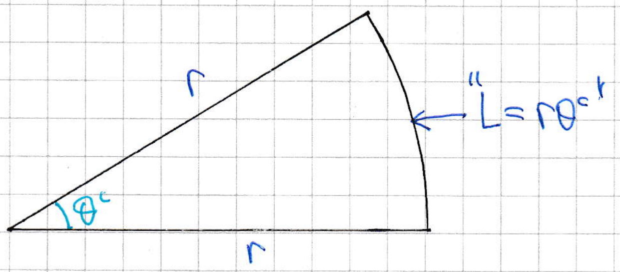
IYGB - SYNOPTIC PAPER S - QUESTION 1



SETTING UP A SIMPLE EQUATION

$$L = \frac{2}{9} \times \text{PERIMETER}$$

$$L = \frac{2}{9} \times (2r + L)$$

$$9L = 4r + 2L$$

$$7L = 4r$$

$$7\cancel{r}\theta = 4\cancel{r}$$

$$\theta = \frac{4}{7}$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER S - QUESTION 2

a) USING THE FORMULA GIVEN $u_k = 15625 \times 1.25^{-k}$

$$u_1 = 15625 \times 1.25^{-1} = 12500$$
$$u_2 = 15625 \times 1.25^{-2} = 10000$$
$$u_3 = 15625 \times 1.25^{-3} = 8000$$

b) USING THE STANDARD FORMULA WITH $a = 12500$, $r = 0.8$

$$S_\infty = \frac{a}{1-r} = \frac{12500}{1-0.8} = \frac{12500}{0.2} = 62500$$

c) USING $S_n = \frac{a(1-r^n)}{1-r}$ WITH $n = 10$

$$\sum_{k=1}^{10} u_k = u_1 + u_2 + u_3 + \dots + u_{10}$$
$$= S_{10}$$
$$= \frac{12500(1-0.8^{10})}{1-0.8}$$
$$= 55789.1136\dots$$
$$\simeq 55789$$

IYGB – SYNOPTIC PAPER S – QUESTION 3

a) OBTAINING THE x & y INTERCEPTS OF EACH LINE

- $l_1: 2x + y = 10$

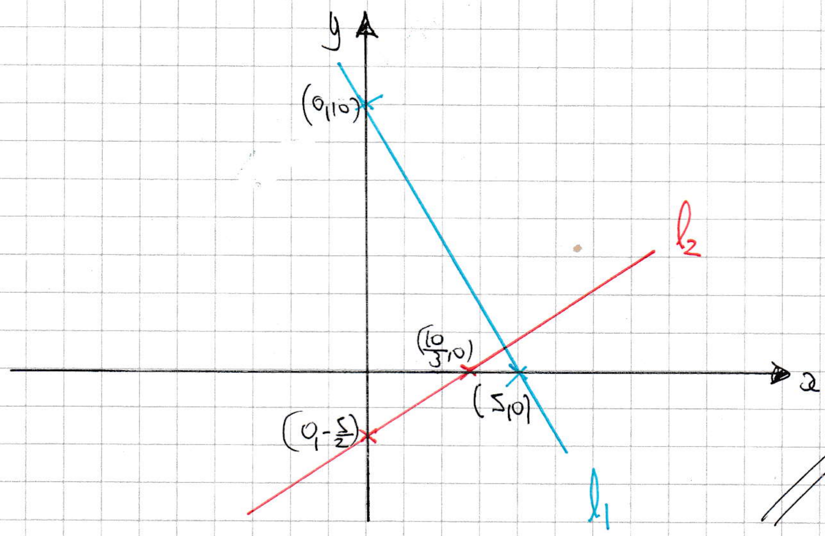
$x=0, y=10 \quad (0,10)$

$y=0, x=5 \quad (5,0)$

- $l_2: 3x - 4y = 10$

$x=0, y=-\frac{5}{2} \quad (0,-\frac{5}{2})$

$y=0, x=\frac{10}{3} \quad (\frac{10}{3},0)$



b) SOLVING SIMULTANEOUSLY – BY SUBSTITUTION

$$\left.\begin{array}{l} 2x + y = 10 \\ 3x - 4y = 10 \end{array}\right\} \Rightarrow \boxed{y = 10 - 2x}$$

