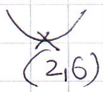
1. a)

$$f(x) = 2x^2 - 8x + 14 = 2\left[x^2 - 4x + 7\right]$$
$$= 2\left[(x-2)^2 - 4 + 7\right]$$
$$= 2\left[(x-2)^2 + 3\right]$$
$$= 2(x-2)^2 + 6$$

b)

MIN OF $f(x)$ IS AT $(2,6)$



MIN OF $f\left(\frac{1}{2}x\right)$ IS AT $(4,6)$ — HORIZONTAL STRETCH BY SCALE FACTOR 2

MIN OF $f(x+1) - 4$ IS AT $(1,2)$ — TRANSLATION BY 1 UNIT TO THE LEFT, 4 UNITS DOWN

2. a)

$$\left.\begin{array}{l} u_{12} = 760 \\ u_{25} = 240 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a + (12-1)d = 760 \\ a + (25-1)d = 240 \end{array}\right\} \Rightarrow \begin{array}{l} a + 11d = 760 \\ a + 24d = 240 \end{array}$$

SUBTRACT $-13d = 520$

$$\boxed{d = -40}$$

$\&$ $a + 24d = 240$
$$a + 24(-40) = 240$$
$$a - 960 = 240$$
$$a = 1200$$

IE 1200 HOUSES

b)

$a = 1200$
$d = -40$
$n = 25$

USING $S_n = \frac{n}{2}(2a + (n-1)d)$

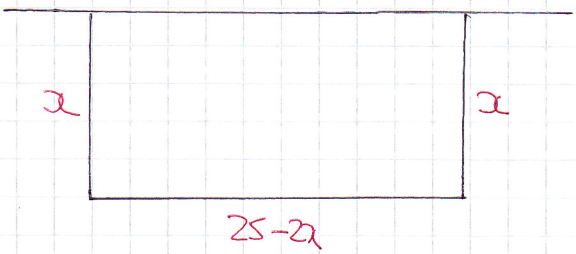
$$S_{25} = \frac{25}{2}\left[2 \times 1200 + 24(-40)\right]$$
$$S_{25} = \frac{25}{2}\left[2400 - 960\right]$$
$$= 25\left[1200 - 480\right]$$
$$= 25 \times 720$$
$$= 18000$$

7200
7200
3600
18000

ALTERNATIVE

$$S_n = \frac{n}{2}\left[a + L\right]$$
$$S_{25} = \frac{25}{2}\left[1200 + 240\right]$$
$$S_{25} = \frac{25}{2} \times 1440$$
$$S_{25} = 25 \times 720$$
$$S_{25} = 18000$$

3.



AREA < 75 (OR $\leq$)

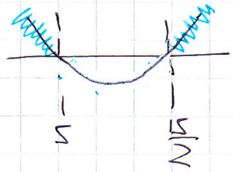
$x(25-2x) < 75$

$25x - 2x^2 < 75$

$-2x^2 + 25x - 75 < 0$

$2x^2 - 25x + 75 > 0$

$(2x-15)(x-5) > 0$



$x < 5$ OR $x > \frac{15}{2}$

BUT x MUST ALSO SATISFY $3 < x < 9$ (OR $\leq \geq$)

$\therefore$ REQUIRED ANSWER IS $3 < x < 5$ OR $\frac{15}{2} < x < 9$

4. a) $f'(x) = \frac{x-6}{\sqrt{x}}$

$f'(16) = \frac{16-6}{\sqrt{16}} = \frac{10}{4} = \frac{5}{2}$

EQUATION OF TANGENT AT $(16,-5)$

$y - y_0 = m(x - x_0)$

$y + 5 = \frac{5}{2}(x-16)$

$2y + 10 = 5x - 80$

$2y = 5x - 90$

b) $f'(x) = \frac{x}{x^{\frac{1}{2}}} - \frac{6}{x^{\frac{1}{2}}} = x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

$f(x) = \int x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}\,dx$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} - 12x^{\frac{1}{2}} + C$

USING $(16,-5)$

$-5 = \frac{2}{3}\times 16^{\frac{3}{2}} - 12\times 16^{\frac{1}{2}} + C$

$-5 = \frac{2}{3}\times 64 - 12\times 4 + C$

$-5 = \frac{128}{3} - 48 + C$

$-15 = 128 - 144 + 3C$

$-15 = -16 + 3C$

$C = \frac{1}{3}$

$\therefore f(x) = \frac{2}{3}x^{\frac{3}{2}} - 12x^{\frac{1}{2}} + \frac{1}{3}$

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c) $\dfrac{x-6}{\sqrt{x}} = -1 \implies x-6 = -\sqrt{x}$

