C1, IYGB, PAPER D

1. $$(\sqrt{8}+\sqrt{50})(\sqrt{24}+\sqrt{54}) = (2\sqrt{2}+5\sqrt{2})(2\sqrt{6}+3\sqrt{6})$$
$$= 7\sqrt{2} \times 5\sqrt{6} = 35\sqrt{12}$$
$$= 35 \times 2\sqrt{3} = 70\sqrt{3}$$

2. $$\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{-\frac{2}{3}} = \left(\sqrt{36}+\sqrt[4]{16}\right)^{-\frac{2}{3}} = (6+2)^{-\frac{2}{3}} = 8^{-\frac{2}{3}}$$
$$= \frac{1}{8^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$$

3. (a) $f(x) = 3x^2+12x+8$

$$\Rightarrow f(x) = 3\left[x^2+4x+\frac{8}{3}\right]$$
$$\Rightarrow f(x) = 3\left[(x+2)^2-4+\frac{8}{3}\right]$$
$$\Rightarrow f(x) = 3\left[(x+2)^2-\frac{12}{3}+\frac{8}{3}\right]$$
$$\Rightarrow f(x) = 3\left[(x+2)^2-\frac{4}{3}\right]$$
$$\Rightarrow f(x) = 3(x+2)^2-4$$

(b) MINIMUM VALUE IS -4

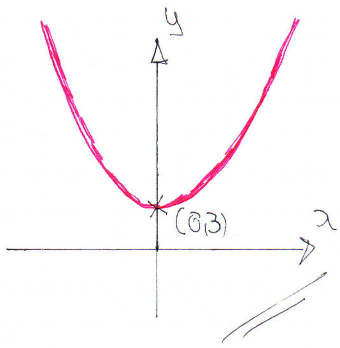
(c) $f(x)=0$

$$\Rightarrow 3(x+2)^2-4=0$$
$$\Rightarrow 3(x+2)^2 = 4$$
$$\Rightarrow (x+2)^2 = \frac{4}{3}$$
$$\Rightarrow x+2 = \pm\sqrt{\frac{4}{3}}$$
$$\Rightarrow x = -2 \pm \frac{2}{\sqrt{3}}$$

OR

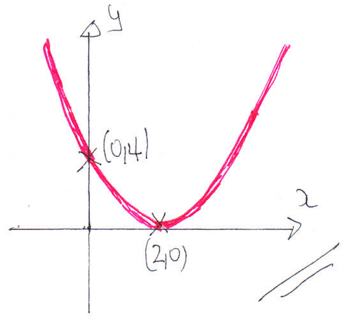
$$x = -2 \pm \frac{2}{3}\sqrt{3}$$

4.



