IYGB - SYNOPTIC PAPER O - QUESTION 1

a) START WITH THE GRADIENT OF BC, B(0,7) & C(8,3)

$$m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-7}{8-0} = \frac{-4}{8} = -\frac{1}{2}$$

$$\Rightarrow y-y_0 = m(x-x_0)$$
$$\Rightarrow y-5 = -\frac{1}{2}(x+6)$$ (SAME GRADIENT, PASSING THROUGH A(-6,5))
$$\Rightarrow 2y-10 = -x-6$$
$$\Rightarrow x+2y=4$$

AS REQUIRED

b) THE GRADIENT OF L_2 MUST BE +2 & PASSES THROUGH C(8,3)

$$\Rightarrow y-y_0 = m(x-x_0)$$
$$\Rightarrow y-3 = 2(x-8)$$
$$\Rightarrow y-3 = 2x-16$$

or $L_2: \; y = 2x-13$

c) FIND THE POINT OF INTERSECTION D

$L_1: \; x+2y=4$
$L_2: \; y=2x-13$

$$\Rightarrow x+2(2x-13)=4$$
$$\Rightarrow x+4x-26=4$$
$$\Rightarrow 5x=30$$
$$\Rightarrow x=6$$
$$\Rightarrow y=-1$$

$\therefore D(6,-1)$

FINALLY THE DISTANCE BD, B(0,7) & D(6,-1)

$$d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$$
$$|BD| = \sqrt{(-1-7)^2+(6-0)^2}$$
$$|BD| = \sqrt{64+36}$$
$$|BD| = 10$$

AS REQUIRED

IYGB - SYNOPTIC PAPER O - QUESTION 2

a) I) JUST SUBSTITUTING

$$f(\ln 5) = \sqrt{e^{\ln 5} - 1} = \sqrt{5-1} = 2$$

II) DIFFERENTIATING FIRST

$$f(x) = (e^x - 1)^{\frac{1}{2}}$$

$$f'(x) = \tfrac{1}{2}(e^x - 1)^{-\frac{1}{2}} \times e^x$$

$$f'(x) = \frac{e^x}{2\sqrt{e^x - 1}}$$

$$f'(\ln 5) = \frac{e^{\ln 5}}{2\sqrt{e^{\ln 5} - 1}} = \frac{5}{2\sqrt{5-1}} = \frac{5}{4}$$

b) BY THE "STANDARD" METHOD

$$\Rightarrow y = \sqrt{e^x - 1}$$

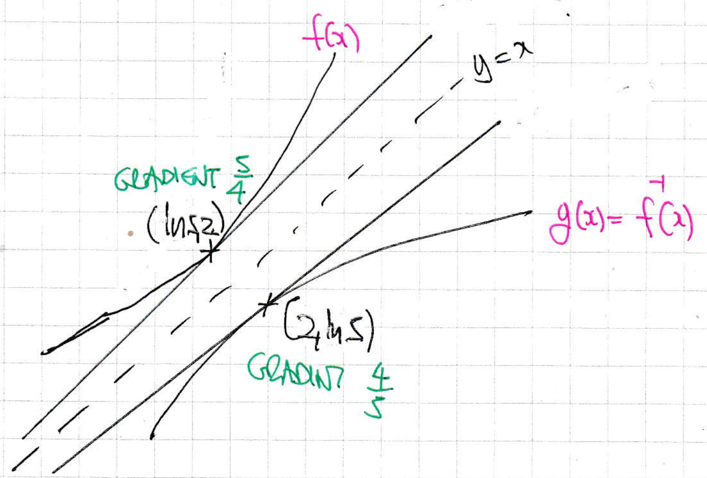
$$\Rightarrow y^2 = e^x - 1$$

$$\Rightarrow y^2 + 1 = e^x$$

$$\Rightarrow x = \ln(1 + y^2)$$

$$\therefore \underset{g(x)}{f^{-1}(x)} = \ln(1 + x^2)$$

c) IT WILL BE THE RECIPROCAL OF $\frac{5}{4}$, REFLECTION IN THE LINE $y = x$



$$\therefore g'(2) = \frac{4}{5}$$

IYGB - SYNOPTIC PAPER O - QUESTION 3

a) BY THE "factor theorem"

$$f\left(\frac{k}{2}\right) = 13 \implies 2\left(\frac{k}{2}\right)^2 + 9\left(\frac{k}{2}\right) - 5 = 13$$

