C1, IYGB, PAPER S

1. a) $y = 2x^3 - 3x + \frac{4}{x}$

$\Rightarrow y = 2x^3 - 3x + 4x^{-1}$

$\Rightarrow \frac{dy}{dx} = 6x^2 - 3 - 4x^{-2}$

$\Rightarrow \frac{dy}{dx} = 6x^2 - 3 - \frac{4}{x^2}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = 6 \times 2^2 - 3 - \frac{4}{2^2}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = 24 - 3 - 1 = 20$

b) $\frac{dy}{dx} = 20$

$\Rightarrow 20 = 6x^2 - 3 - \frac{4}{x^2}$

$\Rightarrow 0 = 6x^2 - 23 - \frac{4}{x^2}$

$\Rightarrow 0 = 6x^4 - 23x^2 - 4$

$\Rightarrow 0 = (6x^2+1)(x^2-4)$

$\Rightarrow \cancel{6x^2+1=0}$

OR

$x^2 = 4$

$x = \begin{cases} 2 & \leftarrow P \\ -2 & \leftarrow Q \end{cases}$

$\therefore y = 2(-2)^3 - 3(-2) + \frac{4}{-2}$

$y = -16 + 6 - 2 = -12$

$\therefore Q(-2, -12)$

NOTE OR SIMPLY $(-2, -12)$ SINCE CURVE IS ODD!!

2. a) $f(x) = x^2 - 10x + 50$

$f(x) = (x-5)^2 - 25 + 50$

$f(x) = (x-5)^2 + 25$

b) MIN VALUE OF $f(x)$ IS 25

c) $A(20, -3)$

$B(x, 3x-13) \leftarrow$ LIES ON $y = 3x - 13$

$|AB| = \sqrt{(3x-13+3)^2 + (x-20)^2}$

$|AB|^2 = (3x-10)^2 + (x-20)^2$

$|AB|^2 = 9x^2 - 60x + 100 + x^2 - 40x + 400$

$|AB|^2 = 10x^2 - 100x + 500$

AS REQUIRED

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d)

$$|AB|^2 = 10x^2 - 100x + 500$$
$$|AB|^2 = 10\left[x^2 - 10x + 50\right]$$
$$|AB| = \sqrt{10 \times 25}$$
$$|AB| = 5\sqrt{10}$$

e) THIS OCCURS WHEN $x=5$

FROM $(x-5)^2+25$

$\therefore B(5,2)$

$3x-13$

3.

$$6^{x+2} \times 2^{1-x} = \frac{8}{3}$$
$$\Rightarrow 6^x \times 6^2 \times 2^1 \times 2^{-x} = \frac{8}{3}$$
$$\Rightarrow 6^x \times 36 \times 2 \times \frac{1}{2^x} = \frac{8}{3}$$
$$\Rightarrow \frac{6^x}{2^x} \times 72 = \frac{8}{3}$$
$$\Rightarrow 72 \times \left(\frac{6}{2}\right)^x = \frac{8}{3}$$
$$\Rightarrow 72 \times 3^x = \frac{8}{3}$$

NEXT DIVIDE BY 8

$$\Rightarrow 9 \times 3^x = \frac{1}{3}$$
$$\Rightarrow 3^x = \frac{1}{27}$$
$$\Rightarrow 3^x = 3^{-3}$$
$$\therefore x = -3$$

4.

$$u_n = \frac{n+2}{2n+1} \Rightarrow u_{n+1} = \frac{(n+1)+2}{2(n+1)+1}$$
$$u_{n+1} = \frac{n+3}{2n+3}$$

REARRANGE FOR n

$$2nu_n + u_n = n+2$$
$$2nu_n - n = 2 - u_n$$
$$n(2u_n - 1) = 2 - u_n$$
$$n = \frac{2-u_n}{2u_n-1}$$

THUS

$$u_{n+1} = \frac{\frac{2-u_n}{2u_n-1}+3}{2\left(\frac{2-u_n}{2u_n-1}\right)+3}$$

