

Created by T. Madas

# **3-DIMENSIONAL SKETCHING SURFACES and CURVES**

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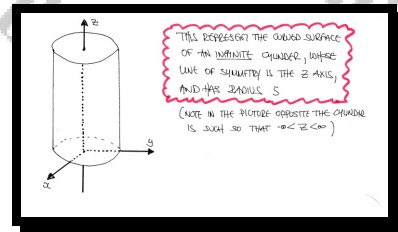
**Question 1**

A surface  $S$  is given by the Cartesian equation

$$x^2 + y^2 = 25.$$

Draw a sketch of  $S$ , and describe it geometrically.

, cylinder



**Question 2**

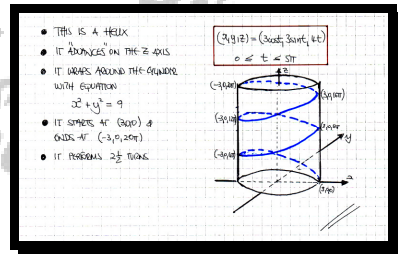
A curve  $C$  is defined parametrically

$$(x, y, z) = (3 \cos t, 3 \sin t, 4t), \quad 0 \leq t \leq 5\pi.$$

where  $t$  is a parameter.

Sketch the graph of  $C$ .

, graph



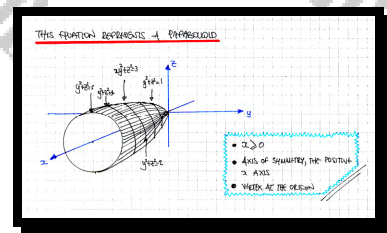
**Question 3**

The surface  $S$  has Cartesian equation

$$x = y^2 + z^2.$$

Sketch the graph of  $S$ .

, graph



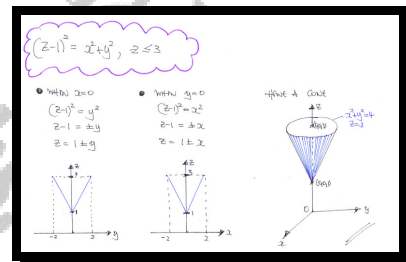
**Question 4**

The surface  $S$  has Cartesian equation

$$(z-1)^2 = x^2 + y^2, 1 \leq z \leq 3.$$

Sketch the graph of  $S$ .

graph



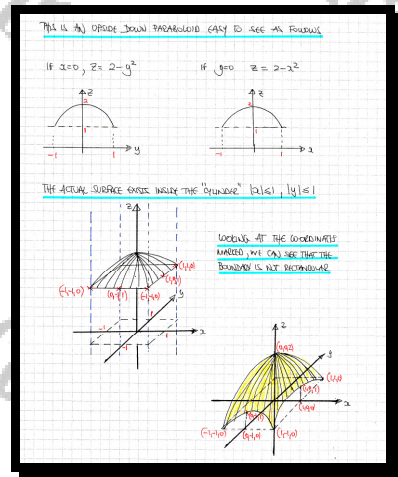
**Question 5**

The surface  $S$  has Cartesian equation

$$z = 2 - x^2 - y^2, \quad |x| \leq 1, \quad |y| \leq 1.$$

Sketch the graph of  $S$ .

graph



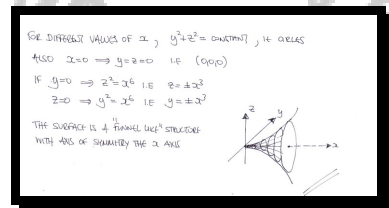
**Question 6**

A surface  $S$  has Cartesian equation

$$y^2 + z^2 = x^6, \quad 0 \leq x \leq \sqrt[4]{\frac{5}{3}}.$$

Sketch the graph of  $S$ .