$$\implies 2\left(\frac{k^2}{4}\right) + \frac{9}{2}k - 18 = 0$$

$$\implies \frac{1}{2}k^2 + \frac{9}{2}k - 18 = 0$$

$$\implies k^2 + 9k - 36 = 0$$

$$\implies (k + 12)(k - 3) = 0$$

$$k = \begin{cases} 3 \\ -12 \end{cases}$$

b) BY THE FACTOR THEOREM AGAIN

$$f(2k) = 121 \implies 2(2k)^2 + 9(2k) - 5 = 121$$

$$\implies 8k^2 + 18k - 126 = 0$$

$$\implies 4k^2 + 9k - 63 = 0$$

$$\implies (4k + 21)(k - 3) = 0$$

$$\implies k = \begin{cases} 3 \\ -\frac{21}{4} \end{cases}$$

$$\therefore \; k = 3$$

IYGB - SYNOPTIC PAPER O - QUESTION 4

a) IF AB IS A DIAMETER, THE MIDPOINT OF AB MUST BE THE CENTRE

$$\text{CENTRE } C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = C\left(\frac{-1+7}{2}, \frac{10+4}{2}\right) = C(3,7)$$

THE RADIUS WILL BE $|CB|$ OR $|CA|$, SAY $A(-1,10)$

$$r = |AC| = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(7-10)^2+(3+1)^2} = \sqrt{9+16} = 5$$

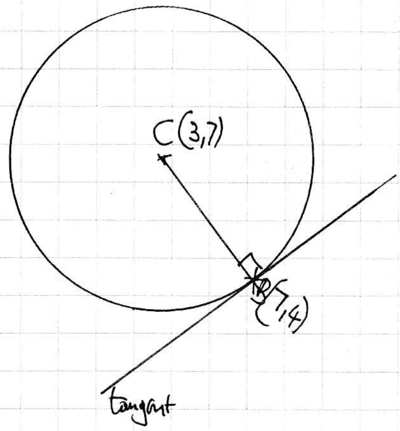
$$\therefore \quad (x-3)^2+(y-7)^2 = 5^2$$

$$x^2-6x+9+y^2-14y+49 = 25$$

$$x^2+y^2-6x-14y+33 = 0$$

AS REQUIRED

b) LOOKING AT THE DIAGRAM



- $m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{4-7}{7-3} = \frac{-3}{4} = -\frac{3}{4}$

- $m_{\text{tangent}} = +\frac{4}{3}$

EQUATION OF TANGENT

$$\Rightarrow y-y_0 = m(x-x_0)$$

$$\Rightarrow y-4 = \frac{4}{3}(x-7)$$

CROSSES y AXIS $\Rightarrow x=0$

$$\Rightarrow y-4 = +\frac{4}{3}(0-7)$$

$$\Rightarrow 3y-12 = -28$$

$$\Rightarrow 3y = -16$$

$$\Rightarrow y = -\frac{16}{3}$$

$$\therefore D\left(0, -\frac{16}{3}\right)$$

IYGB - SYNOPTIC PAPER O - QUESTION 5

LET $y = x^2 - x - 3$

$\Rightarrow (x^2-x-3)^2 - 12(x^2-x-3) + 27 = 0$

$\Rightarrow y^2 - 12y + 27 = 0$

$\Rightarrow (y-3)(y-9) = 0$

$\Rightarrow y = \begin{cases} 3 \\ 9 \end{cases}$

$\Rightarrow x^2 - x - 3 = \begin{cases} 3 \\ 9 \end{cases}$

$\Rightarrow \begin{cases} x^2 - x - 6 = 0 \\ x^2 - x - 12 = 0 \end{cases}$

$\Rightarrow \begin{cases} (x+2)(x-3) = 0 \\ (x-4)(x+3) = 0 \end{cases}$

$\therefore x = \begin{cases} -3 \\ -2 \\ 3 \\ 4 \end{cases}$ //

IYGB – SYNOPTIC PAPER O – QUESTION 6

a) REWRITE AS AN INDEX AND EXPAND

$$\sqrt[3]{1-3x} = (1-3x)^{\frac{1}{3}}$$

$$= 1 + \frac{\frac{1}{3}}{1}(-3x)^1 + \frac{\frac{1}{3}(-\frac{2}{3})}{1\times2}(-3x)^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{1\times2\times3}(-3x)^3 + O(x^4)$$

$$= 1 - x - x^2 - \frac{5}{3}x^3 + O(x^4)$$

b) VALIDITY IS $|3x| < 1$

$$\therefore \quad -\frac{1}{3} < x < \frac{1}{3}$$

c) LET $x = 0.001$

$$\sqrt[3]{1-3(0.001)} = 1 - (0.001) - (0.001)^2 - \frac{5}{3}(0.001)^3 + O((0.001)^4)$$

$$\sqrt[3]{0.997} = 0.9989989983\ldots$$

$$\sqrt[3]{\frac{997}{1000}} = 0.9989989983\ldots$$

$$\frac{\sqrt[3]{997}}{10} = 0.9989989983\ldots$$

$$\sqrt[3]{997} \simeq 9.989989983$$

IYGB - SYNOPTIC PAPER O - QUESTION 7

CHANGE THE LOG BASE 2 INTO NATURAL LOGS

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \text{HERE} \qquad \log_2 x = \frac{\log_e x}{\log_e 2} = \frac{\ln x}{\ln 2}$$