TIDY BY MULTIPLYING TOP & BOTTOM OF THE FRACTION (RHS) BY $(2u_n-1)$

$$\therefore\ u_{n+1} = \frac{2-u_n+3(2u_n-1)}{2(2-u_n)+3(2u_n-1)} = \frac{2-u_n+6u_n-3}{4-2u_n+6u_n-3} = \frac{5u_n-1}{4u_n+1}$$

$$u_{n+1} = \frac{5u_n-1}{4u_n+1}$$

ie $A=5$, $B=4$

5.

$$\left.\begin{array}{l}15y-8x=39\\(x+3)^2+(y-1)^2=289\end{array}\right\} \Rightarrow \left\{\begin{array}{l}8x=15y-39\\64(x+3)^2+64(y-1)^2=289\times64\end{array}\right\}$$

$$\left.\begin{array}{l}8x=15y-39\\(8x+24)^2+64(y-1)^2=289\times64\end{array}\right\} \Rightarrow \text{... BY SUBSTITUTION NOW ...}$$

THUS WE OBTAIN

$$\Rightarrow (15y-39+24)^2+64(y-1)^2=289\times64$$

$$\Rightarrow (15y-15)^2+64(y-1)^2=289\times64$$

$$\Rightarrow 225(y-1)^2+64(y-1)^2=289\times64$$

$$\Rightarrow 289(y-1)^2=289\times64$$

$$\Rightarrow (y-1)^2=64$$

$$\Rightarrow y-1 = \begin{cases}8\\-8\end{cases}$$

$$\Rightarrow y = \begin{cases}9\\-7\end{cases}$$

$$x_9 = \frac{15\times9-39}{8} = \frac{135-39}{8} = \frac{96}{8} = 12$$

$$x_7 = \frac{15(-7)-39}{8} = \frac{-105-39}{8} = \frac{-144}{8} = -18$$

$\therefore (12,9)$ & $(-18,-7)$

6.