$\Rightarrow 3x - 4(10-2x) = 10$

$\Rightarrow 3x - 40 + 8x = 10$

$\Rightarrow 11x = 50$

$\Rightarrow x = \frac{50}{11}$

AND USING $y = 10 - 2x$

$\Rightarrow y = 10 - 2 \times \frac{50}{11}$

$\Rightarrow y = 10 - \frac{100}{11}$

$\Rightarrow y = \frac{110-100}{11}$

$\Rightarrow y = \frac{10}{11}$

$\therefore P\left(\frac{50}{11}, \frac{10}{11}\right)$

IYGB - SYNOPTIC PAPER S - QUESTION 4

a) FOLLOWING THE USUAL METHODOLOGY

$$\Rightarrow f(x) = \frac{2x-3}{x-2}$$

$$\Rightarrow y = \frac{2x-3}{x-2}$$

$$\Rightarrow y(x-2) = 2x-3$$

$$\Rightarrow yx - 2y = 2x - 3$$

$$\Rightarrow yx - 2x = 2y - 3$$

$$\Rightarrow x(y-2) = 2y-3$$

$$\Rightarrow x = \frac{2y-3}{y-2}$$

$$\therefore f^{-1}(x) = \frac{2x-3}{x-2}$$

b) AS $f(x)$ IS SELF INVERSE, IE $f(x) = f^{-1}(x)$ THEN WE HAVE

$$\Rightarrow f(f^{-1}(x)) = x$$

$$\Rightarrow f(f(x)) = x$$

$$\Rightarrow f(f(k+2)) = k+2$$

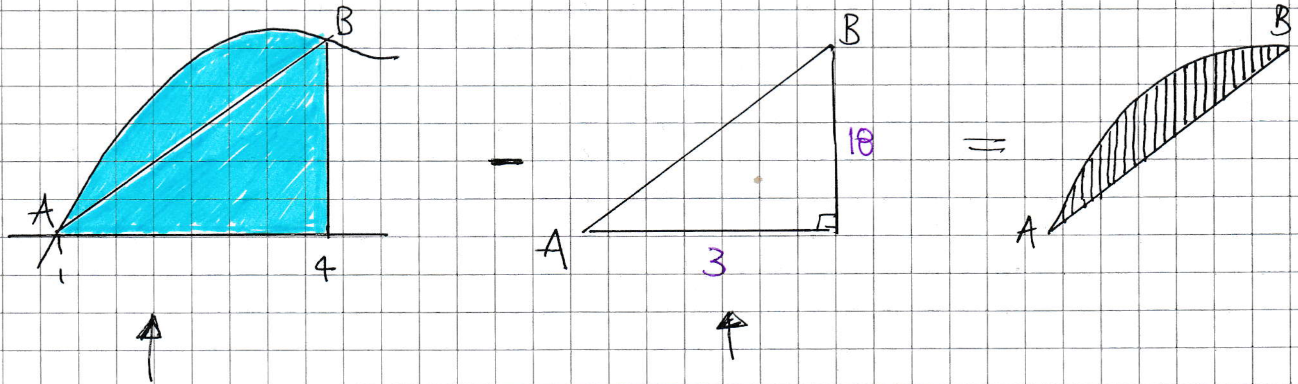
IYGB - SYNOPTIC PAPER S - QUESTION 5

START BY COMPUTING THE y CO-ORDS OF A & B

$y(1) = 1 - 12 + 45 - 34 = 0$ IE $A(1,0)$

$y(4) = 64 - 192 + 180 - 34 = 18$ IE $B(4,18)$

LOOKING AT THE DIAGRAM BELOW



$$\int_1^4 x^3 - 12x^2 + 45x - 34 \, dx$$

$\frac{1}{2} \times 3 \times 18 = 27$

$$= \left[\frac{1}{4}x^4 - 4x^3 + \frac{45}{2}x^2 - 34x\right]_1^4$$

$$= (64 - 256 + 360 + 136) - \left(\frac{1}{4} - 4 + \frac{45}{2} - 34\right)$$

$$= \frac{189}{4} = 47.25$$

$\therefore$ REQUIRED AREA $= 47.25 - 27 = 20.25 = \frac{81}{4}$ // AS REQUIRED

IYGB – SYN PAPER S – QUESTION 6

a) $f(x) = 2x^3 - 9x^2 - 11x + 30$

$f(5) = 2\times5^3 - 9\times5^2 - 11\times5 + 30$

$= 250 - 225 - 55 + 30 = 280 - 280 = 0$

INDEED $(x-5)$ IS A FACTOR

∴ BY LONG DIVISION / MANIPULATION

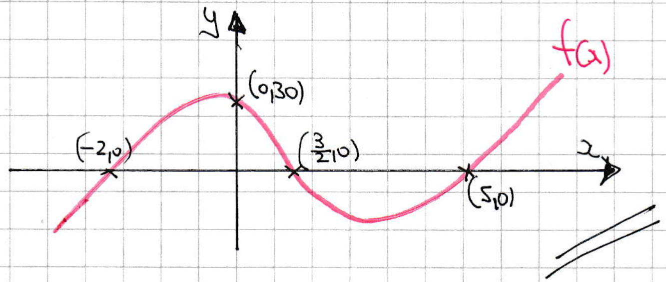
$f(x) = 2x^3 - 9x^2 - 11x + 30$

$= 2x^2(x-5) + x(x-5) - 6(x-5)$

$= (x-5)(2x^2 + x - 6)$

$= (x-5)(2x-3)(x+2)$

b)



$+2x^3 \ldots \Rightarrow \sim$

$x=0, y=30 \Rightarrow (0,30)$

$y=0, x = 5, \frac{3}{2}, -2 \Rightarrow (5,0), (\frac{3}{2},0), (-2,0)$

c) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 7x + 30 \\ y = 2x^3 - 9x^2 - 11x + 30 \end{array}\right\} \Rightarrow 2x^3 - 9x^2 - 11x + \cancel{30} = 7x + \cancel{30}$$

$\Rightarrow 2x^3 - 9x^2 - 18x = 0$

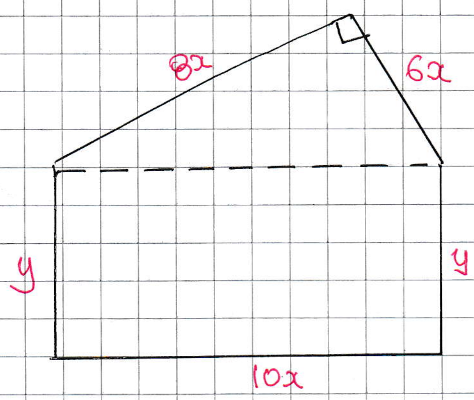
$\Rightarrow x(2x^2 - 9x - 18) = 0$

$\Rightarrow x(2x+3)(x-6)$

$\Rightarrow x = 0, -\frac{3}{2}, 6$

IYGB - SYNOPTIC PAPER S - QUESTION 7

a)