$$\Longrightarrow \quad 2 + \log_2 x = 2\ln\left(\frac{x}{\sqrt{e}}\right)$$

$$\Longrightarrow \quad 2 + \frac{\ln x}{\ln 2} = 2\ln x - 2\ln\sqrt{e}$$

$$\Longrightarrow \quad 2 + \frac{\ln x}{\ln 2} = 2\ln x - 2\ln e^{\frac{1}{2}}$$

$$\Longrightarrow \quad 2 + \frac{\ln x}{\ln 2} = 2\ln x - \ln e$$

$$\Longrightarrow \quad 2 + \frac{\ln x}{\ln 2} = 2\ln x - 1$$

$$\Longrightarrow \quad 3 + \frac{\ln x}{\ln 2} = 2\ln x$$

GETTING RID OF THE FRACTIONS AND TIDY

$$\Longrightarrow \quad 3\ln 2 + \ln x = 2\ln x \ln 2$$

$$\Longrightarrow \quad 3\ln 2 = 2\ln x \ln 2 - \ln x$$

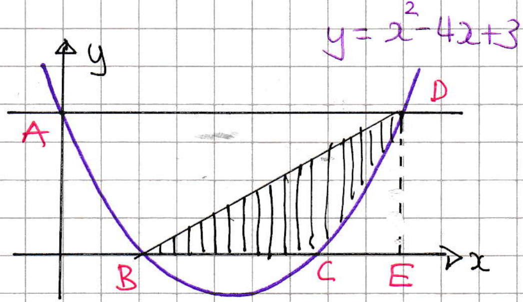
$$\Longrightarrow \quad 3\ln 2 = \ln x\left[2\ln 2 - 1\right]$$

$$\Longrightarrow \quad \ln x = \frac{3\ln 2}{2\ln 2 - 1}$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER O - QUESTION 8

START COLLECTING INFORMATION



- BY INSPECTION $A(0,3)$

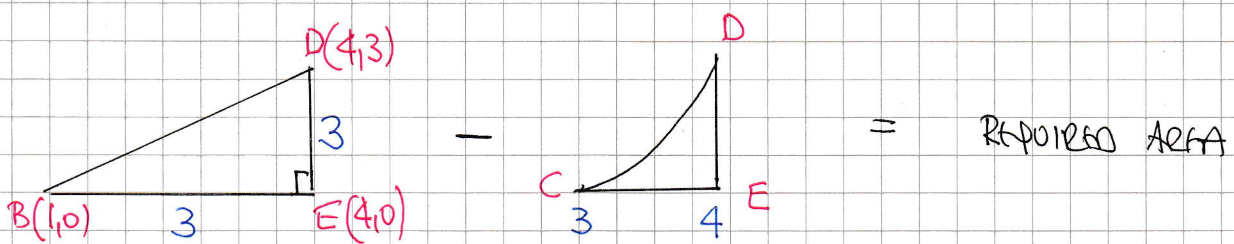
- $y = 0$

$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = \begin{cases} 1 \\ 3 \end{cases}$$

$\therefore B(1,0) \quad C(3,0)$

- BY INSPECTION (DUE TO SYMMETRY) $D(4,3)$

LOOKING AT THE DIAGRAM BELOW



$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

$$\int_3^4 x^2 - 4x + 3 \, dx = \left[\frac{1}{3}x^3 - 2x^2 + 3x\right]_3^4$$
$$= \left(\frac{64}{3} - 32 + 12\right) - \cancel{(9 - 18 + 9)}$$
$$= \frac{4}{3}$$

THE REQUIRED AREA IS $\frac{9}{2} - \frac{4}{3} = \frac{19}{6}$

IYGB - SYNOPTIC PAPER O - QUESTION 9

FORM TWO EQUATIONS

$$u_1 + u_3 = 35$$

$$a + ar^2 = 35$$

$$a(1+r^2) = 35$$

$$\boxed{a = \frac{35}{1+r^2}}$$

$$\frac{a + ar + ar^2 + ar^3 + ar^4}{\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}} = 49$$

$$\Rightarrow \frac{a(1+r+r^2+r^3+r^4)}{\frac{1}{a}\left(1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}\right)} = 49$$

$$\Rightarrow \frac{a^2(1+r+r^2+r^3+r^4)}{1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}} = 49$$

