C1, IYGB, PAPER A

1. (a) (I) $2^{-4}+8^{-1}=\frac{1}{2^4}+\frac{1}{8}=\frac{1}{16}+\frac{1}{8}=\frac{1}{16}+\frac{2}{16}=\frac{3}{16}$ //

(II) $\left(\frac{81}{16}\right)^{\frac{3}{4}}=\left[\sqrt[4]{\frac{81}{16}}\right]^3=\left(\frac{3}{2}\right)^3=\frac{27}{8}$ //

(b) $\frac{(4xy^2)^2}{(2x)^3}=\frac{16x^2y^4}{8x^3}=2x^{-1}y^4$ // OR $\frac{2y^4}{x}$

2. (a) $\sqrt{150}-\sqrt{54}=\sqrt{25}\sqrt{6}-\sqrt{9}\sqrt{6}=5\sqrt{6}-3\sqrt{6}=2\sqrt{6}$ //

(b) $\frac{21}{\sqrt{7}}=\frac{21\sqrt{7}}{\sqrt{7}\sqrt{7}}=\frac{21\sqrt{7}}{7}=3\sqrt{7}$ //

3. (a)
$$f(x)=x^2-4x-16$$
$$=(x-2)^2-2^2-16$$
$$=(x-2)^2-4-16$$
$$=(x-2)^2-20$$
//

(b)
$$f(x)=0$$
$$x^2-4x-16=0$$
$$(x-2)^2-20=0$$
$$(x-2)^2=20$$
$$x-2=\pm\sqrt{20}$$
$$x=2\pm\sqrt{20}$$
// OR $2\pm2\sqrt{5}$

4. $5x+y=7$
$3x^2+y^2=21$ $\Rightarrow$ $\boxed{y=7-5x}$

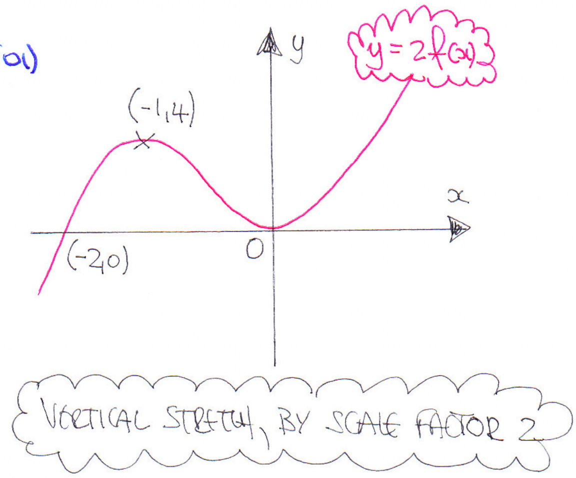
SUBSTITUTE INTO THE QUADRATIC

$$\Rightarrow 3x^2+(7-5x)^2=21$$
$$\Rightarrow 3x^2+49-70x+25x^2=21$$
$$\Rightarrow 28x^2-70x+28=0$$
$$\Rightarrow 4x^2-10x+4=0$$
$$\Rightarrow 2x^2-5x+2=0$$
$$\Rightarrow (2x-1)(x-2)=0$$

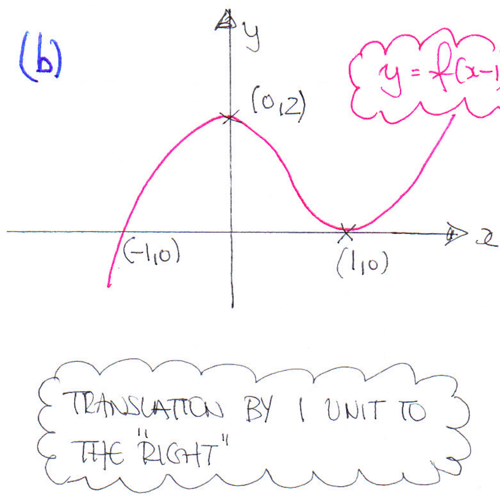
$\Rightarrow x = \begin{cases} 2 \\ \frac{1}{2} \end{cases}$ $\quad y = \begin{cases} 7-5\times2=-3 \\ 7-5\times\frac{1}{2}=7-\frac{5}{2}=\frac{14}{2}-\frac{5}{2}=\frac{9}{2} \end{cases}$