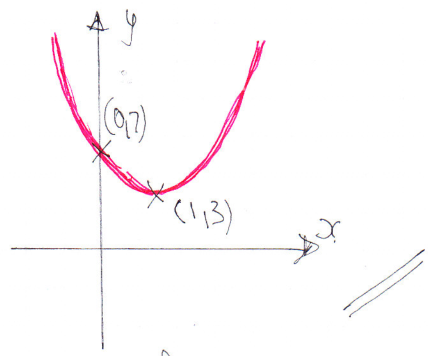
• $f(x+2)$

TRANSLATION "LEFT" BY 2 UNITS



• $f(x)-3$

TRANSLATION DOWN BY 3 UNITS



$f(2x)$

HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

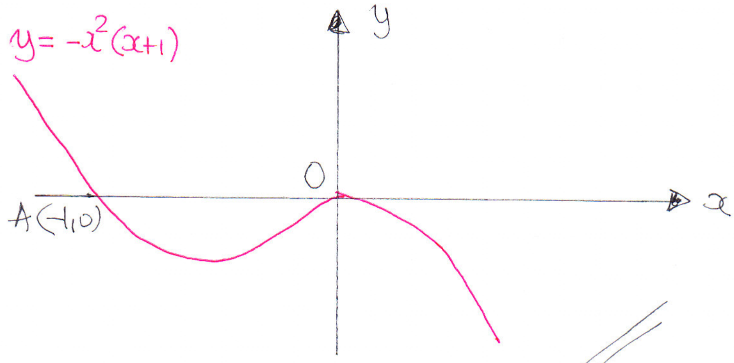
5. (a) $y = -x^2(x+1)$

$y = -x^3 - x^2$

$-x^3 \Rightarrow$

TOUCHES AT $(0,0)$

CROSSES AT $(-1,0)$



(b) $y = -x^3 - x^2$

$\frac{dy}{dx} = -3x^2 - 2x$

$\left.\frac{dy}{dx}\right|_{x=-1} = -3(-1)^2 - 2(-1) = -3+2 = -1$

EQUATION OF TANGENT $\Rightarrow$ $y - y_0 = m(x - x_0)$

$y - 0 = -1(x+1)$

$y = -x - 1$

$y + x + 1 = 0$ AS REQUIRED

6. (a) $u_{n+1} = a + \frac{1}{2}u_n$

- $u_1 = 520$
- $u_2 = a + \frac{1}{2}u_1 = a + \frac{1}{2} \times 520 = a + 260$
- $u_3 = a + \frac{1}{2}u_2 = a + \frac{1}{2}(a+260) = a + \frac{1}{2}a + 130 = \frac{3}{2}a + 130$
- $u_4 = a + \frac{1}{2}u_3 = a + \frac{1}{2}\left(\frac{3}{2}a + 130\right) = a + \frac{3}{4}a + 65 = \frac{7}{4}a + 65$

NOW $\frac{7}{4}a + 65 = 72$

$\Rightarrow \frac{7}{4}a = 7$

$\Rightarrow 7a = 28$

$\Rightarrow a = 4$

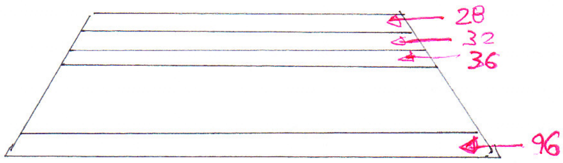
(b) $u_{n+1} = 4 + \frac{1}{2}u_n \Rightarrow$ $u_{10} = 4 + \frac{1}{2}u_9$

$9 = 4 + \frac{1}{2}u_9$

$5 = \frac{1}{2}u_9$

$u_9 = 10$

7.



$a = 28$
$d = 4$
$L = u_n = 96$

- $u_n = a + (n-1)d$

$\Rightarrow 96 = 28 + (n-1) \times 4$

$\Rightarrow 68 = 4(n-1)$

$\Rightarrow 17 = n - 1$

$\Rightarrow \boxed{n = 18}$ ← NUMBER OF ROWS

- $S_n = \frac{n}{2}[a + L]$

$\Rightarrow S_{18} = \frac{18}{2}[28 + 96]$

$\Rightarrow S_{18} = 9 \times 124$

$\Rightarrow S_{18} = 900 + 180 + 36$

$\Rightarrow S_{18} = 1116$ // AS REQUIRED

8. (a) $f'(x) = -\frac{4}{x^2}$

$\Rightarrow f'(x) = -4x^{-2}$

$\Rightarrow f(x) = \int -4x^{-2}\,dx$

$\Rightarrow f(x) = 4x^{-1} + C$

$\Rightarrow \boxed{f(x) = \frac{4}{x} + C}$

$f(1) = 2$

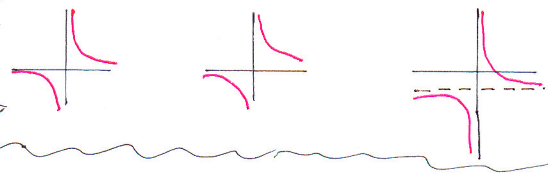
$2 = \frac{4}{1} + C$

$2 = 4 + C$

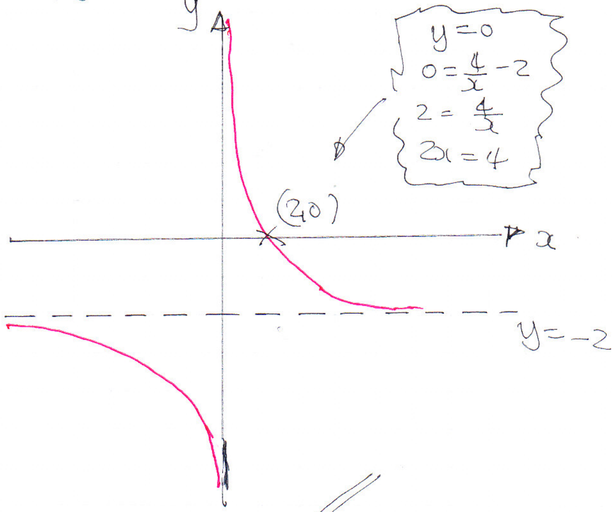
$\boxed{C = -2}$

$\therefore f(x) = \frac{4}{x} - 2$ //

(b)



THUS



$y = 0$
$0 = \frac{4}{x} - 2$
$2 = \frac{4}{x}$
$2x = 4$

//

9.

$$\left.\begin{aligned} y &= k(2x^2+1) \\ y &= x^2-2x \end{aligned}\right\} \Rightarrow \text{SOLVING SIMULTANEOUSLY}$$

$\Rightarrow k(2x^2+1) = x^2-2x$

$\Rightarrow 2kx^2+k = x^2-2x$

$\Rightarrow 2kx^2-x^2+2x+k=0$

$\Rightarrow (2k-1)x^2+2x+k=0$

BUT
CURVES TOUCH!

