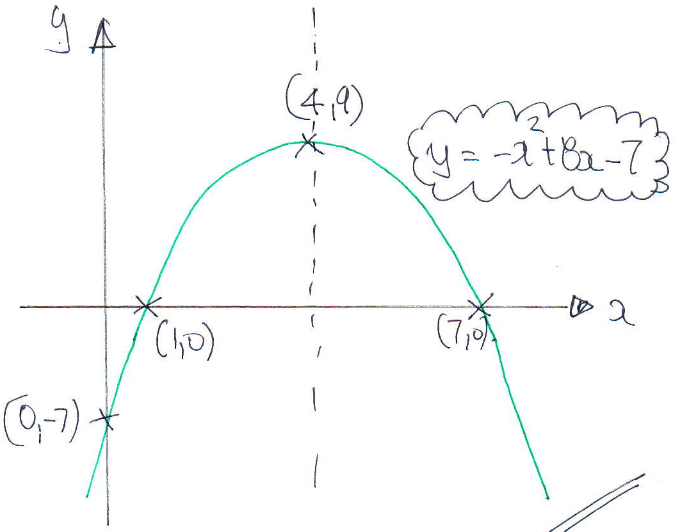
C1, IYGB, PAPER I

1. a) $x^2 - 8x + 7 = (x-4)^2 - 16 + 7 = (x-4)^2 - 9$

b) MAX AT $(4,9)$

c)



- $-x^2 \Longrightarrow \frown$
- $x=0,\ y=-7$ $(0,-7)$
- $y=0,\ -x^2+8x-7=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x = \begin{cases} 1 \\ 7 \end{cases}$

$\therefore (1,0)\ (7,0)$

2. $\left.\begin{array}{l} x^2 - 3xy + y^2 = 11 \\ 3y - x = 1 \end{array}\right\} \Longrightarrow \boxed{3y - 1 = x}$ SUB INTO THE QUADRATIC

$\Longrightarrow (3y-1)^2 - 3(3y-1)y + y^2 = 11$

$\Longrightarrow \cancel{9y^2} - 6y + 1 - \cancel{9y^2} + 3y + y^2 = 11$

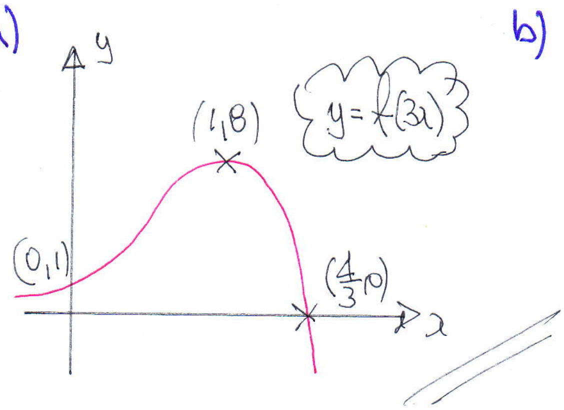
$\Longrightarrow y^2 - 3y - 10 = 0$

$\Longrightarrow (y-5)(y+2) = 0$

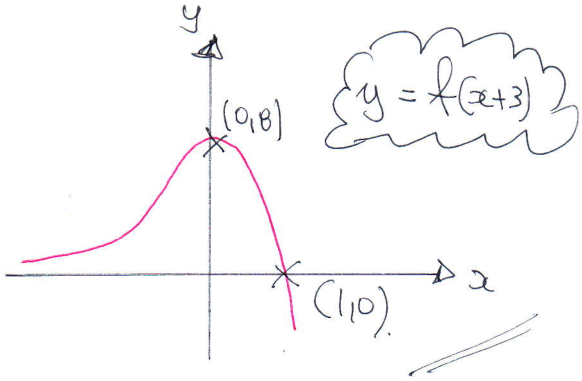
$y = \begin{cases} -2 \\ 5 \end{cases} \qquad x = \begin{cases} -7 \\ 14 \end{cases}$

$\therefore (-7,-2)$ & $(14,5)$

3. a)



b)



C1, IYGB, PAPER I

4.

