C1, IYGB, PAPER R

1. $x+\frac{9}{x}=\frac{15}{2}$

$\Rightarrow x^2+9=\frac{15}{2}x$

$\Rightarrow 2x^2+18=15x$

$\Rightarrow 2x^2-15x+18=0$

$\Rightarrow (2x-3)(x-6)=0$

$\Rightarrow x=\begin{cases}3/2\\6\end{cases}$

2. a) $4^{\frac{3}{2}}+4^{-\frac{1}{2}}=\left(\sqrt{4}\right)^3+\frac{1}{\sqrt{4}}$

$=8+\frac{1}{2}$

$=\frac{17}{2}$

b) $\frac{12y^{-5}}{3y^{-2}}=4y^{-3}=\frac{4}{y^3}$

3. $\int y\,dx=\int x(6x-5\sqrt{x})\,dx=\int x(6x-5x^{\frac{1}{2}})\,dx$

$=\int 6x^2-5x^{\frac{3}{2}}dx=\frac{6}{3}x^3-\frac{5}{\frac{5}{2}}x^{\frac{5}{2}}+C$

$=2x^3-2x^{\frac{5}{2}}+C$

4. $\sum_{k=1}^{100}a_k=\sum_{k=1}^{100}(5k-3)=\underbrace{2+7+12+\cdots+497}$

THIS IS AN A.P WITH $a=2$, $d=5$, $L=497$, $n=100$

USING $S_n=\frac{n}{2}(a+L)$

$S_{100}=\frac{100}{2}(2+497)$

$S_{100}=50\times 499$

$S_{100}=(50\times 500)-50$

$S_{100}=24950$

5. $x^2 - kx + (k+3) = 0$

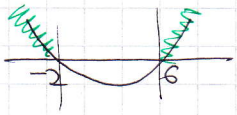
REAL ROOTS (ONE OR TWO) $\Rightarrow b^2 - 4ac \geqslant 0$

$\Rightarrow (-k)^2 - 4 \times 1 \times (k+3) \geqslant 0$

$\Rightarrow k^2 - 4(k+3) \geqslant 0$

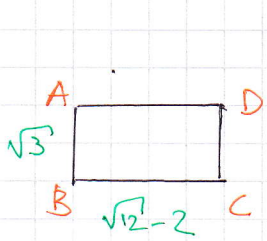
$\Rightarrow k^2 - 4k - 12 \geqslant 0$

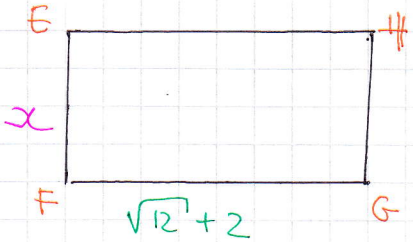
$\Rightarrow (k-6)(k+2) \geqslant 0$



$k \leqslant -2$ OR $k \geqslant 6$

6.





$$\frac{x}{\sqrt{3}} = \frac{\sqrt{12}+2}{\sqrt{12}-2}$$

$$\Rightarrow x = \frac{\sqrt{3}(\sqrt{12}+2)}{\sqrt{12}-2}$$

$$\Rightarrow x = \frac{6+2\sqrt{3}}{2\sqrt{3}-2}$$

$$\Rightarrow x = \frac{(6+2\sqrt{3})(2\sqrt{3}+2)}{(2\sqrt{3}-2)(2\sqrt{3}+2)}$$

$$\Rightarrow x = \frac{12\sqrt{3}+12+4\times 3+4\sqrt{3}}{4\times 3 + \cancel{4\sqrt{3}} - \cancel{4\sqrt{3}} - 4}$$

$$\Rightarrow x = \frac{24+16\sqrt{3}}{8}$$

$$\Rightarrow x = 3+2\sqrt{3}$$

C1, IYGB, PAPER R

7. $g(x) = f(\frac{1}{3}x) = \sqrt{27(\frac{1}{3}x)^3+1} = \sqrt{27\times\frac{1}{27}x^3+1} = \sqrt{x^3+1}$