$\Rightarrow b^2-4ac=0$

$\Rightarrow 2^2-4(2k-1)\times k=0$

$\Rightarrow 4-4k(2k-1)=0$

$\Rightarrow 4-8k^2+4k=0$

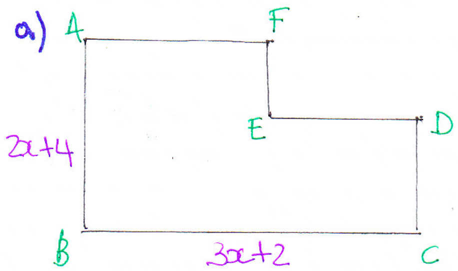
$\Rightarrow 0=8k^2-4k-4$

$\Rightarrow 2k^2-k-1=0$

$\Rightarrow (2k+1)(k-1)=0$

$\therefore k = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$

10. a)



- $P = 2(2x+4)+2(3x+2)$
  $P = 4x+8+6x+4$
  $P = 10x+12$

- $27 < P < 52$
  $27 < 10x+12 < 52$
  $15 < 10x < 40$
  $1.5 < x < 4$

b) $(2x+4)(3x+2) - 4x < 98$

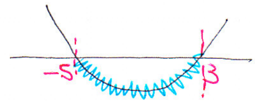
$\Rightarrow 6x^2 + 16x + 8 - 4x - 98 < 0$

$\Rightarrow 6x^2 + 12x - 90 < 0$

$\Rightarrow x^2 + 2x - 15 < 0$

$\Rightarrow (x-3)(x+5) < 0$

C.V = $\begin{cases} 3 \\ -5 \end{cases}$



$-5 < x < 3$

BUT FROM PART (a)

$1.5 < x < 4$

$\therefore \quad 1.5 < x < 3$

11. a) $y - y_0 = m(x - x_0)$

$y - 4 = \frac{1}{2}(x-3)$

$y - 4 = \frac{1}{2}x - \frac{3}{2}$

$y = \frac{1}{2}x - \frac{3}{2} + 4$

$y = \frac{1}{2}x + \frac{5}{2}$

b) IF $x = -3$

$y = \frac{1}{2}(-3) + \frac{5}{2}$

$y = -\frac{3}{2} + \frac{5}{2}$

$y = 1$

$\therefore B(-3,1)$ IS ON l

(c) $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$A(3,4)$ & $B(-3,1)$

$|AB| = \sqrt{(1-4)^2 + (-3-3)^2}$

$|AB| = \sqrt{9+36}$

$|AB| = \sqrt{45}$

OR $3\sqrt{5}$

(d) $P\left(p, \frac{1}{2}p + \frac{5}{2}\right) \quad A(3,4)$

SINCE IT LIES ON THE LINE $y = \frac{1}{2}x + \frac{5}{2}$

$\Rightarrow |AP| = \sqrt{\left(\frac{1}{2}p + \frac{5}{2} - 4\right)^2 + (p-3)^2}$

$\Rightarrow \sqrt{125} = \sqrt{\left(\frac{1}{2}p - \frac{3}{2}\right)^2 + (p-3)^2}$

$\Rightarrow 125 = \left(\frac{1}{2}p - \frac{3}{2}\right)^2 + (p-3)^2$

$\Rightarrow 125 = \frac{1}{4}p^2 - \frac{3}{2}p + \frac{9}{4} + p^2 - 6p + 9$

$\Rightarrow 500 = p^2 - 6p + 9 + 4p^2 - 24p + 36$

$\Rightarrow 500 = 5p^2 - 30p + 45$

$\Rightarrow 100 = p^2 - 6p + 9$

$\Rightarrow 0 = p^2 - 6p - 91$

$\Rightarrow 0 = (p+7)(p-13)$

$p = \begin{cases} -7 \\ 13 \end{cases}$