MULTIPLY TOP & BOTTOM OF THE FRACTION IN THE L.H.S BY r^4

$$\Rightarrow \frac{a^2(1+r+r^2+r^3+r^4)\,r^4}{\left(1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}\right)r^4} = 49$$

$$\Rightarrow \frac{a^2r^4\cancel{(1+r+r^2+r^3+r^4)}}{\cancel{r^4+r^3+r^2+r+1}} = 49$$

$$\Rightarrow \boxed{a^2r^4 = 49}$$

$$\Rightarrow ar^2 = \begin{cases} 7 \\ -7 \end{cases}$$

$$\Rightarrow \frac{35}{1+r^2} \times r^2 = \begin{cases} 7 \\ \cancel{-7} \end{cases}$$

$$\Rightarrow \frac{35r^2}{1+r^2} = 7$$

$$\Rightarrow 35r^2 = 7 + 7r^2$$

$$\Rightarrow 28r^2 = 7$$

$$\Rightarrow r^2 = \frac{1}{4}$$

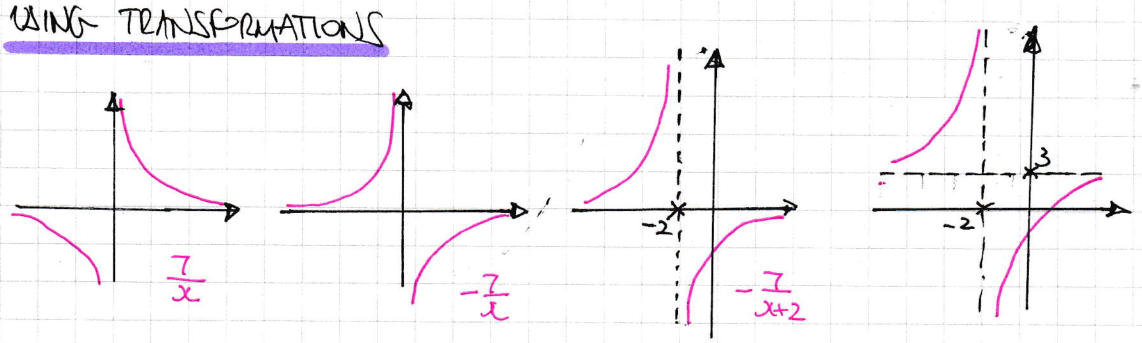
$$\Rightarrow r = \pm\frac{1}{2} \quad \& \quad a = 28$$