8. a) GRAD $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{6-4}{2-(-4)} = \frac{2}{6} = \frac{1}{3}$

REQUIRED GRADIENT IS -3

$\therefore y-y_0 = m(x-x_0)$

$y-6 = -3(x-2)$

$y-6 = -3x+6$

$y = -3x+12$

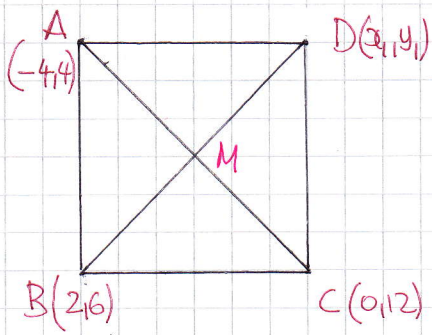
b) $A(-4,4)$ $B(2,6)$ $C(0,12)$

$|AB| = \sqrt{(6-4)^2+(2+4)^2} = \sqrt{4+36} = \sqrt{40}$

$|BC| = \sqrt{(12-6)^2+(0-2)^2} = \sqrt{36+4} = \sqrt{40}$

$\therefore |AB| = |BC|$

c)



$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-4+0}{2}, \frac{4+12}{2}\right)$

$= M(-2,8)$

AND $\left(\frac{x_1+2}{2}, \frac{y_1+6}{2}\right) = (-2,8)$

$\therefore \frac{x_1+2}{2} = -2 \Rightarrow x_1+2 = -4$

$\frac{y_1+6}{2} = 8 \Rightarrow y_1+6 = 16$

$\therefore x_1 = -6$

$y_1 = 10$

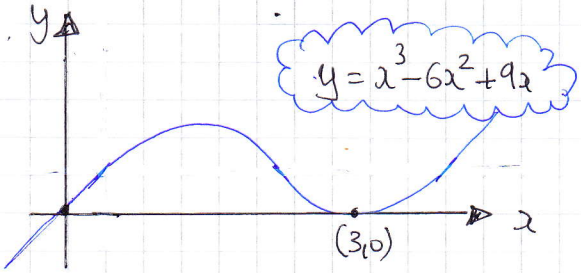
$\therefore D(-6,10)$

C1, IYGB, PAPER R

9. a) $y = x^3 - 6x^2 + 9x$

$y = x(x^2 - 6x + 9)$

$y = x(x-3)(x-3)$



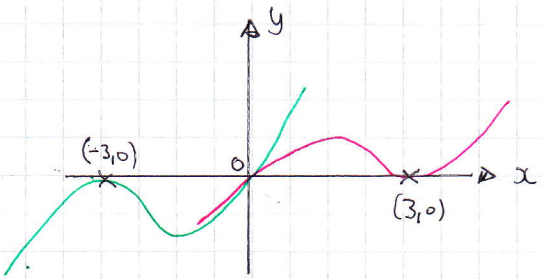
b) A ROTATION BY 180° IS A REFLECTION ABOUT x & ABOUT y

$\therefore f(x) = x^3 - 6x^2 + 9x$

$-f(x) = -x^3 + 6x^2 - 9x$

$-f(-x) = -(-x)^3 + 6(-x)^2 - 9(-x)$

$\therefore y = x^3 + 6x^2 + 9x$



OR SIMPLY ROTATE IT & PRODUCE EQUATION FROM THE GREEN SKETCH BY

$y = x(x+3)^2$

10. $a = 800$

$d = 100$

a) $u_4 = a + (n-1)d$

$u_{10} = 800 + 9 \times 100$

$u_{10} = 1700$

1st £1700

b) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 800 + 19 \times 100]$

$S_{20} = 10[1600 + 1900]$

$S_{20} = 35000$

$\therefore$ £35000

c) ANDREW $\frac{40}{2}[2 \times 800 + 39 \times 100] =$ BEATRICE $\frac{40}{2}[2 \times 1580 + 39d]$