$f(x)$

$\implies x+\sqrt{x}-6=0$

$\implies (\sqrt{x})^2+\sqrt{x}-6=0$

$\implies (\sqrt{x}+3)(\sqrt{x}-2)=0$

$\implies \sqrt{x} = \cancel{-3}$ or 2

$\boxed{x=4}$

ALTERNATIVE

$(x-6)^2 = (-\sqrt{x})^2$

$x^2-12x+36 = x$

$x^2-13x+36=0$

$(x-4)(x-9)=0$

$x = 4$ or $\cancel{9}$

DOES NOT SATISFY THE ORIGINAL EQUATION

$\therefore f(4) = \frac{2}{3}\times 4^{\frac{3}{2}} - 12\times 4^{\frac{1}{2}} + \frac{1}{3}$

$f(4) = \frac{2}{3}\times 8 - 12\times 2 + \frac{1}{3}$

$f(4) = \frac{16}{3} - 24 + \frac{1}{3}$

$f(4) = \frac{17}{3} - 24$

$f(4) = \frac{17}{3} - \frac{72}{3}$

$f(4) = -\frac{55}{3}$

$\therefore Q\left(4, -\frac{55}{3}\right)$ //

5. $\left.\begin{array}{l}(x+y\sqrt{3})^2 = 56+12\sqrt{3} \\ y=3x\end{array}\right\} \implies (x+3x\sqrt{3})^2 = 56+12\sqrt{3}$

FACTORIZE x OUT OF THE BRACKET

$\implies x^2(1+3\sqrt{3})^2 = 56+12\sqrt{3}$

$\implies x^2(1+6\sqrt{3}+27) = 56+12\sqrt{3}$

$\implies x^2(28+6\sqrt{3}) = 56+12\sqrt{3}$

$\implies x^2 = \dfrac{56+12\sqrt{3}}{28+6\sqrt{3}} = \dfrac{2\cancel{(28+6\sqrt{3})}}{\cancel{28+6\sqrt{3}}}$

$\implies x^2 = 2$

$\implies x = \pm\sqrt{2}$

$\implies y = \pm 3\sqrt{2}$

$\therefore (\sqrt{2}, 3\sqrt{2})$ OR $(-\sqrt{2}, -3\sqrt{2})$ //

6. a) $u_{n+1} = \frac{u_n + 1}{2} \implies 2u_{n+1} = u_n + 1$

$2u_{n+1} - 1 = u_n$

$\boxed{u_n = 2u_{n+1} - 1}$

b)

$u_4 = 21$

$u_3 = 2u_4 - 1 = 2\times21 - 1 = 42 - 1 = 41$

$u_2 = 2u_3 - 1 = 2\times41 - 1 = 81$

$u_1 = 2u_2 - 1 = 2\times81 - 1 = 161$

$\therefore k = 161$

7.

$$\left.\begin{array}{l} y = kx - 9 \\ y = 3(x+1)^2 \end{array}\right\} \implies kx - 9 = 3(x+1)^2$$

$kx - 9 = 3x^2 + 6x + 3$

$0 = 3x^2 + 6x - kx + 12$

$\boxed{0 = 3x^2 + (6-k)x + 12}$ (#)

TANGENT $\implies$ REPEATED ROOTS IE $b^2 - 4ac = 0$

$(6-k)^2 - 4\times3\times12 = 0$

$(6-k)^2 = 144$

$6 - k = \begin{cases} 12 \\ -12 \end{cases}$

$-k = \begin{cases} 6 \\ -18 \end{cases}$

$k = \begin{cases} -6 \\ 18 \end{cases}$

- IF $k = -6$ (#) BECOMES

$3x^2 + 12x + 12 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2, \; y = 3$

- IF $k = 18$ (#) BECOMES

$3x^2 - 12x + 12$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2 \quad y = 27$

$\therefore P(-2, 3)$

OR $P(2, 27)$

8\. a) GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{-2-(-4)}{3-0} = \frac{2}{3}$

SINCE IT PASSES THROUGH $(0,-4)$ $y = \frac{2}{3}x - 4$

$3y = 2x - 12$

$0 = 2x - 3y - 12$

$2x - 3y - 12 = 0$

b) LET $C\left(x, \frac{2}{3}x - 4\right)$ $A(0,-4)$

$|AC| = 3\sqrt{13}$

$\Rightarrow \sqrt{\left[-4-\left(\frac{2}{3}x-4\right)\right]^2 + (0-x)^2} = 3\sqrt{13}$

$\Rightarrow \sqrt{\left(-\frac{2}{3}x\right)^2 + x^2} = 3\sqrt{13}$

$\Rightarrow \sqrt{\frac{4}{9}x^2 + x^2} = 3\sqrt{13}$

$\Rightarrow \sqrt{\frac{13}{9}x^2} = 3\sqrt{13}$

$\Rightarrow \frac{13}{9}x^2 = 9 \times 13$

$\Rightarrow \frac{1}{9}x^2 = 9$

$\Rightarrow x^2 = 81$

$\Rightarrow x = \begin{cases} -9 \\ 9 \end{cases}$ BUT $y = \frac{2}{3}x - 4 \begin{cases} -10 \\ 2 \end{cases}$

$\therefore C(9,2)$ OR $(-9,-10)$