$$z\sqrt{8} - 6 = \frac{2z}{\sqrt{2}}$$

$$\Rightarrow z\sqrt{8}\sqrt{2} - 6\sqrt{2} = 2z$$

$$\Rightarrow z\sqrt{16} - 6\sqrt{2} = 2z$$

$$\Rightarrow 4z - 6\sqrt{2} = 2z$$

$$\Rightarrow 2z = 6\sqrt{2}$$

$$\Rightarrow z = 3\sqrt{2}$$

ALTERNATIVE

$$z\sqrt{8} - 6 = \frac{2z}{\sqrt{2}}$$

$$2\sqrt{2}z - 6 = \frac{2z\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$2\sqrt{2}z - 6 = \frac{2\sqrt{2}z}{2}$$

$$2\sqrt{2}z - 6 = \sqrt{2}z$$

$$\sqrt{2}z = 6$$

$$z = \frac{6}{\sqrt{2}}$$

$$z = \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{6\sqrt{2}}{2}$$

$$z = 3\sqrt{2}$$

5. a) • GRAD OF $L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-14)}{3-(-1)} = \frac{12}{4} = 3$

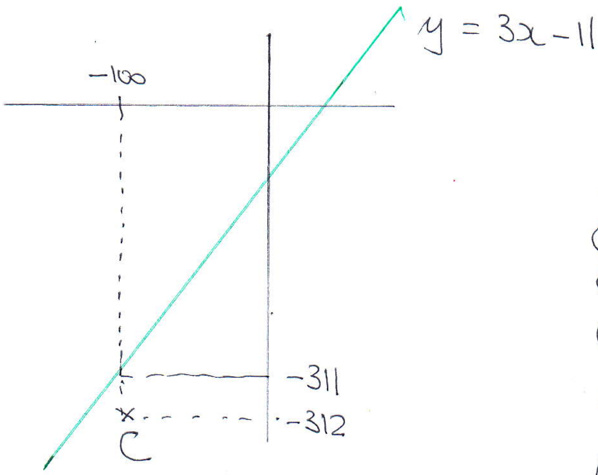
$\therefore$ $y - y_0 = m(x - x_0)$ with $m = 3$ $A(-1,-14)$

$$y + 14 = 3(x+1)$$

$$y + 14 = 3x + 3$$

$$y = 3x - 11$$

b)



when $x = -100$

$$y = 3(-100) - 11$$

$$y = -311$$

AS $-312 < -311$ C LIES BELOW L

C1, IYGB, PAPER I

6. a)

$$f(x) = 4x\sqrt{x} - \frac{25}{16}x^2$$
$$f(x) = 4x\left(x^{\frac{1}{2}}\right) - \frac{25}{16}x^2$$
$$f(x) = 4x^{\frac{3}{2}} - \frac{25}{16}x^2$$
$$f'(x) = 6x^{\frac{1}{2}} - \frac{25}{8}x$$

b)

$$f(4) = 4\times4\times\sqrt{4} - \frac{25}{16}\times4^2$$
$$= 16\times2 - \frac{25}{\cancel{16}}\times\cancel{16}$$
$$= 32 - 25$$
$$= 7$$
$$\therefore (4, 7)$$

$$f'(4) = 6\times4^{\frac{1}{2}} - \frac{25}{\cancel{8}_2}\times\cancel{4}$$
$$= 6\times2 - \frac{25}{2}$$
$$= 12 - \frac{25}{2}$$
$$= -\frac{1}{2}$$

$$\therefore \; y - y_0 = m(x - x_0)$$
$$y - 7 = -\frac{1}{2}(x - 4)$$
$$2y - 14 = -x + 4$$
$$x + 2y = 18$$

7.

FIRST SEQUENCE

$a = 50$

$d = 3$

SECOND SEQUENCE

$a = 200$

$d = -2$

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

$$S_n = \frac{n}{2}\left[2\times50 + (n-1)\times3\right] \qquad T_n = \frac{n}{2}\left[2\times200 + (n-1)(-2)\right]$$
$$S_n = \frac{n}{2}\left[100 + 3n - 3\right] \qquad T_n = \frac{n}{2}\left[400 + 2 - 2n\right]$$
$$S_n = \frac{n}{2}(97 + 3n) \qquad T_n = \frac{n}{2}(402 - 2n)$$

$$\frac{n}{2}(97 + 3n) > \frac{n}{2}(402 - 2n)$$
$$97 + 3n > 402 - 2n \qquad (n > 0)$$
$$5n > 305$$
$$n > 61$$