$$y = 2x^2 + 4(p+2)x + 8p + q + 8$$

"MEET THE y AXIS AT $(0,18)$" $\Rightarrow$ $18 = 8p + q + 8$

$$\boxed{10 = 8p + q}$$

"CURVE HAS NO x INTERCEPTS" $\Rightarrow$ $b^2 - 4ac < 0$

THUS $\left[4(p+2)\right]^2 - 4 \times 2 \times (8p + q + 8) < 0$

$$16(p+2)^2 - 8(8p + q + 8) < 0$$

$$2(p+2)^2 - (8p + q + 8) < 0$$

$$2p^2 + \cancel{8p} + \cancel{8} - \cancel{8p} - q - \cancel{8} < 0$$

$$\boxed{2p^2 - q < 0}$$

BUT $\boxed{q = 10 - 8p}$

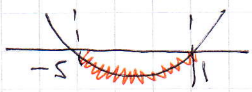
$$2p^2 - (10 - 8p) < 0$$

$$2p^2 + 8p - 10 < 0$$

$$p^2 + 4p - 5 < 0$$

$$(p+5)(p-1) < 0$$

C.V. $= \begin{cases} 1 \\ -5 \end{cases}$



$$-5 < p < 1$$

$$-40 < 8p < 8$$

$$-8 < -8p < 40$$

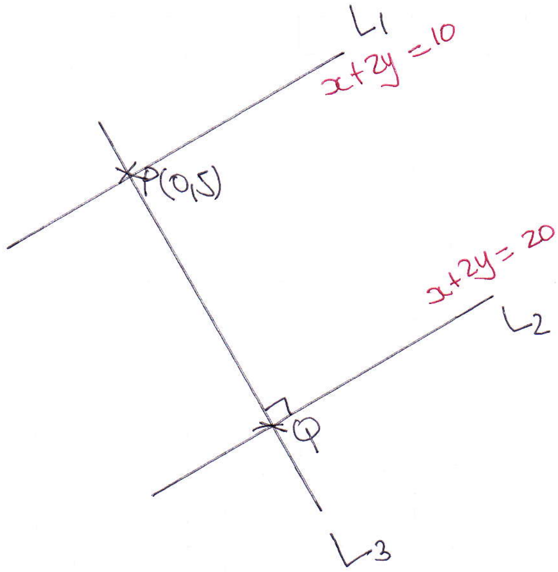
$$2 < -8p + 10 < 50$$

$$2 < q < 50$$

AS REQUIRED

C1, IYGB, PAPER S

7. $\begin{cases} L_1: x+2y=10 \\ L_2: x+2y=20 \end{cases}$



- PICK A RANDOM POINT ON L_1
  SAY $P(0,5)$
- GRADIENT OF BOTH LINES IS $-\frac{1}{2}$
- PERPENDICULAR LINE L_3 TO BOTH LINES HAS GRAD 2 AND PASSES THROUGH $(0,5)$
- EQUATION OF L_3 $\quad y = 2x+5$
- INTERSECT L & L_3 $\quad \left.\begin{array}{l} y=2x+5 \\ x+2y=20 \end{array}\right\} \Rightarrow$

$$x+2(2x+5)=20$$
$$5x+10=20$$
$$5x=10$$
$$x=2$$
$$\& \quad y=9$$
$$\therefore \; Q(2,9)$$

- DISTANCE $|PQ| = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$

8. $(\sqrt{3}-1)x^2 - 2\sqrt{3}x = 3+3\sqrt{3}$

$$\Rightarrow x^2 - \frac{2\sqrt{3}}{\sqrt{3}-1}x = \frac{3+3\sqrt{3}}{\sqrt{3}-1}$$

$$\Rightarrow x^2 - \frac{2\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}x = \frac{(3+3\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$\Rightarrow x^2 - \frac{6+2\sqrt{3}}{3-1}x = \frac{3\sqrt{3}+3+9+3\sqrt{3}}{3-1}$$

$$\Rightarrow x^2 - (3+\sqrt{3})x = 6+3\sqrt{3}$$

$$\Rightarrow x^2 - (3+\sqrt{3})x - (6+3\sqrt{3}) = 0$$

BY THE QUADRATIC FORMULA

$$x = \frac{3+\sqrt{3} \pm \sqrt{(3+\sqrt{3})^2 + 4\times1\times(6+3\sqrt{3})}}{2\times1}$$

$$x = \frac{3+\sqrt{3} \pm \sqrt{9+6\sqrt{3}+3+24+12\sqrt{3}}}{2}$$

$$x = \frac{3+\sqrt{3} \pm \sqrt{36+18\sqrt{3}}}{2} = \frac{3+\sqrt{3} \pm 3\sqrt{4+2\sqrt{3}}}{2}$$

$$x = \frac{3+\sqrt{3} \pm 3\sqrt{\sqrt{3}^2+2\times1\times\sqrt{3}+1^2}}{2}$$

$$x = \frac{3+\sqrt{3} \pm 3\sqrt{(\sqrt{3}+1)^2}}{2} = \frac{3+\sqrt{3} \pm 3(\sqrt{3}+1)}{2}$$

$$x = \begin{cases} \dfrac{3+\sqrt{3}+3\sqrt{3}+3}{2} = \dfrac{6+4\sqrt{3}}{2} = 3+2\sqrt{3} \\[2ex] \dfrac{3+\sqrt{3}-3\sqrt{3}-3}{2} = \dfrac{-2\sqrt{3}}{2} = -\sqrt{3} \end{cases}$$

(I.E. $p=3$, $q=2$, $r=-1$)

9.

$$x^2+y^2 = 2xy + z^2$$

$$x^2 - 2xy + y^2 = z^2$$

$$(x-y)^2 = z^2$$

$$x - y = \pm z$$

$$x = y \pm z$$

(P.T.O)

10. $u_r = 4r - 7$ → ARITHMETIC SEQUENCE $\boxed{\begin{array}{l} a = -3 \\ d = 4 \end{array}}$

- $\displaystyle\sum_{r=k+1}^{N} u_r - \sum_{r=1}^{k} u_r = 400$ ← DIFFERENCE IN SUMS BETWEEN k^{TH} TO N^{TH} & 1^{ST} TO k^{TH} IS 400