CONSTRAINT

$P = 120$

$2y + 10x + 8x + 6x = 120$

$2y + 24x = 120$

$y + 12x = 60$

$y = 60 - 12x$

"MAIN EQUATION"

$\text{AREA} = A = 10xy + \frac{1}{2}(8x)(6x)$

$A = 10xy + 24x^2$

$A = 10x(60 - 12x) + 24x^2$

$A = 600x - 120x + 24x^2$

$A = 600x - 96x^2$ // AS REQUIRED

b) DIFFERENTIATE W.R.T x & SOLVE FOR ZERO

$\frac{dA}{dx} = 600 - 192x$

$0 = 600 - 192x$

$192x = 600$

$x = \frac{25}{8} = 3.125$

$\therefore A_{MAX} = 600(3.125) - 96(3.125)^2 = 937.5$

JUSTIFYING IT IS A MAX

$\frac{d^2A}{dx^2} = -192$

$\left.\frac{d^2A}{dx^2}\right|_{x=3.125} = -192 < 0$ INDEED A MAX //

IYGB - SYNOPTIC PAPER S - QUESTION 8

STARTING WITH A STANDARD EXPONENTIALLY DECAYING MODEL

$$\Longrightarrow M = m_0 e^{-kt} \qquad k > 0 \qquad (m_0 = \text{INITIAL MASS})$$

$$\Longrightarrow 10.24 = 12 e^{-k \times 30}$$

$$\Longrightarrow e^{-30k} = \frac{64}{75}$$

$$\Longrightarrow e^{30k} = \frac{75}{64}$$

$$\Longrightarrow 30k = \ln \frac{75}{64}$$

$$\Longrightarrow \boxed{k = \frac{1}{30} \ln \frac{75}{64}}$$

NOW WHEN $t = T$, $M = \frac{1}{2} m_0 = 6$

$$\Longrightarrow M = 12 e^{-\left(\frac{1}{30} \ln \frac{75}{64}\right) t}$$

$$\Longrightarrow 6 = 12 e^{-\left(\frac{1}{30} \ln \frac{75}{64}\right) T}$$

$$\Longrightarrow \frac{1}{2} = e^{-\left(\frac{1}{30} \ln \frac{75}{64}\right) T}$$

$$\Longrightarrow 2 = e^{\left(\frac{1}{30} \ln \frac{75}{64}\right) T}$$

$$\Longrightarrow \ln 2 = \left(\frac{1}{30} \ln \frac{75}{64}\right) T$$

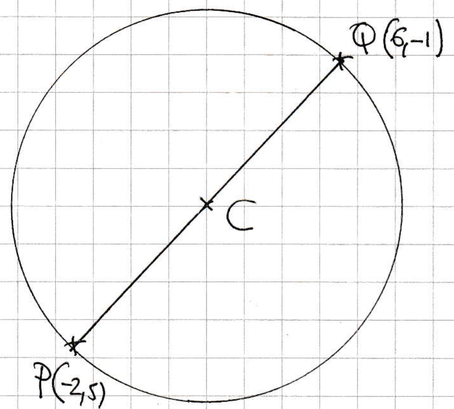
$$\Longrightarrow T = \frac{30 \ln 2}{\ln 75/64}$$

$$\Longrightarrow T = 131.108187\ldots$$

$\therefore$ HALF LIFE $\simeq$ 131 days

IYGB - SYNOPTIC PAPER S - QUESTION 9

a) **LOOKING AT THE DIAGRAM**

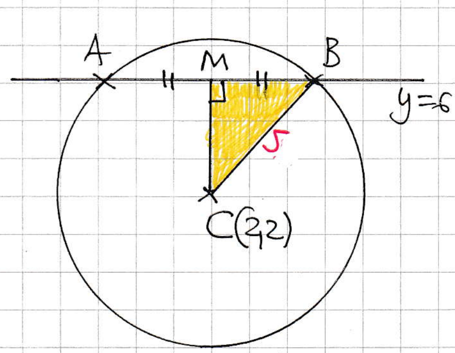


- THE CENTRE C MUST BE THE MIDPOINT OF PQ.
- $C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = C\left(\frac{6-2}{2}, \frac{5-1}{2}\right)$

  $= C(2,2)$
- RADIUS $= |PC|$ OR $|PQ|$
- $r = d = \sqrt{[6-2]^2+[-1-2]^2} = \sqrt{16+9} = 5$
- **EQUATION IS GIVEN BY**

$$(x-2)^2+(y-2)^2 = 25$$

b) **LOOKING AT THE DIAGRAM OPPOSITE**



- M IS THE MIDPOINT OF AB
- M(2,6) BY INSPECTION (CIRCLE THEOREM)
- $|MC| = 4$ (BY INSPECTION OF THE y CO-ORDS)