$$\therefore n = 62$$

8. a) $f(x) = 0$

$x^2 - 2mx - 5 = 0$

NOW

$b^2 - 4ac = (-2m)^2 - 4\times1(-5)$

$= 4m^2 + 20 \geqslant 20$

FOR ALL VALUES OF m

$\therefore$ ALWAYS TWO DISTINCT ROOTS AS THE DISCRIMINANT IS POSITIVE

b) COMPLETING THE SQUARE OR USE THE QUADRATIC FORMULA

$x = \frac{-(-2m) \pm \sqrt{4m^2+20}}{2}$

$x = \frac{2m \pm 2\sqrt{m^2+5}}{2}$

$x = m \pm \sqrt{m^2+5}$

OR

$x^2 - 2mx - 5 = 0$

$(x-m)^2 - m^2 - 5 = 0$

$(x-m)^2 = m^2+5$

$x - m = \pm\sqrt{m^2+5}$

$x = m \pm \sqrt{m^2+5}$

9. a) $u_{n+2} = u_{n+1} + 6u_n$ $\quad u_1 = 1 \quad u_2 = 13$

$u_3 = u_2 + 6u_1 = 13 + 6\times1 = 19$

$u_4 = u_3 + 6u_2 = 19 + 6\times13 = 19 + 78 = 97$

$u_5 = u_4 + 6u_3 = 97 + 6\times19 = 97 + 114 = 211$

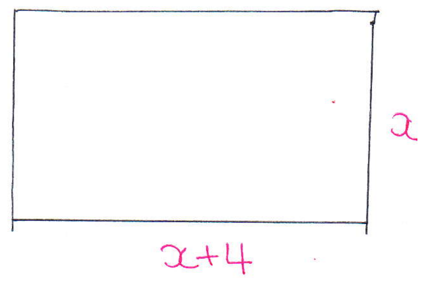
b)

1	13	19	97	211
3−2	9+4	27−8	81+16	243−32
3^1-2^1	3^2+2^2	3^3-2^3	3^4+2^4	3^5-2^5
$3^1+(-2)^1$	$3^2+(-2)^2$	$3^3+(-2)^3$	$3^4+(-2)^4$	$3^5+(-2)^5$

$\therefore \quad u_n = 3^n + (-2)^n$

C1, IYGB, PAPER I

10.



FRAME

$\circ$ PERIMETER $= (x + x + 4) \times 2$
$= 4x + 8$

$\circ$ AREA $= x(x+4)$
$= x^2 + 4x$

GLASS

THUS PERIMETER $\times$ 5 PENCE $+$ AREA $\times$ 2 PENCE $\leqslant 1000$ ← £10

$(4x+8) \times 5 + (x^2 + 4x) \times 2 \leqslant 1000$

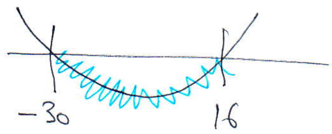
$20x + 40 + 2x^2 + 8x \leqslant 1000$

$2x^2 + 28x - 960 \leqslant 0$

$x^2 + 14x - 480 \leqslant 0$

$(x - 16)(x + 30) \leqslant 0$

C.V $= \begin{cases} 16 \\ -30 \end{cases}$



$-30 \leqslant x \leqslant 16$

$\therefore \quad 0 < x \leqslant 16$

11. a) $\frac{dy}{dx} = 3x^2 - 12x + 9$

$\Rightarrow y = \int 3x^2 - 12x + 9 \, dx$

$\Rightarrow y = x^3 - 6x^2 + 9x + C$

WHEN $x = 1 \quad y = 0$

$0 = 1 - 6 + 9 + C$

$C = -4$

$\Rightarrow y = x^3 - 6x^2 + 9x - 4$

b) $(1,0)$ IS A TOUCHING POINT

SO

$y = (x-1)^2(x-4)$

$\therefore R(4,0)$

$P(0,-4)$

CHECK $(x-4)(x^2 - 2x + 1)$

$= x^3 - 2x^2 + x$
$\quad - 4x^2 + 8x - 4$

$x^3 - 6x^2 + 9x - 4$

12. • EQUATION AD: $y+2x=6$

$y=-2x+6$

$\therefore$ GRAD IS -2

• GRAD OF "PAB" MUST BE $\frac{1}{2}$

• POINT A HAS CO.ORDS $(3,0)$ ← $y+2x=6$, $0+2x=6$, $x=3$
• POINT D HAS CO.ORDS $(0,6)$
• EQUATION "PAB"

$y-y_0=m(x-x_0)$

$y-0=\frac{1}{2}(x-3)$

$2y=x-3$

• WHEN $x=0$ $\quad 2y=-3$

$y=-\frac{3}{2}$ $\quad \therefore P\left(0,-\frac{3}{2}\right)$

• $|AP|=6+\frac{3}{2}=7.5$

AS REQUIRED