INTEGRATION BY PARTS

(student version)

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Question 1

Carry out the following integrations:

1. $\int x\mathrm{e}^{2x}\ dx$

2. $\int 3x\cos 2x\ dx$

3. $\int x\sin 4x\ dx$

4. $\int -2x\sin 5x\ dx$

5. $\int (1-2x)\mathrm{e}^{-x}\ dx$

6. $\int x^2\,\mathrm{e}^{-3x}\ dx$

7. $\int 16x^3\ln x\ dx$

8. $\int \ln x\ dx$

9. $\int x\cos\left(\frac{1}{2}x\right)\ dx$

10. $\int (3x-1)\sin(3x-1)\ dx$

Question 2

Carry out the following integrations:

1. $\int 6x\mathrm{e}^{3x}\ dx$

2. $\int 12x\cos 3x\ dx$

3. $\int x\sin 6x\ dx$

4. $\int -x\sin 2x\ dx$

5. $\int (2-x)\mathrm{e}^{-3x}\ dx$

6. $\int x^2\mathrm{e}^{4x}\ dx$

7. $\int x^2\mathrm{e}^{-\frac{1}{2}x}\ dx$

8. $\int 25x^4\ln x\ dx$

9. $\int 24x\cos\left(\frac{2}{3}x\right)\ dx$

10. $\int x^2\sin(1-x)\ dx$

Question 3

Carry out the following integrations:

1. $\int \frac{1}{2} x e^{4x} \, dx$

2. $\int 5x \sin 4x \, dx$

3. $\int (2x+1) \cos 2x \, dx$

4. $\int -3x \cos 4x \, dx$

5. $\int x^2 e^{-2x} \, dx$

6. $\int x^2 \sin 5x \, dx$

7. $\int x^2 \cos \frac{1}{3} x \, dx$

8. $\int \frac{1}{2} x^3 \ln x \, dx$

9. $\int x \ln 3x \, dx$

10. $\int \frac{\ln x}{x^3} \, dx$

Question 4

Carry out the following integrations:

1. $\int x e^{5x} \, dx$

2. $\int 2x \cos 3x \, dx$

3. $\int x \sin 3x \, dx$

4. $\int x \sin 4x \, dx$

5. $\int 2 \ln x \, dx$

6. $\int x^2 \ln x \, dx$

7. $\int x \sin\left(\frac{1}{2}x\right) dx$

8. $\int x \sin(2x-1) \, dx$

9. $\int \frac{\ln x}{x^2} \, dx$

10. $\int x \sec^2 x \, dx$

Question 5

Carry out the following integrations:

1. $\int x^2 \sin x \, dx$

2. $\int x^3 \ln x \, dx$

3. $\int \sin x \ln(\sec x) \, dx$

4. $\int x \cos 5x \, dx$

5. $\int x^2 \sin 3x \, dx$

6. $\int 4x \mathrm{e}^{-\frac{2}{3}x} \, dx$

7. $\int x^2 \cos\left(\frac{1}{3}x\right) dx$

8. $\int 2x^2 \sec^2 x \, dx$

9. $\int x^2 \mathrm{e}^{\frac{1}{2}x} \, dx$

10. $\int x \sec x \tan x \, dx$

Question 6

Carry out the following integrations:

1. $\int x^2 \mathrm{e}^{-\frac{1}{4}x} \, dx$

2. $\int x^2 \mathrm{e}^{-x} \, dx$

3. $\int \mathrm{e}^x \cos x \, dx$

4. $\int (\ln x)^2 \, dx$

5. $\int \mathrm{e}^x \sin x \, dx$

6. $\int (x^3 + 5x^2 - 2)\mathrm{e}^{2x} \, dx$

7. $\int x \cos^2 x \, dx$

8. $\int x \ln 2x^3 \, dx$

Question 7

Carry out the following integrations, to the answer given:

1. $\int_0^{\ln 2} x\mathrm{e}^{2x} \; dx = \ln 4 - \frac{3}{4}$

2. $\int_0^{\frac{\pi}{3}} 6x\sin 3x \; dx = \frac{2\pi}{3}$

3. $\int_0^{\frac{\pi}{2}} x^2\cos x \; dx = \frac{1}{4}\left(\pi^2 - 8\right)$

4. $\int_1^{\mathrm{e}} x\ln x \; dx = \frac{1}{4}\left(\mathrm{e}^2 + 1\right)$

5. $\int_0^{1} 4x\mathrm{e}^{3x} \; dx = \frac{4}{9}\left(2\mathrm{e}^3 + 1\right)$

6. $\int_0^{\frac{\pi}{4}} x\sin 4x \; dx = \frac{\pi}{16}$

7. $\int_1^{2} x^3\ln x \; dx = 4\ln 2 - \frac{15}{16}$

8. $\int_0^{1} x\mathrm{e}^{-2x} \; dx = \frac{1}{4}\left(1 - 3\mathrm{e}^{-2}\right)$

9. $\int_0^{\frac{\pi}{4}} 12x\cos 2x \; dx = \frac{3}{2}\left(\pi - 2\right)$

10. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4x\sin 2x \; dx = \pi - 1$

Question 8

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{3}} x\sin 3x \ dx = \frac{\pi}{9}$

2. $\displaystyle\int_0^{\frac{\pi}{4}} 2x\cos 4x \ dx = -\frac{1}{4}$

3. $\displaystyle\int_0^{\ln 2} 4x\mathrm{e}^{-x} \ dx = 2-\ln 4$

4. $\displaystyle\int_1^{\mathrm{e}} \ln x \ dx = 1$

5. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin 2x \ dx = \frac{\pi}{4}$

6. $\displaystyle\int_0^{\ln 4} x\mathrm{e}^{\frac{1}{2}x} \ dx = 8\ln 2-4$

7. $\displaystyle\int_0^{\pi} x\cos\left(\frac{1}{4}x\right) \ dx = 2\sqrt{2}(\pi+4)-16$

8. $\displaystyle\int_0^{1} (2x+1)\mathrm{e}^{2x} \ dx = \mathrm{e}^2$

9. $\displaystyle\int_{\frac{1}{\mathrm{e}}}^{1} x\ln x \ dx = \frac{1}{4}\left(\frac{3}{\mathrm{e}^2}-1\right)$

10. $\displaystyle\int_{-1}^{0} 3\ln(2x+3) \ dx = \tfrac{3}{2}(\ln 27-2)$ **REQUIRES ADDITIONAL TECHNIQUES**

Question 9

Carry out the following integrations, to the answer given:

1. $\displaystyle\int_0^{\frac{\pi}{4}} x\sec^2 x \ \ dx = \frac{1}{4}(\pi - \ln 4)$

2. $\displaystyle\int_1^2 \frac{\ln x}{x} \ \ dx = \frac{1}{2}(\ln 2)^2$

3. $\displaystyle\int_0^{\frac{\pi}{2}} x\sin^2 x \ \ dx = \frac{1}{16}(\pi^2 + 4)$