IYGB - SYNOPTIC PAPER O - QUESTION 10

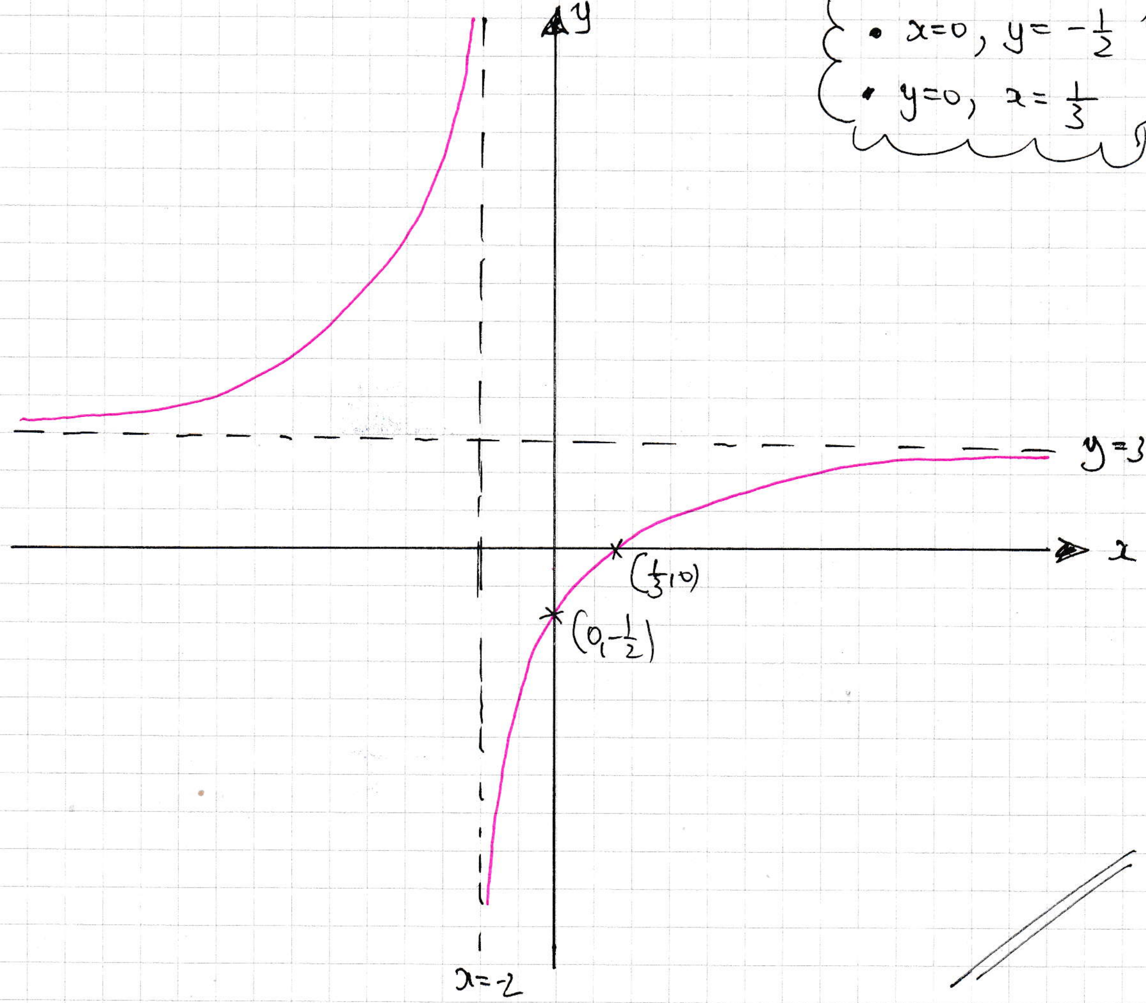
a) REWRITE THE FUNCTION AS FOLLOWS

$$f(x) = \frac{3x-1}{x+2} = \frac{3(x+2)-7}{x+2} = 3 - \frac{7}{x+2}$$

USING TRANSFORMATIONS



HENCE WE HAVE



- $x=0, \ y=-\frac{1}{2}$
- $y=0, \ x=\frac{1}{3}$

IYGB - SYNOPTIC PAPER O - QUESTION 10

b) I) SINCE THE TWO GRAPHS MEET ON THE x AXIS, THEY MUST MEET ON $A\left(\frac{1}{3},0\right)$.

$$\Rightarrow 0 = \frac{1}{\frac{1}{3}} + k$$

$$\Rightarrow 0 = 3 + k$$

$$\Rightarrow k = -3$$

(II) SOLVING SIMULTANEOUSLY WITH $k=-3$

$$\Rightarrow \frac{1}{x} - 3 = \frac{3x-1}{x+2}$$

$$\Rightarrow (x+2) - 3x(x+2) = x(3x-1)$$

$$\Rightarrow x+2-3x^2-6x = 3x^2-x$$

$$\Rightarrow 0 = 6x^2+4x-2$$

$$\Rightarrow 0 = 3x^2+2x-1$$

$$\Rightarrow 0 = (3x-1)(x+1)$$

$$\Rightarrow x = \begin{cases} -1 \leftarrow B \\ \frac{1}{3} \leftarrow A \end{cases}$$

& $g(-1) = \frac{1}{-1} - 3 = -4$

$\therefore B(-1,-4)$

IYGB - SYNOPTIC PAPER O - QUESTION 11

a) AT P & Q $x=0$

$27-3t^2=0$

$27=3t^2$

$t^2=9$

$t = \begin{cases} 3 \\ -3 \end{cases}$

$t=3 \quad x=0 \quad y=-75$

$t=-3 \quad x=0 \quad y=75$

AT S & R, $y=0$

$0 = 5t(4-t^2)$

$0 = 5t(2-t)(2+t)$

$t = \begin{cases} 0 \\ 2 \\ -2 \end{cases}$

$t=0 \quad x=27 \quad y=0$

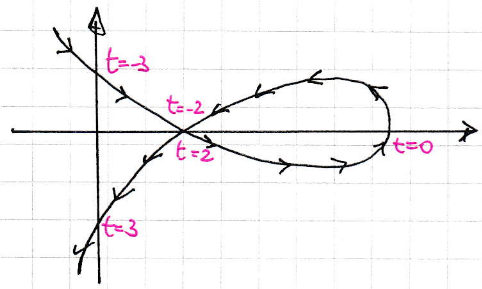
$t=2 \quad x=15 \quad y=0$

$t=-2 \quad x=15 \quad y=0.$

$\therefore \quad t_P=-3, \; t_S=\mp 2, \; t_R=0, \; t_Q=3$

$\therefore \quad P(0,75) \quad Q(0,-75) \quad R(27,0) \quad S(15,0)$

b) LOOKING AT THE DIAGRAM & CONSIDERING THE TOP HALF OF THE LOOP



TOTAL LOOP AREA = 2 × TOP HALF $= 2\int_{x_S}^{x_R} y(x)\,dx$

$= 2\int_{t_S}^{t_R} y(t)\frac{dx}{dt}\,dt$

$= 2\int_{2}^{0} 5t(4-t^2)(-6t)\,dt$

IYGB - SYNOPTIC PAPER O - QUESTION 11

$$= 60\int_{2}^{0} -t^2(4-t^2)\,dt = 60\int_{0}^{2} t^2(4-t^2)\,dt$$

$$= 60\int_{0}^{2} 4t^2 - t^4\,dt = 60\left[\frac{4}{3}t^3 - \frac{1}{5}t^5\right]_{0}^{2}$$