graph



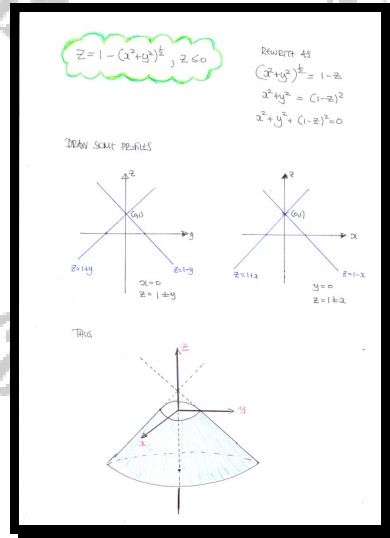
Question 7

A surface  $S$  is has Cartesian equation

$$z = 1 - \sqrt{x^2 + y^2}, \quad z \leq 0.$$

Sketch the graph of  $S$ .

graph



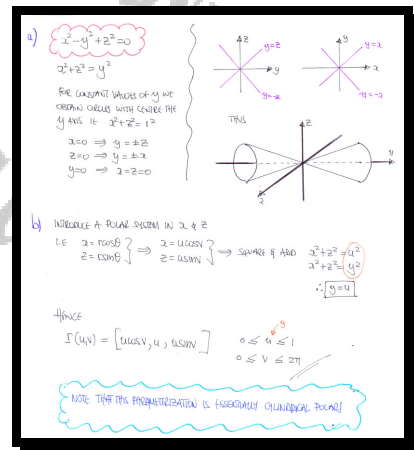
**Question 8**

A surface  $S$  has Cartesian equation

$$x^2 - y^2 + z^2 = 0.$$

- Sketch the graph of  $S$ .
- Find a parameterization for the equation of  $S$ , for  $0 \leq y \leq 1$ , in terms of the parameters  $u$  and  $v$ .

$$\mathbf{r}(u, v) = \langle u \cos v, u, u \sin v \rangle \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 2\pi$$



## Question 9

A surface  $S$  has Cartesian equation

$$x^2 + y^2 + z^2 = 2x.$$

Describe fully the graph of  $S$ , and hence find a parameterization for its equation in terms of the parameters  $u$  and  $v$ .

$$\mathbf{r}(u, v) = \langle 1 + \sin u \cos v, \sin u \sin v, \cos u \rangle, \quad 0 \leq u \leq \pi, \quad 0 \leq v \leq 2\pi$$

Handwritten solution for Question 9:

d)  $x^2 + y^2 + z^2 = 2x$   
 $x^2 - 2x + y^2 + z^2 = 0$   
 $(x-1)^2 + y^2 + z^2 = 1$   
 i.e. A sphere centre at  $(1, 0, 0)$  radius 1

$x-1 = 1 \sin u \cos v$   
 $y = 1 \sin u \sin v$   
 $z = 1 \cos u$

i.e. spherical coordinates with  $r=1$

$x = 1 + \sin u \cos v$   
 $y = \sin u \sin v$   
 $z = \cos u$

Hence  $\mathbf{r}(u, v) = [1 + \sin u \cos v, \sin u \sin v, \cos u]$   $0 \leq u \leq \pi$   
 $0 \leq v \leq 2\pi$

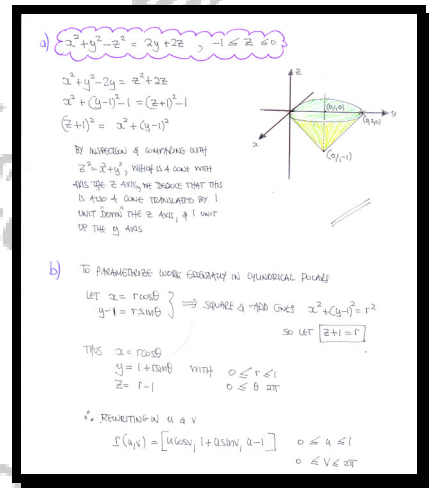
**Question 10**

A surface  $S$  has Cartesian equation

$$x^2 + y^2 - z^2 = 2y + 2z, \quad -1 \leq z \leq 0.$$

- a) Sketch the graph of  $S$ .
- b) Find a parameterization for the equation of  $S$ , in terms of the parameters  $u$  and  $v$ .