BY PYTHAGORAS

$$|MB|^2 + |MC|^2 = |BC|^2$$
$$|MB|^2 + 4^2 = 5^2$$
$$|MB|^2 + 16 = 25$$
$$|MB|^2 = 9$$
$$|MB| = 3$$

$$\therefore AB = 2|MB| = 6$$

IYGB - SYN PAPER S - QUESTION 10

START WITH THE SUGGESTED SIMPLIFICATION

$$\begin{aligned}(\tan x + \cot x)\sin 2x &= \tan x \sin 2x + \cot x \sin 2x\\ &= \frac{\sin x}{\cancel{\cos x}} \times 2\sin x\cos x + \frac{\cos x}{\cancel{\sin x}} \times 2\sin x\cos x\\ &= 2\sin^2 x + 2\cos^2 x\\ &= 2(\sin^2 x + \cos^2 x)\\ &= 2\end{aligned}$$

$$\therefore (\tan x + \cot x)\sin 2x \equiv 2$$

NOW SPLIT THE EXPRESSION AS FOLLOWS, NOTING $\sin\frac{5\pi}{6} = \sin\frac{\pi}{6}$

$$\left(\tan\frac{\pi}{8} + \cot\frac{\pi}{8}\right)\sin\frac{\pi}{4} = 2$$
$$\left(\tan\frac{\pi}{8} + \cot\frac{\pi}{8}\right) \times \frac{1}{\sqrt{2}} = 2$$
$$\tan\frac{\pi}{8} + \cot\frac{\pi}{8} = 2\sqrt{2}$$

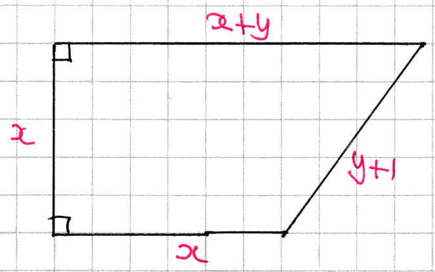
$$\left(\tan\frac{5\pi}{12} + \cot\frac{5\pi}{12}\right)\sin\frac{5\pi}{6} = 2$$
$$\left(\tan\frac{5\pi}{12} + \cot\frac{5\pi}{12}\right)\sin\frac{\pi}{6} = 2$$
$$\left(\tan\frac{5\pi}{12} + \cot\frac{5\pi}{12}\right) \times \frac{1}{2} = 2$$
$$\tan\frac{5\pi}{12} + \cot\frac{5\pi}{12} = 4$$

$$\therefore \tan\frac{\pi}{8} + \tan\frac{5\pi}{12} + \cot\frac{\pi}{8} + \cot\frac{5\pi}{12} = 4 + 2\sqrt{2}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER S - QUESTION 11

SETTING UP TWO EQUATIONS



- AREA = 30

$\Rightarrow \frac{(x+y)+x}{2} \times x = 30$

$\Rightarrow \frac{(2x+y)x}{2} = 30$

$\Rightarrow (2x+y)x = 60$

$\Rightarrow 2x^2+xy = 60$

$\Rightarrow 4x^2+2xy = 120$

- PERIMETER = 27

$\Rightarrow x+x+x+y+y+1=27$

$\Rightarrow 3x+2y=26$

$\Rightarrow 2y=26-3x$

$\Rightarrow 2xy=26x-3x^2$

SOLVING SIMULTANEOUSLY BY SUBSTITUTION

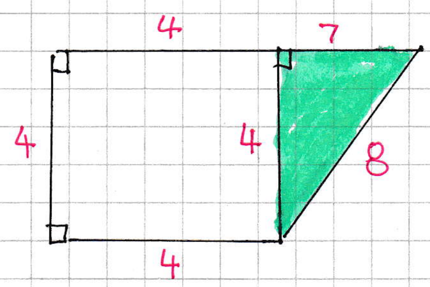
$\Rightarrow 4x^2+(26x-3x^2)=120$

$\Rightarrow x^2+26x-120=0$

$\Rightarrow (x-4)(x+30)=0$

$\Rightarrow x = \cancel{-30}$ or 4 & $y=\frac{26-3x}{2}=7$ $\therefore$ $x=4$ & $y=7$

BUT THERE IS A CONTRADICTION WITH THESE VALUES



$4^2+7^2 = 16+49 = 65 \neq 8^2$

$\therefore$ THIS TRAPEZIUM DOES NOT EXIST

IYGB - SYNOPTIC PAPER S - QUESTION 12

WORK AS FOLLOWS

$$x^2 + 2y^2 < 3xy$$
$$x^2 - 3xy + 2y^2 < 0$$
$$(x - y)(x - 2y) < 0$$

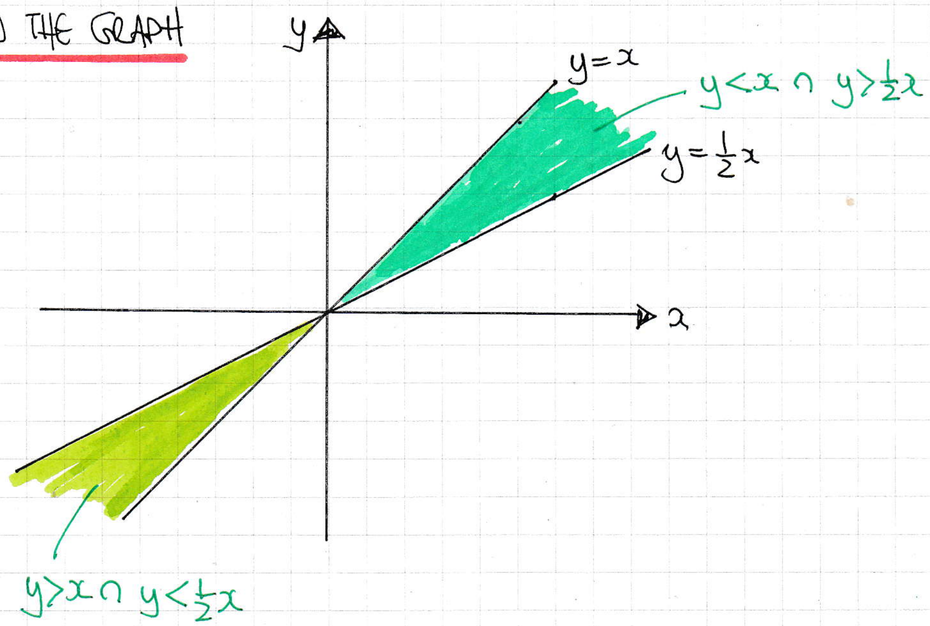
EITHER $\quad x - y > 0 \quad$ AND $\quad x - 2y < 0$
$\qquad\qquad -y > -x \quad$ AND $\quad -2y < -x$
$\qquad\qquad y < x \quad$ AND $\quad y > \frac{1}{2}x$