$$= \left[80t^3 - 12t^5\right]_{0}^{2} = (640 - 384) - (0)$$

$$= 256$$

AS REQUIRED

c) WORK AS FOLLOWS

- $y = 5t(4-t^2)$
- $x = 27 - 3t^2$

$3t^2 = 27 - x$

$$y^2 = 25t^2(4-t^2)^2$$

×27

$$27y^2 = 27\left[25t^2(4-t^2)^2\right]$$

$$27y^2 = 75t^2(12-3t^2)^2$$

$$27y^2 = 25(27-x)(12-(27-x))^2$$

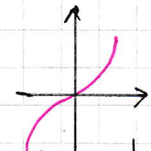
$$27y^2 = 25(27-x)(x-15)^2$$

$$y^2 = \frac{25}{27}(27-x)(x-15)^2$$

IYGB - SYNOPTIC PAPER O - QUESTION 12

a) BY "INVERSE" RULE

$$\Rightarrow y = \arcsin x$$
$$\Rightarrow \sin y = x$$
$$\Rightarrow x = \sin y$$
$$\Rightarrow \frac{dx}{dy} = \cos y$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{\cos^2 y}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{1-\sin^2 y}$$
$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{1-x^2}$$
$$\Rightarrow \frac{dy}{dx} = \pm \frac{1}{\sqrt{1-x^2}}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad //$$



(arcsinx has positive gradient)

b) FIRSTLY FIND THE VALUE OF k

$$\Rightarrow \arcsin(3x) + 2\arcsin y = \frac{\pi}{2}$$
$$\Rightarrow \arcsin\frac{1}{2} + 2\arcsin y = \frac{\pi}{2}$$
$$\Rightarrow \frac{\pi}{6} + 2\arcsin y = \frac{\pi}{2}$$
$$\Rightarrow 2\arcsin y = \frac{\pi}{3}$$
$$\Rightarrow \arcsin y = \frac{\pi}{6}$$
$$\Rightarrow y = \frac{1}{2}$$

i.e. $k = \frac{1}{2}$

IYGB – SYNOPTIC PAPER O – QUESTION 12

NOW DIFFERENTIATING THE EQUATION IMPLICITLY

$$\Longrightarrow \arcsin 3x + 2\arcsin y = \frac{\pi}{2}$$

$$\Longrightarrow \frac{d}{dx}\left(\arcsin 3x\right) + \frac{d}{dx}\left(2\arcsin y\right) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Longrightarrow \frac{1}{\sqrt{1-(3x)^2}} \times 3 + \frac{2}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Longrightarrow \frac{3}{\sqrt{1-9x^2}} + \frac{2}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

AT $\left(\frac{1}{6}, \frac{1}{2}\right)$

$$\Longrightarrow \frac{3}{\sqrt{1-9\left(\frac{1}{6}\right)^2}} + \frac{2}{\sqrt{1-\left(\frac{1}{2}\right)^2}}\frac{dy}{dx} = 0$$

$$\Longrightarrow \frac{3}{\sqrt{\frac{3}{4}}} + \frac{2}{\sqrt{\frac{3}{4}}}\frac{dy}{dx} = 0$$

$$\Longrightarrow 3 + 2\frac{dy}{dx} = 0$$

$$\Longrightarrow \frac{dy}{dx} = -\frac{3}{2}$$

IYGB - SYNOPTIC PAPER O - QUESTION 13

a) USING THE SUBSTITUTION GIVEN

$$x = \sec\theta \qquad x=1 \longmapsto \theta = 0$$

$$\frac{dx}{d\theta} = \sec\theta\tan\theta \qquad x=2 \longmapsto \theta = \frac{\pi}{3}$$

$$dx = \sec\theta\tan\theta\, d\theta$$

TRANSFORMING THE INTEGRAL

$$\int_1^2 \frac{1}{x^2 - x\sqrt{x^2-1}}\, dx = \int_0^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta}{\sec^2\theta - \sec\theta\sqrt{\sec^2\theta - 1}}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta}{\sec^2\theta - \sec\theta\sqrt{\tan^2\theta}}\, d\theta = \int_0^{\frac{\pi}{3}} \frac{\cancel{\sec\theta}\tan\theta}{\sec^{\cancel{2}}\theta - \cancel{\sec\theta}|\tan\theta|}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sec\theta - \tan\theta}\, d\theta$$