$$\mathbf{r}(u, v) = \langle u \cos v, 1 + u \sin v, u - 1 \rangle, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 2\pi$$





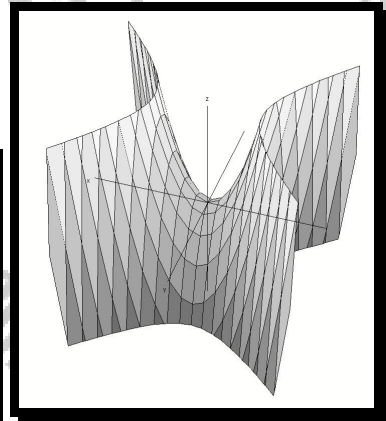
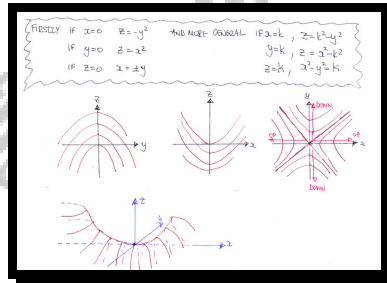
Question 11

A surface  $S$  has Cartesian equation

$$z = x^2 - y^2.$$

Sketch contour profiles of  $S$ , parallel to the  $y$ - $z$  plane, parallel to the  $x$ - $z$  plane, and parallel to the  $x$ - $y$  plane.

graph



**Question 12**

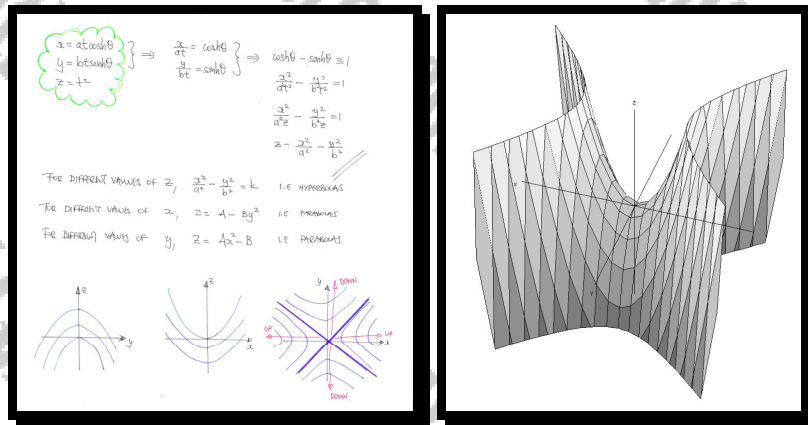
A surface  $S$  is given parametrically by

$$x = at \cosh \theta, \quad y = bt \sinh \theta, \quad z = t^2,$$

where  $t$  and  $\theta$  are real parameters, and  $a$  and  $b$  are non zero constants.

- Find a Cartesian equation for  $S$ .
- Sketch profiles of  $S$  parallel to the  $y$ - $z$  plane, parallel to the  $x$ - $z$  plane, and parallel to the  $x$ - $y$  plane,

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$



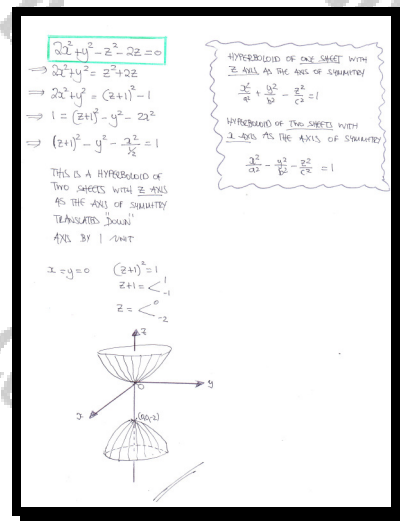
Question 13

A surface  $S$  is has Cartesian equation

$$2x^2 + y^2 - z^2 - 2z = 0.$$

Sketch the graph of  $S$ .