OR $\quad x - y < 0 \quad$ AND $\quad x - 2y > 0$
$\qquad\qquad y > x \quad$ AND $\quad y < \frac{1}{2}x$

$\Rightarrow y < x$ AND $y > \frac{1}{2}x$ OR $y > x$ AND $y < \frac{1}{2}x$

$$\left(y < x \cap y > \tfrac{1}{2}x\right) \cup \left(y > x \cap y < \tfrac{1}{2}x\right)$$

AND THE GRAPH



IYGB - SYNOPTIC PAPER S - QUESTION 13

WE ARE GIVEN THAT

$$\frac{dx}{dt} = kx \qquad \text{AND} \qquad 2(x^2+y^2) = 5xy$$

FIRSTLY DIFFERENTIATE THE IMPLICIT RELATIONSHIP W.R.T x.

$$\Longrightarrow 2\left(2x + 2y\frac{dy}{dx}\right) = 5y + 5x\frac{dy}{dx}$$

$$\Longrightarrow 4x + 4y\frac{dy}{dx} = 5y + 5x\frac{dy}{dx}$$

$$\Longrightarrow (4y - 5x)\frac{dy}{dx} = 5y - 4x$$

$$\Longrightarrow \frac{dy}{dx} = \frac{5y-4x}{4y-5x}$$

NOW WE HAVE

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{5y-4x}{4y-5x} \times kx$$

NOW WHEN $x=2$

$$\Longrightarrow 2(2^2+y^2) = 5 \times 2 \times y$$

$$\Longrightarrow 2(4+y^2) = 10y$$

$$\Longrightarrow 4 + y^2 = 5y$$

$$\Longrightarrow y^2 - 5y + 4 = 0$$

$$\Longrightarrow (y-4)(y-1)$$

$$\Longrightarrow y = \begin{cases} 1 \\ 4 \end{cases}$$

FINALLY WE HAVE

$$\left.\frac{dy}{dt}\right|_{(2,1)} = \frac{5-8}{4-10} \times (k \times 2) = 2k \times \frac{-3}{-6}$$

$$\left.\frac{dy}{dt}\right|_{(2,4)} = \frac{20-8}{16-10} \times (k \times 2) = \frac{12}{6} \times 2k$$

$$\therefore \frac{dy}{dt} = \begin{cases} k \\ 4k \end{cases}$$

IYGB - SYN PAPER S - QUESTION 14

PROCEED BY PARTIAL FRACTIONS

$$\Rightarrow \frac{10x^2-x-6}{(2+3x)(1-2x^2)} \equiv \frac{A}{2+3x} + \frac{Bx+C}{1-2x^2}$$

$$\Rightarrow 10x^2-x-6 \equiv A(1-2x^2) + (2+3x)(Bx+C)$$

$$\Rightarrow 10x^2-x-6 \equiv (3B-2A)x^2 + (2B+3C)x + (A+2C)$$

- $3B-2A=10$
- $2B+3C=-1$, $\boxed{6B+9C=-3}$
- $A+2C=-6$, $\boxed{A=-6-2C}$

THUS WE HAVE

$$3B-2(-6-2C)=10$$
$$3B+12+4C=10$$
$$3B+4C=-2$$
$$\boxed{-6B-8C=4}$$

$C=1, \quad A=-8, \quad B=-2$

HENCE WE NOW HAVE

$$\frac{10x^2-x-6}{(2+3x)(1-2x^2)} = \frac{-8}{2+3x} + \frac{-2x+1}{1-2x^2}$$

$$= (1-2x)(1-2x^2)^{-1} - 8(2+3x)^{-1}$$

$$= (1-2x)(1-2x^2)^{-1} - 8\times 2^{-1}\left(1+\tfrac{3}{2}x\right)^{-1}$$

$$= (1-2x)(1-2x^2)^{-1} - 4\left(1+\tfrac{3}{2}x\right)^{-1}$$

USING $(1-x)^{-1} = 1+x+x^2+x^3+O(x^4)$
USING $(1+x)^{-1} = 1-x+x^2+x^3+O(x^4)$

IYGB

$$\frac{10x^2-x-6}{(2+3x)(1-2x^2)} = \left\{\begin{array}{l}(1-2x)\left[1+2x^2+O(x^4)\right] \\ -4\left(1-\frac{3}{2}x+\frac{9}{4}x^2-\frac{27}{8}x^3+O(x^4)\right)\end{array}\right\}$$

$$= \left\{\begin{array}{l}1-2x+2x^2-4x^3+O(x^4) \\ -4+6x-9x^2+\frac{27}{2}x^3+O(x^4)\end{array}\right\}$$

$$= -3+4x-7x^2+\frac{19}{2}x^3+O(x^4)$$

IYGB – SYNOPTIC PAPER X – QUESTION 15

START BY FINDING THE GRADIENT FUNCTION IN PARAMETRIC

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t}{\frac{-6}{t^3}} = 10t \times \frac{t^3}{-6} = -\frac{5}{3}t^4$$