$\therefore (2,-3)$ & $\left(\frac{1}{2},\frac{9}{2}\right)$

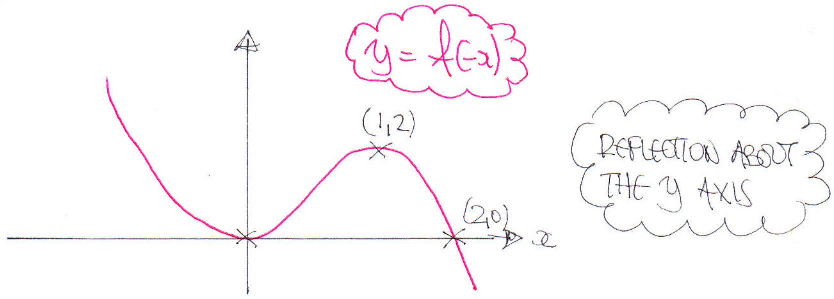
5. (a)



(b)



(c)



6. $b^2 - 4ac < 0$ (NO REAL ROOTS)

$\Rightarrow 8^2 - 4 \times (3p-2) \times p < 0$

$\Rightarrow 64 - 4p(3p-2) < 0$

$\Rightarrow 64 - 12p^2 + 8p < 0$

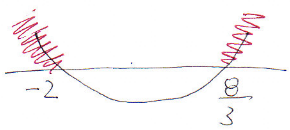
$\Rightarrow -12p^2 + 8p + 64 < 0$ (MULTIPLY BY −1 REVERSES DIRECTION)

$\Rightarrow 12p^2 - 8p - 64 > 0$

$\Rightarrow 3p^2 - 2p - 16 > 0$

$\Rightarrow (3p-8)(p+2) > 0$

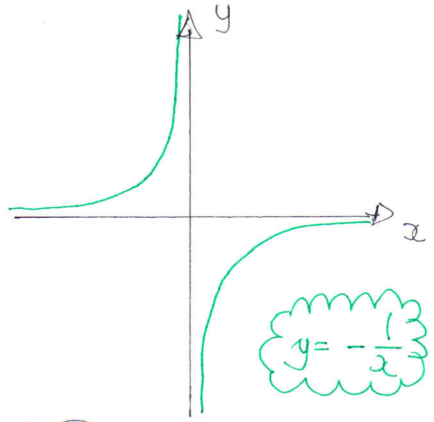
C.V = -2, $\frac{8}{3}$



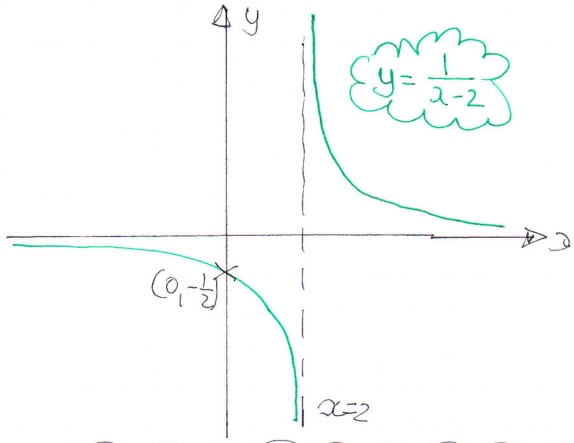
$\therefore p < -2$ OR $p > \frac{8}{3}$ //

C1, IYGB, PAPER A

7.



REFLECTION OF $y=\frac{1}{x}$ IN THE x AXIS



TRANSLATION 2 UNITS TO THE "RIGHT"

8. (a)

$$\text{GRAD } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{1-3}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$$

$$\text{GRAD } BC = \frac{y_2-y_1}{x_2-x_1} = \frac{5-1}{5-3} = \frac{4}{2} = 2$$

AS GRADIENTS ARE NEGATIVE RECIPROCALS OF EACH OTHER $AB \perp BC$

(b) $m = 2$, $B(3,1)$

$$y - y_0 = m(x - x_0)$$
$$y - 1 = 2(x-3)$$
$$y - 1 = 2x - 6$$
$$y = 2x - 5$$

(c) MIDPOINT $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+3}{2}, \frac{3+1}{2}\right) = (1,2)$