graph



**Question 14**

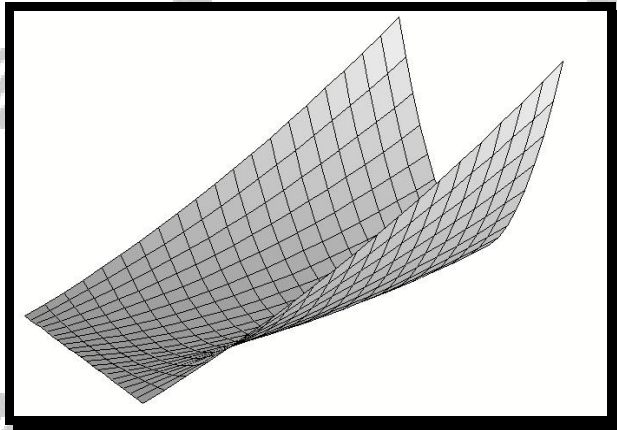
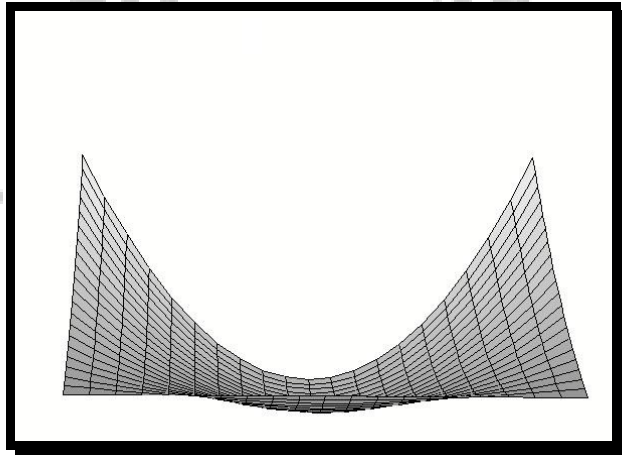
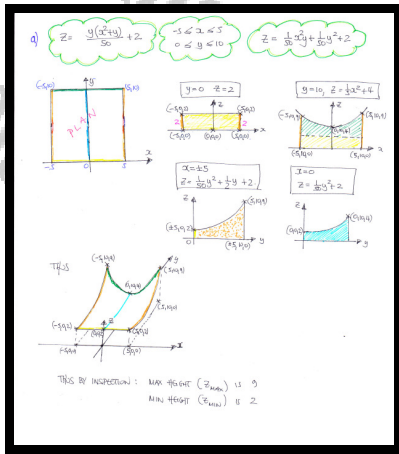
A building whose plan measures 10 m long by 10 m wide has vertical walls and a suspended fabric roof. The height,  $z$  m, of the roof above the ground is modelled in three dimensional Cartesian space by the equation

$$z = \frac{y(x^2 + y)}{50} + 2, \quad -5 \leq x \leq 5, \quad 0 \leq y \leq 10.$$

Sketch the graph of the surface which models the roof of the building.

Give a brief description of its shape including its key features with relevant coordinates such as the maximum height and minimum height of the roof.

graph



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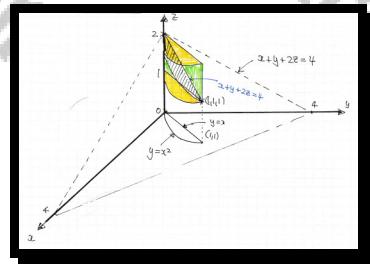
**Question 15**

A solid is bounded by the surfaces

$$y = x^2, \quad y = x, \quad z = 1 \quad \text{and} \quad z = 2.$$

Sketch in the same set of axes the solid and the plane with equation  $x + y + 2z = 4$ .

graph



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**Question 16**

The curve  $C$  is the intersection of the surfaces with respective Cartesian equations

$$x^2 + y^2 + z^2 = 1, \quad z \geq 0 \quad \text{and} \quad x^2 + y^2 = x, \quad z \geq 0.$$

Find a suitable clockwise parameterisation for  $C$ , in the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + tk,$$

fully defining the functions  $f$  and  $g$ , and the range of the parameter  $t$  in a suitably labelled graph or diagram.

graph