AS REQUIRED

b) USING $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$

$$\ldots = \int_0^{\frac{\pi}{3}} \frac{\tan\theta(\tan\theta + \sec\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{\tan^2\theta + \tan\theta\sec\theta}{1}\, d\theta$$

$$= \int \tan^2\theta + \tan\theta\sec\theta \, d\theta$$

$$= \int \sec^2\theta - 1 + \tan\theta\sec\theta \, d\theta$$

IYGB - SYNOPTIC PAPER O - QUESTION 13

$$= \left[\tan\theta - \theta + \sec\theta \right]_0^{\frac{\pi}{3}}$$

$$= \left(\sqrt{3} - \frac{\pi}{3} + 2 \right) - \left(0 - 0 + 1 \right)$$

$$= \sqrt{3} - \frac{\pi}{3} + 1$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER O - QUESTION 14

START BY WRITING DOWN THE EQUATION OF THE CUBIC

$$\Rightarrow y = (x-a)(x-b)^2$$

$$\Rightarrow y = (x-a)(x^2-2bx+b^2) = x^3-2bx^2+b^2x - ax^2+2abx+ab^2$$

$$\Rightarrow y = x^3-(a+2b)x^2+(2ab+b^2)x+ab^2$$

DIFFERENTIATING & SETTING TO ZERO

$$\Rightarrow \frac{dy}{dx} = 3x^2-2(a+2b)x+(2ab+b^2)$$

$$\Rightarrow 0 = 3x^2-2(a+2b)x+b(2a+b)$$

AS $x=b$ IS A SOLUTION, BEING A LOCAL MINIMUM

$$\Rightarrow (x-b)\left[3x-(2a+b)\right]=0$$

$$\Rightarrow x = \begin{cases} b & \leftarrow \text{POINT B} \\ \frac{2a+b}{3} & \leftarrow \text{POINT C} \end{cases}$$

NOW WE HAVE $A(a,0)$, $B(b,0)$, $D\left(\frac{2a+b}{3},0\right)$

- $|AB| = b-a$
- $|AD| = \frac{2a+b}{3}-a = \frac{2a+b-3a}{3} = \frac{b-a}{3}$

$$\therefore |AD| = \frac{b-a}{3}$$

$$|AD| = \frac{|AB|}{3}$$

$$|AB| = 3|AD|$$

AS REQUIRED

IYGB - SYNOPTIC PAPER O - QUESTION 15

START BY FINDING THE GRADIENT FUNCTION BY THE QUOTIENT RULE

$$y = \frac{1+\cos x}{1+\sin x}$$

$$\frac{dy}{dx} = \frac{(1+\sin x)(-\sin x) - (1+\cos x)(\cos x)}{(1+\sin x)^2}$$

$$\frac{dy}{dx} = \frac{-\sin x - \sin^2 x - \cos x - \cos^2 x}{(1+\sin x)^2}$$

$$\frac{dy}{dx} = -\frac{\cos^2 x + \sin^2 x + \cos x + \sin x}{(1+\sin x)^2}$$

$$\frac{dy}{dx} = -\frac{1+\cos x + \sin x}{(1+\sin x)^2}$$

SOLVING FOR ZERO YIELDS

$$\Rightarrow 1 + \cos x + \sin x = 0$$

$$\Rightarrow \cos x + \sin x = -1$$

BY "R-TRANSFORMATION" OF THE L.H.S OR MANIPULATION

$$\Rightarrow \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x = -\frac{\sqrt{2}}{2}$$

NOTE THAT

$$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

$$\left(\begin{array}{l} x - \frac{\pi}{4} = 3\pi/4 \pm 2n\pi \\ x - \frac{\pi}{4} = 5\pi/4 \pm 2n\pi \end{array}\right. \qquad n = 0,1,2,3,\ldots$$

$$\left(\begin{array}{l} x = \pi \pm 2n\pi \\ x = \frac{3\pi}{2} \pm 2n\pi \end{array}\right.$$

IYGB – SYNOPTIC PAPER O – QUESTION 15

$$\Rightarrow x = \begin{cases} \pi \\ \cancel{\frac{3\pi}{2}} \end{cases}$$ UNDEFINED AS DENOMINATOR IS ZERO

USING $x=\pi$

$$y = \frac{1+\cos\pi}{1+\sin\pi} = \frac{1-1}{1+0} = 0 \qquad \therefore (\pi, 0)$$

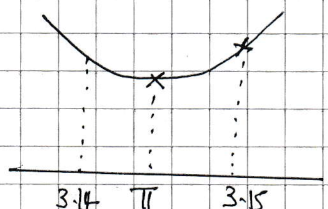
TO CHECK THE NATURE USE THE FUNCTION VALUE IN THE NEIGHBOURHOOD OF $x=\pi$ (OR THE GRADIENT FUNCTION IN THE SAME NEIGHBOURHOOD)