$1600 + 3900 = 3160 + 39d$

$5500 = 3160 + 39d$

$2340 = 39d$

$d = 60$

$39 \times 6 = 234$

C1, IYGB, PAPER R

11. a) $f(x) = 4x^2 + 12kx$ NOW $f(x) = 9$

$\Rightarrow 4x^2 + 12kx - 9 = 0$

$\Rightarrow b^2 - 4ac = (12k)^2 - 4\times 4 \times (-9) = 144k^2 + 144 \geqslant 144 > 0$

$\therefore$ ALWAYS 2 DISTINCT REAL ROOTS

b) BY QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-12k \pm \sqrt{144k^2+144}}{2\times 4}$$

$$x = \frac{-12k \pm \sqrt{144(k^2+1)}}{8} = \frac{-12k \pm 12\sqrt{k^2+1}}{8} = -\frac{3}{2}k \pm \frac{3}{2}\sqrt{k^2+1}$$

OR COMPLETING THE SQUARE

$4x^2 + 12kx - 9 = 0$

$4\left[x^2 + 3kx - \frac{9}{4}\right] = 0$

$\left(x + \frac{3}{2}k\right)^2 - \frac{9}{4}k^2 - \frac{9}{4}$

$\left(x + \frac{3}{2}k\right)^2 = \frac{9}{4}k^2 + \frac{9}{4}$

$x + \frac{3}{2}k = \pm\sqrt{\frac{9}{4}(k^2+1)}$

$x + \frac{3}{2}k = \pm\frac{3}{2}(k^2+1)$

12. a) $y = x^3 - 3x^2 + 2x + 9$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$\left.\frac{dy}{dx}\right|_{x=2} = 3\times 2^2 - 6\times 2 + 2 = 12 - 12 + 2 = 2$

EQUATION OF TANGENT: $y - y_0 = m(x - x_0)$

$y - 9 = 2(x-2)$

$y - 9 = 2x - 4$

$y = 2x + 5$

C1, IYGB, PAPER R

b) GRAD REQUIRED IS $-\frac{1}{2}$

$$\Rightarrow 3x^2-6x+2=-\frac{1}{2}$$
$$\Rightarrow 6x^2-12x+4=-1$$
$$\Rightarrow 6x^2-12x+5=0$$
$$\Rightarrow x=\frac{-(-12)\pm\sqrt{(-12)^2-4\times6\times5}}{2\times6}$$
$$\Rightarrow x=\frac{12\pm\sqrt{144-120}}{12}$$
$$\Rightarrow x=\frac{12\pm\sqrt{24}}{12}$$
$$\Rightarrow x=\frac{12\pm2\sqrt{6}}{12}$$
$$\Rightarrow x=\frac{6\pm\sqrt{6}}{6}$$
$$\Rightarrow x=\frac{6+\sqrt{6}}{6} \quad (x\geqslant 1)$$

$$\frac{6-\sqrt{6}}{6}=\frac{1}{2}-\frac{1}{6}\sqrt{6}<\frac{1}{2}$$

OR COMPLETE THE SQUARE

$$\Rightarrow x^2-2x+\frac{5}{6}=0$$
$$\Rightarrow (x-1)^2-1+\frac{5}{6}=0$$
$$\Rightarrow (x-1)^2=\frac{1}{6}$$
$$\Rightarrow x-1=\pm\frac{1}{\sqrt{6}}$$
$$\Rightarrow x-1=\pm\frac{\sqrt{6}}{6}$$
$$\Rightarrow x=1\pm\frac{\sqrt{6}}{6}$$
$$\Rightarrow x=1+\frac{\sqrt{6}}{6} \quad x\geqslant 1$$
$$\Rightarrow x=\frac{6+\sqrt{6}}{6}$$