- $u_N - u_k = 40$ ← DIFFERENCE BETWEEN THE N^{TH} & k^{TH} TERM IS 40

$\therefore u_N - u_k = 40$

$$\left[-3+(N-1)\times 4\right] - \left[-3+(k-1)\times 4\right] = 40$$

$$\cancel{-3} + 4(N-1) \cancel{+3} - 4(k-1) = 40$$

$$(N-1) - (k-1) = 10$$

$$\boxed{N - k = 10}$$

THUS $\displaystyle\sum_{r=k+1}^{N} u_r - \sum_{r=1}^{k} u_r = 400$

$$\Rightarrow \left[\sum_{r=1}^{N} u_r - \sum_{r=1}^{k} u_r\right] - \sum_{r=1}^{k} u_r = 400$$

$$\Rightarrow S_N - S_k - S_k = 400$$

$$\Rightarrow S_N - 2S_k = 400$$

$$\Rightarrow \frac{N}{2}\left[2(-3) + (N-1)\times 4\right] - 2\times\frac{k}{2}\left[2(-3) + (k-1)\times 4\right] = 400$$

$$\Rightarrow \frac{N}{2}\left[-6 + 4N - 4\right] - k\left[-6 + 4k - 4\right] = 400$$

$$\Rightarrow \frac{N}{2}\left[4N - 10\right] - k\left[4k - 10\right] = 400$$

$$\Rightarrow N\left[2N - 5\right] - k\left[4k - 10\right] = 400$$

$$\Rightarrow 2N^2 - 5N - 4k^2 + 10k = 400$$

NOW $\boxed{N = k + 10}$

$$\Rightarrow 2(k+10)^2 - 5(k+10) - 4k^2 + 10k = 400$$

$$\Rightarrow 2k^2 + 40k + 200 - 5k - 50 - 4k^2 + 10k = 400$$

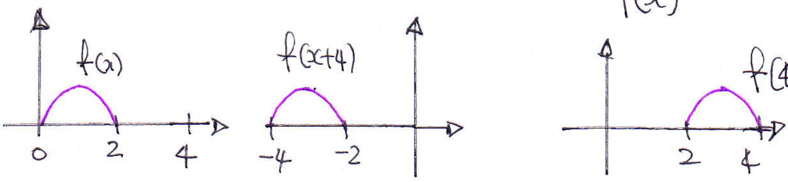
$$\Rightarrow -2k^2 + 45k - 250 = 0$$

$$\Rightarrow 2k^2 - 45k + 250 = 0$$

$$\Rightarrow (2k-25)(k-10)$$

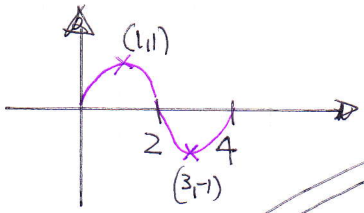
$k = 10$ or $\cancel{\frac{25}{2}}$ $\quad \therefore N = 20$

11. a) $f(x) \longmapsto f(x+4) \longmapsto f(-x+4)$
$-f(x)$

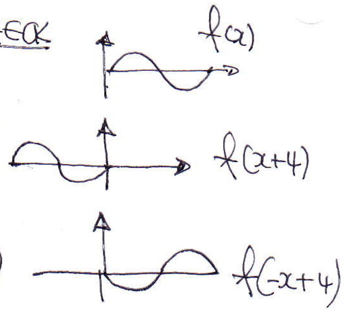


CONSIDER THE TRANSFORMATION SEQUENCE ABOVE
THE SECTION SEEN AT THE END IS $-f(x)$

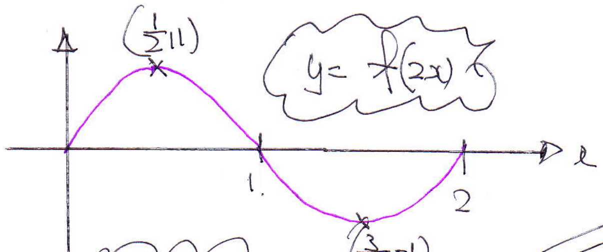
$\therefore f(x):$



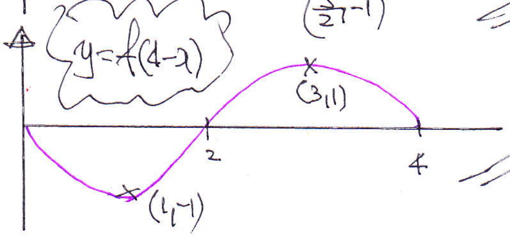
CHECK



b)



c)



(SINCE $f(4-x) = -f(x)$)