FIND THE EQUATION OF THE TANGENT AT A GENERAL POINT SAY AT THE POINT WHERE $t=p$, $\left(\frac{3}{p^2}, 5p^2\right)$

$$\Rightarrow y - 5p^2 = -\frac{5}{3}p^4\left(x - \frac{3}{p^2}\right)$$
$$\Rightarrow 3y - 15p^2 = -5p^4\left(x - \frac{3}{p^2}\right)$$
$$\Rightarrow 3y - 15p^2 = -5p^4x + 15p^2$$
$$\Rightarrow 3y + 5p^4x = 30p^2$$

NOW THIS TANGENT PASSES THROUGH $\left(\frac{9}{2}, \frac{5}{2}\right)$

$$\Rightarrow \frac{15}{2} + \frac{45}{2}p^4 = 30p^2 \quad \times \frac{2}{15}$$
$$\Rightarrow 1 + 3p^4 = p^2$$
$$\Rightarrow 3p^4 - 4p^2 + 1 = 0$$
$$\Rightarrow (p^2-1)(3p^2-1) = 0$$

NO NEED TO FIND p, p^2 WILL SUFFICE

$$\Rightarrow p = \begin{cases} 1 \\ 1/3 \end{cases}$$

FINALLY CO-ORDINATES

IF $p^2 = 1 \Rightarrow (3, 5)$

IF $p^2 = \frac{1}{3} \Rightarrow \left(9, \frac{5}{3}\right)$

IYGB - SYN PAPER S - QUESTION 16

a)

$f(2x+3)$

REPLACE x WITH $\left(\frac{1}{2}x\right)$

$f\left(2\left(\frac{1}{2}x\right)+3\right) = f(x+3)$

REPLACE x WITH $(x-3)$

$f\left((x-3)+3\right) = f(x)$

STRETCH PARALLEL TO THE x AXIS BY SCALE FACTOR 2

FOLLOWED BY

TRANSLATION BY VECTOR $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

b)

$f(2x+3)$

REPLACE x WITH $\left(x-\frac{3}{2}\right)$

$f\left(2\left(x-\frac{3}{2}\right)+3\right) = f(2x)$

REPLACE x WITH $\left(\frac{1}{2}x\right)$

$f\left(2\left(\frac{1}{2}x\right)\right) = f(x)$

TRANSLATION BY THE VECTOR $\begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}$

FOLLOWED BY

STRETCH PARALLEL TO THE x AXIS BY SCALE FACTOR 2

IYGB - SYNOPTIC PAPER $ - QUESTION 17

APPLY THE GIVEN TRANSFORMATION TO $f(x) = 2^{4x}$

$$\Rightarrow f(x-1) = \frac{5}{8}$$

$$\Rightarrow 2^{4(x-1)} = \frac{5}{8}$$

$$\Rightarrow 2^{4x} \times 2^{-4} = \frac{5}{8}$$

$$\Rightarrow \frac{2^{4x}}{16} = \frac{5}{8}$$

$$\Rightarrow 2^{4x} = 10$$

$$\Rightarrow \log_{10} 2^{4x} = \log_{10} 10$$

$$\Rightarrow 4x \log_{10} 2 = 1$$

$$\Rightarrow x = \frac{1}{4 \log_{10} 2}$$

AS REQUIRED

IYGB – SYNOPTIC PAPER S – QUESTION 19

WRITE THE EQUATION EXPLICITLY

$$\Rightarrow f(x) = \frac{1}{2} f\left(x+\frac{\pi}{4}\right)$$

$$\Rightarrow 2f(x) = f\left(x+\frac{\pi}{4}\right)$$

$$\Rightarrow 2\sec^2 x = \sec^2\left(x+\frac{\pi}{4}\right)$$

$$\Rightarrow \frac{2}{\cos^2 x} = \frac{1}{\cos^2\left(x+\frac{\pi}{4}\right)}$$

$$\Rightarrow \frac{\cos^2 x}{2} = \cos^2\left(x+\frac{\pi}{4}\right).$$

USING THE COMPOUND ANGLE IDENTITY, $\cos(A+B)$

$$\Rightarrow \frac{1}{2}\cos^2 x = \left[\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4}\right]^2$$

$$\Rightarrow \frac{1}{2}\cos^2 x = \left[\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right]^2$$

$$\left\{\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\right\}$$

$$\Rightarrow \frac{1}{2}\cos^2 x = \left(\frac{1}{\sqrt{2}}\right)^2(\cos x - \sin x)^2$$

$$\Rightarrow \cancel{\frac{1}{2}}\cos^2 x = \cancel{\frac{1}{2}}(\cos x - \sin x)^2$$

FINALLY TWO DIFFERENT APPROACHES TO FOLLOW

$$\cancel{\cos^2 x} = \cancel{\cos^2 x} + \sin^2 x - 2\sin x\cos x$$

$$0 = \sin^2 x - 2\sin x\cos x$$

$$0 = \sin x(\sin x - 2\cos x)$$

$$\frac{0}{\cos x} = \frac{\sin x(\sin x - 2\cos x)}{\cos x}$$

$$0 = \sin x(\tan x - 2)$$

$$\sin x = 0 \quad \text{OR} \quad \tan x = 2$$

$$\cos x = \pm \cos x \mp \sin x$$

$$\mp\cos x + \cos x = -\sin x$$

$$\pm\cos x - \cos x = \pm\sin x$$

$$\left(\begin{array}{l}\cos x - \cos x = \sin x \\ -\cos x - \cos x = -\sin x.\end{array}\right.$$