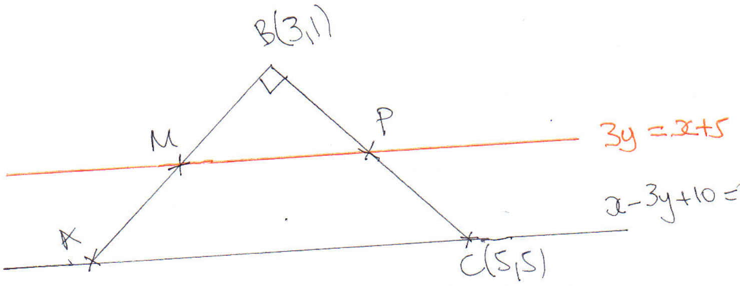
GRADIENT OF AC: $x - 3y + 10 = 0$

$$x + 10 = 3y$$
$$y = \frac{1}{3}x + \frac{10}{3}$$

"PARALLEL" TO AC $\Rightarrow$ GRADIENT $\frac{1}{3}$ & $M(1,2)$

$$y - y_0 = m(x - x_0)$$
$$y - 2 = \frac{1}{3}(x-1)$$
$$3y - 6 = x - 1$$
$$3y = x + 5$$

(d)



P IS THE MIDPOINT OF OF BC (BY SIMILAR TRIANGLES)

$$P\left(\frac{3+5}{2}, \frac{1+5}{2}\right)$$

$\therefore P(4,3)$

9. (a)

$$\left.\begin{array}{l} u_3 - u_2 = d \\ u_2 - u_1 = d \end{array}\right\} \Rightarrow u_3 - u_2 = u_2 - u_1$$

$$\Rightarrow (4k+1) - (2k+5) = (2k+5) - (k-2)$$

$$\Rightarrow 4k+1-2k-5 = 2k+5-k+2$$

$$\Rightarrow 2k - 4 = k + 7$$

$$\Rightarrow k = 11$$

AS REQUIRED

(b) $k = 11 \Rightarrow \begin{cases} u_1 = 9 \\ u_2 = 27 \\ u_3 = 45 \end{cases}$

$\therefore a = 9 \quad d = 18$

$$u_n = a + (n-1)d$$

$$u_{41} = 9 + 40 \times 18$$

$$u_{41} = 9 + 720$$

$$u_{41} = 729$$

(c)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2 \times 9 + (n-1) \times 18\right]$$

$$S_n = \frac{n}{2}\left(\cancel{18} + 18n - \cancel{18}\right)$$

$$S_n = \frac{n}{2} \times 18n$$

$$S_n = 9n^2$$

$$S_n = (3n)^2$$

I.E INDEED A SQUARE NUMBER FOR ALL n

10. (a)

$$f'(x) = \frac{8x^3 - 1}{x^2}$$

$$f'(-1) = \frac{8(-1)^3 - 1}{(-1)^2}$$

$$f'(-1) = \frac{-8-1}{1} = -9$$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = -9(x + 1)$$

$$y = -9x - 9$$

(b) $f'(x) = \frac{8x^3-1}{x^2} = \frac{8x^3}{x^2} - \frac{1}{x^2} = 8x - x^{-2}$

$\therefore f''(x) = 8 + 2x^{-3}$ OR $8 + \frac{2}{x^3}$

(c) (I) $f(x) = \int 8x - x^{-2}\,dx$

$f(x) = 4x^2 + x^{-1} + C$

$$\boxed{f(x) = 4x^2 + \frac{1}{x} + C}$$

WHEN $x=-1$ $f(x)=0$ SINCE $P(-1,0)$

$0 = 4(-1)^2 + \frac{1}{-1} + C$

$0 = 4 - 1 + C$

$C = -3$

$\therefore f(x) = 4x^2 + \frac{1}{x} - 3$

(II) $y=0$

$0 = 4x^2 + \frac{1}{x} - 3 = 0$

$0 = 4x^3 + 1 - 3x = 0$

$0 = 4x^3 - 3x + 1$

BUT A SOLUTION IS $x=1$ FROM $P(-1,0)$

$\therefore 0 = (x+1)(4x^2 + Ax + 1)$

$4x^2$

Ax^2

COMPARE x^2, IE NO x^2 WHEN MULTIPLIED OUT

$\therefore 4x^2 + Ax^2 = 0x^2$

$\therefore A = -4$

$\therefore 0 = (x+1)(4x^2 - 4x + 1)$

$0 = (x+1)(2x-1)^2$ ← EXPECTED TO BE A BRACKET SQUARED AS IT TOUCHES THE x AXIS

$x = -1 \leftarrow P$

$x = \frac{1}{2} \leftarrow Q$

$\therefore Q(\frac{1}{2}, 0)$