$$x^2 = 144$$

$$x = 12 \text{ m}$$

(III) AREA OF SECTOR $= \frac{1}{2}r^2\theta^c = \frac{1}{2} \times 37^2 \times 2.481\ldots = 1698.243\ldots$

AREA OF TRIANGLE $= \frac{1}{2} \times B \times H = \frac{1}{2} \times 70 \times 12 = 420$

$\therefore$ AREA OF SEGMENT $= 1698.243 - 420 \simeq 1278 \text{ m}^2$

5.

$$(2+kx)^6 = \binom{6}{0}(2)^6(kx)^0 + \binom{6}{1}(2)^5(kx)^1 + \binom{6}{2}(2)^4(kx)^2 + \binom{6}{3}(2)^3(kx)^3 + \ldots$$

$$= (1 \times 64 \times 1) + (6 \times 32 \times kx) + (15 \times 16 \times k^2x^2) + (20 \times 8 \times k^3x^3) + \ldots$$

$$= \underbrace{64}_{a} + \underbrace{192kx}_{b} + \underbrace{240k^2x^2}_{b} + \underbrace{160k^3x^3}_{c} + \ldots$$

◉ $192k = 240k^2 \Rightarrow$ AS $k \neq 0$

$192 = 240k$

$$\boxed{k = \frac{4}{5}}$$

∴ $b = 192k = 192 \times \frac{4}{5} = 153.6$

$C = 160k^3 = 160 \times \left(\frac{4}{5}\right)^3 = 81.92$

THUS

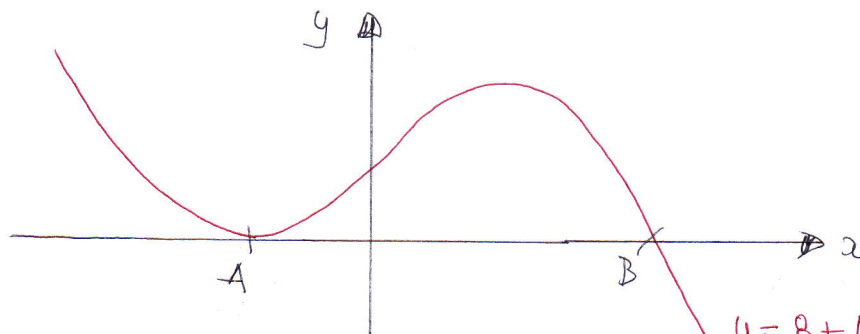$a = 64$

$b = 153.6$

$C = 81.92$ //

6. a)

$y = 8 + 4x - 2x^2 - x^3$

◉ BY VERIFICATION

IF $x = -2$

$y = 8 + 4(-2) - 2(-2)^2 - (-2)^3$

$y = 8 - 8 - 8 + 8$

$y = 0$ //

∴ $A(-2, 0)$

◉

$$\begin{array}{r} -x^2 + 4 \\ x+2 \overline{\smash{\big)}\, -x^3 - 2x^2 + 4x + 8} \\ \underline{x^3 + 2x^2} \\ 4x + 8 \\ \underline{-4x - 8} \\ 0 \end{array}$$

∴ $y = (x+2)(-x^2+4)$

$y = (x+2)(4 - x^2)$

$y = (x+2)(2-x)(2+x)$

$y = (x+2)^2(2-x)$

$\therefore$ $A(-2,0)$ $\leftarrow$ TOUCHING POINT FROM $(x+2)^2$

$B(2,0)$ $\leftarrow$ CROSSING POINT FROM $(2-x)$ //

b) $y = 8 + 4x - 2x^2 - x^3$

$\frac{dy}{dx} = 4 - 4x - 3x^2$

• SOLVE FOR ZERO

$4 - 4x - 3x^2 = 0$

$0 = 3x^2 + 4x - 4$

$0 = (x+2)(3x-2)$

$x = \begin{cases} -2 & \leftarrow A \\ \frac{2}{3} & \leftarrow C \end{cases}$

$y = 8 + 4\left(\frac{2}{3}\right) - 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$y = \frac{256}{27}$

$\therefore C\left(\frac{2}{3}, \frac{256}{27}\right)$ //

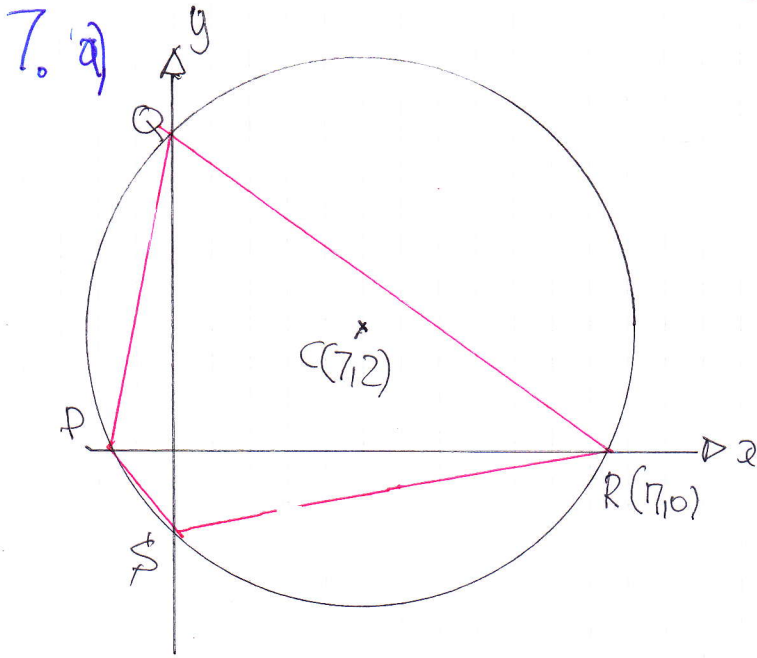c) AREA $= \displaystyle\int_{-2}^{2} 8 + 4x - 2x^2 - x^3 \, dx$