$$\sin x = 0 \quad \text{OR} \quad \sin x = 2\cos x$$

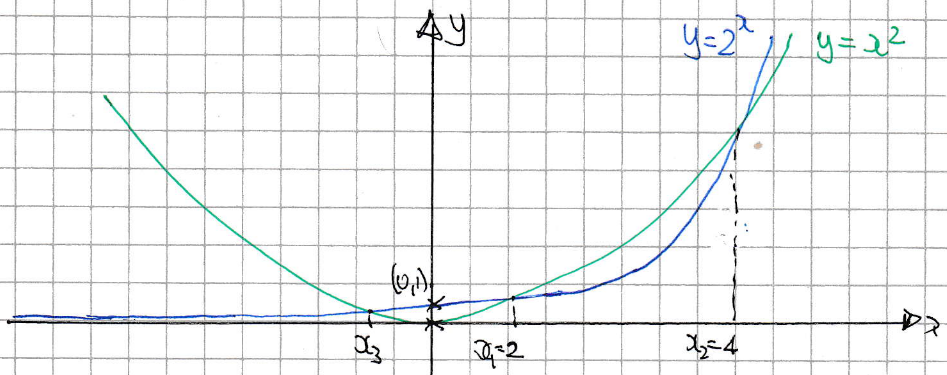
$$\tan x = 2$$

IYGB - SYNOPTIC PAPER S - QUESTION 10

a) EVIDENTLY TWO POSITIVE INTEGER SOLUTIONS

$$2^2 = 2^2 \quad \text{OR} \quad 4^2 = 2^4 \qquad \therefore \ x_1 = 2, \ x_2 = 4$$

b) SKETCHING THESE STANDARD CURVES



c) DEFINE A FUNCTION FIRST AND DIFFERENTIATE IT

$$f(x) = x^2 - 2^x$$
$$f'(x) = 2x - 2^x \ln 2$$

USE THE NEWTON - RAPSON - STARTING VALUE TRY -1, AS $(-1)^2 - 2^{-1} = \frac{1}{2}$ WHICH IS SUFFICIENTLY CLOSE TO ZERO

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \implies x_{n+1} = x_n - \frac{x_n^2 - 2^{x_n}}{2x_n - 2^{x_n}\ln 2}$$

$$x_2 = -1 - \frac{(-1)^2 - 2^{-1}}{2(-1) - 2^{-1}\ln 2} \approx -0.786923\ldots$$

$$x_3 \approx -0.766843\ldots$$

$$x_4 \approx -0.766665\ldots$$

$$x_5 \approx -0.766665\ldots$$

$$\therefore x \approx -0.7667$$

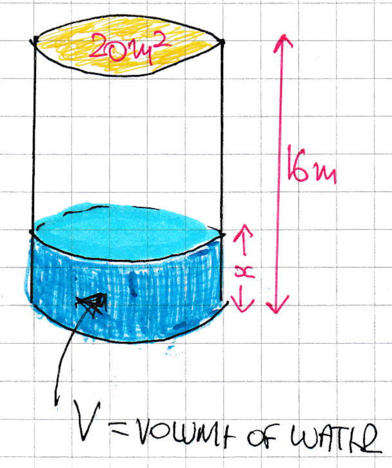
IYGB - SYNOPTIC PAPER S - QUESTION 20

a) START MODELLING AS FOLLOWS

IN FLOW: $\frac{dV}{dt} = 10$

OUT FLOW: $\frac{dV}{dt} = -\sqrt{x}$

NET FLOW: $\frac{dV}{dt} = 10 - \sqrt{x}$



$V = 20x$

$\frac{dV}{dx} = 20$

$t=0,\ x=9$

RELATING VARIABLES, x & V

$$\frac{dV}{dx} \times \frac{dx}{dt} = 10 - \sqrt{x}$$

$$20\frac{dx}{dt} = 10 - \sqrt{x}$$ // AS REQUIRED

b) SOLVING THE O.D.E. BY SEPARATING VARIABLES, & INPUTING THE INITIAL CONDITION & THE REQUIRED ANSWER AS LIMITS

$$\Longrightarrow\ 20\,dx = (10 - \sqrt{x})\,dt$$

$$\Longrightarrow\ \frac{20}{10-\sqrt{x}}\,dx = 1\,dt$$

$$\Longrightarrow\ \int_{x=9}^{x=16} \frac{20}{10-\sqrt{x}}\,dx = \int_{t=0}^{t} 1\,dt$$

BY SUBSTITUTION ON THE INTEGRAL ON THE L.H.S

- $u = 10 - \sqrt{x}$

  $\sqrt{x} = 10 - u$

  $x = (10-u)^2$

  $\frac{dx}{du} = -2(10-u)$

  $dx = -2(10-u)$

- LIMITS $x=9 \longmapsto u=7$

  $x=16 \longmapsto u=6$

IYGB - SYNOPTIC PAPER S - QUESTION 20

$$\Longrightarrow \int_{u=7}^{u=6} \frac{20}{u}\left[-2(10-u)\ du\right] = \left[t\right]_0^t$$

$$\Longrightarrow \int_7^6 \frac{-40(10-u)}{u}\ du = t-0$$

$$\Longrightarrow \int_7^6 \frac{40(u-10)}{u}\ du = t$$

$$\Longrightarrow t = 40\int_7^6 \frac{u-10}{u}\ du$$

$$\Longrightarrow t = 40\int_7^6 1-\frac{10}{u}\ du$$

$$\Longrightarrow t = 40\left[u-10\ln|u|\right]_7^6$$

$$\Longrightarrow t = 40\left[(6-10\ln6)-(7-10\ln7)\right]$$

$$\Longrightarrow t = 40\left[10\ln7-10\ln6-1\right]$$

$$\Longrightarrow t \approx 21.66027193$$

$$\Longrightarrow t \simeq 22 \text{ hours}$$

IYGB - SYNOPTIC PAPER S - QUESTION 21

LOOKING AT THE SUM OF THE FIRST 20 TERMS

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \implies 1360 = \frac{20}{2}\left[22 + 19 \times d\right]$$
$$\implies 1360 = 10(22 + 19d)$$
$$\implies 136 = 22 + 19d$$
$$\implies 114 = 19d$$
$$\implies d = 6$$

NOW SUPPOSE THE SERIES HAS k TERMS — FIND THE LAST TERM

$$u_n = a + (n-1)d \implies u_k = 11 + (k-1) \times 6$$
$$\implies u_k = 6k + 5$$

NOW CONSIDER THE LAST TWENTY TERM — REWRITE THE TERMS BACKWARDS

$a = 6k + 5$

$d = -6$

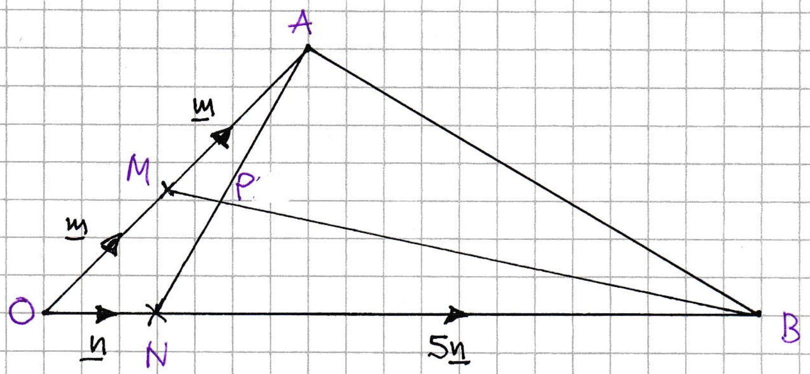
$S_{20} = 4720$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$4720 = \frac{20}{2}\left[2(6k+5) + 19(-6)\right]$$
$$4720 = 10(12k + 10 - 114)$$
$$472 = 12k - 104$$
$$576 = 12k$$
$$k = \frac{576}{12} = \frac{600 - 24}{12} = 50 - 2$$
$$k = 48$$

IE 48 TERMS

IYGB - SYN PAPER S - QUESTION 22

START WITH A DIAGRAM - DEFINE VECTORS $\vec{OM} = \underline{m}$ & $\vec{ON} = \underline{n}$



- $\vec{MB} = \vec{MO} + \vec{OB}$
  $\vec{MB} = -\underline{m} + 6\underline{n}$
- $\vec{AN} = \vec{AO} + \vec{ON}$
  $\vec{AN} = -2\underline{m} + \underline{n}$
- $\vec{OP} = \vec{OM} + \vec{MP}$
  $\vec{OP} = \vec{OM} + \lambda \vec{MB}$ (FOR SOME λ)
- $\vec{OP} = \vec{OA} + \vec{AP}$
  $\vec{OP} = \vec{OA} + \mu \vec{AN}$ (FOR SOME μ)

EQUATING EXPRESSIONS FOR $\vec{OP}$

$$\Rightarrow \vec{OM} + \lambda \vec{MB} = \vec{OA} + \mu \vec{AN}$$
$$\Rightarrow \underline{m} + \lambda(-\underline{m} + 6\underline{n}) = 2\underline{m} + \mu(-2\underline{m} + \underline{n})$$
$$\Rightarrow (1-\lambda)\underline{m} + 6\lambda\underline{n} = (2-2\mu)\underline{m} + \mu\underline{n}$$

EQUATING COEFFICIENTS FOR $\underline{m}$ & $\underline{n}$

$$\left.\begin{array}{l} 1-\lambda = 2-2\mu \\ 6\lambda = \mu \end{array}\right\} \Rightarrow \begin{array}{l} 1-\lambda = 2-2(6\lambda) \\ 1-\lambda = 2-12\lambda \\ 11\lambda = 1 \\ \lambda = \frac{1}{11} \quad \& \quad \mu = \frac{6}{11} \end{array}$$

SO POINT P IS $\frac{6}{11}$ OF THE WAY FROM A TO N

$$\therefore |AP| : |PN| = 6 : 5$$

IYGB – SYNOPTIC PAPER S – QUESTION 23

a) COMPLETING THE SQUARE AS FOLLOWS

$$A^4 + 4B^4 = (A^2)^2 + (2B^2)^2$$

BRACKET IS $x^2+2xy+y^2$

$$= \left[(A^2)^2 + 2(A^2)(2B^2) + (2B^2)^2\right] - 2(A^2)(2B^2)$$
$$= (A^2+2B^2)^2 - 4A^2B^2$$
$$= (A^2+2B^2)^2 - (2AB)^2$$
$$= (A^2+2B^2-2AB)(A^2+2B^2+2AB)$$

DIFFERENCE OF SQUARES

$$= (A^2-2AB+2B^2)(A^2+2AB+2B^2)$$

b) USING PART (a)

$$x^4+64 = x^4+4\times16 = x^4+4\times2^4$$
$$= (x^2-2\times2x+2\times2^2)(x^2+2\times2x+2\times2^2)$$
$$= (x^2-4x+8)(x^2+4x+8)$$