$= \left[8x + 2x^2 - \frac{2}{3}x^3 - \frac{1}{4}x^4 \right]_{-2}^{2}$

$= \left(16 + 8 - \frac{16}{3} - 4 \right) - \left(-16 + 8 + \frac{16}{3} - 4 \right)$

$= \frac{44}{3} - \left(-\frac{20}{3} \right)$

$= \frac{64}{3}$ //

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7. a)

• $R(17,0)$ • $C(7,2)$

$$|RC| = \sqrt{(17-7)^2 + (0-2)^2}$$

$$|RC| = \sqrt{100 + 4}$$

$$|RC| = \sqrt{104}$$

∴ EQUATION

$$(x-7)^2 + (y-2)^2 = \sqrt{104}^2$$

$$(x-7)^2 + (y-2)^2 = 104$$

b) WHEN $x=0$

$$(-7)^2 + (y-2)^2 = 104$$

$$\Rightarrow 49 + (y-2)^2 = 104$$

$$\Rightarrow (y-2)^2 = 55$$

$$\Rightarrow y-2 = \pm\sqrt{55}$$

$$\Rightarrow y = 2 \pm \sqrt{55}$$

∴ $Q(0, 2+\sqrt{55})$

$S(0, 2-\sqrt{55})$

∴ DIFFERENCE OF $2\sqrt{55}$

OR $(2+\sqrt{55}) - (2-\sqrt{55})$

$= 2\sqrt{55}$

c) WHEN $y=0$
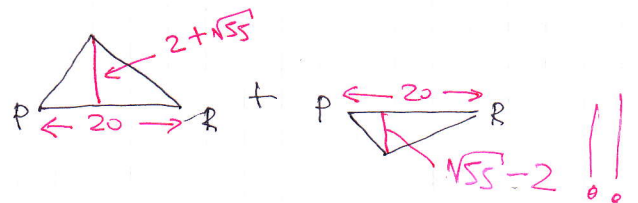
$$(x-7)^2 + (0-2)^2 = 104$$

$$(x-7)^2 + 4 = 104$$

$$(x-7)^2 = 100$$

$x-7 = \begin{cases} 10 \\ -10 \end{cases}$

$x = \begin{cases} 17 \leftarrow R \\ -3 \leftarrow P \end{cases}$

$$= \tfrac{1}{2} \times 20 \times (2+\sqrt{55}) + \tfrac{1}{2} \times 20 \times (2-\sqrt{55})$$

$$= 10(2+\sqrt{55}) + 10(\sqrt{55}-2)$$

$$= 20 + 10\sqrt{55} + 10\sqrt{55} - 20$$

$$= 20\sqrt{55}$$

ALTERNATIVE AREA $= \tfrac{1}{2}|PR||QS|$

$$= \tfrac{1}{2} \times 20 \times 2\sqrt{55}$$

8. $\log_3 8 - 3\log_3 t = 3$

$\Rightarrow \log_3 8 - \log_3 t^3 = 3\log_3 3$

$\Rightarrow \log_3 \left(\frac{8}{t^3}\right) = \log_3 27$

$\Rightarrow \quad \frac{8}{t^3} = 27$

$\Rightarrow 27t^3 = 8$

$\Rightarrow t^3 = \frac{8}{27}$

$\Rightarrow t = \frac{2}{3}$ //

9. a) $f(x) = x^3 - 4x^2 - \frac{1}{2}x + 2$

$f(4) = 64 - 64 - 2 + 2 = 0$

$\therefore (x-4)$ IS A FACTOR of $f(x)$ //

b) BY LONG DIVISION OR INSPECTION

$$
\begin{array}{r}
x^2 - \frac{1}{2} \\
x-4 \overline{\smash{\big)}\ x^3 - 4x^2 - \frac{1}{2}x + 2} \\
\underline{-x^3 + 4x^2} \\
0 \quad 0 \quad -\frac{1}{2}x + 2 \\
\underline{+\frac{1}{2}x - 2} \\
0
\end{array}
$$

$\therefore f(x) = (x-4)\left(x^2 - \frac{1}{2}\right)$ //

c) $\cos^3\theta - 4\cos^2\theta - \frac{1}{2}\cos\theta + 2 = 0$

$(\cos\theta - 4)(\cos^2\theta - \frac{1}{2}) = 0$ ← FROM PART (b)

EITHER $\cancel{\cos\theta = 4}$ OR $\cos^2\theta - \frac{1}{2} = 0$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

- $\arccos\left(\frac{\sqrt{2}}{2}\right) = 45°$

$\begin{cases} \theta = 45 \pm 360u \\ \theta = 315 \pm 360u \end{cases}$

- $\arccos\left(-\frac{\sqrt{2}}{2}\right) = 135$

$\theta = 135 \pm 360u$

$\theta = 225 \pm 360u$

$u = 0,1,2,3...$

$\theta_1 = 45°$
$\theta_2 = 315°$
$\theta_3 = 135°$
$\theta_4 = 225°$