- $f(x) = \dfrac{1+\cos x}{1+\sin x}$

$f(3.14) = 0.00000126... > 0$

$f(\pi) = 0$

$f(3.15) = 0.0000356... > 0$



$\therefore (\pi, 0)$ IS A LOCAL MIN

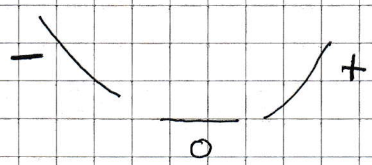
OR

- $f'(x) = -\dfrac{1+\cos x+\sin x}{(1+\sin x)^2}$

$f'(3.14) = -0.001588... < 0$

$f'(\pi) = 0$

$f'(3.15) = 0.008514... > 0$



$\therefore (\pi, 0)$ IS A LOCAL MIN

IYGB - SYNOPTIC PAPER O - QUESTION 16

a) WE ARE GIVEN THAT

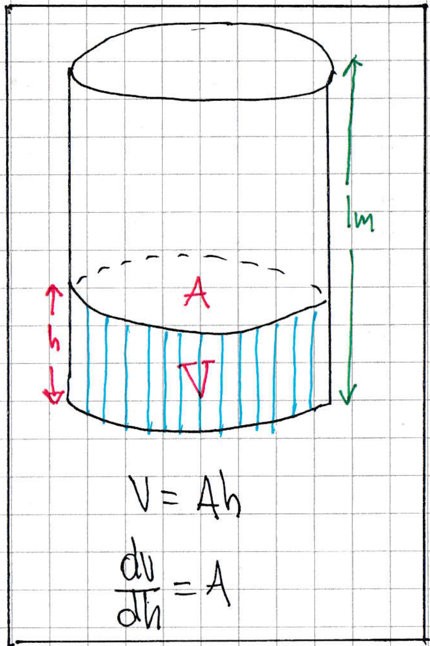
$$\Longrightarrow \frac{dV}{dt} = -kV^{\frac{1}{2}}$$

$$\Longrightarrow \frac{dV}{dh} \times \frac{dh}{dt} = -kV^{\frac{1}{2}}$$

$$\Longrightarrow A\frac{dh}{dt} = -k(Ah)^{\frac{1}{2}}$$

$$\Longrightarrow \frac{dh}{dt} = -k\frac{A^{\frac{1}{2}}h^{\frac{1}{2}}}{A}$$

$$\Longrightarrow \frac{dh}{dt} = -Bh^{\frac{1}{2}} \qquad \left(B = \frac{kA^{\frac{1}{2}}}{A}\right)$$



$V = Ah$

$\frac{dV}{dh} = A$

b) SOLVING THE O.D.E BY SEPARATING VARIABLES

$$\Longrightarrow \frac{1}{h^{\frac{1}{2}}}\,dh = -B\,dt$$

$$\Longrightarrow \int h^{-\frac{1}{2}}\,dh = \int -B\,dt$$

$$\Longrightarrow 2h^{\frac{1}{2}} = -Bt + C$$

$$\Longrightarrow \boxed{h^{\frac{1}{2}} = Pt + Q}$$

• APPLYING CONDITION $t=0,\ h=1 \Longrightarrow Q=1$

$$\boxed{h^{\frac{1}{2}} = 1 + Pt}$$

• APPLY CONDITION $t=5\ h=\frac{1}{2}$

$$\sqrt{\tfrac{1}{2}} = 1 + 5P$$

$$\frac{\sqrt{2}}{2} = 1 + 5P$$

$$\sqrt{2} = 2 + 10P$$

$$10P = \sqrt{2} - 2$$

$$P = \frac{1}{10}(\sqrt{2} - 2)$$

$$\therefore h^{\frac{1}{2}} = 1 - \frac{1}{10}(2 - \sqrt{2})t$$

IYGB - SYNOPTIC PAPER O - QUESTION 16

REARRANGING & TIDYING UP

$$\sqrt{h} = 1 - \frac{1}{10}(2-\sqrt{2})t$$

$$\frac{1}{10}(2-\sqrt{2})t = 1 - \sqrt{h}$$

$$(2-\sqrt{2})t = 10(1-\sqrt{h})$$

$$t = \frac{10(1-\sqrt{h})}{2-\sqrt{2}}$$

$$t = \frac{10(2+\sqrt{2})(1-\sqrt{h})}{(2-\sqrt{2})(2+\sqrt{2})}$$

$$t = \frac{10(2+\sqrt{2})(1-\sqrt{h})}{4-2}$$

$$t = \frac{10(2+\sqrt{2})(1-\sqrt{h})}{2}$$

$$t = 5(2+\sqrt{2})(1-\sqrt{h})$$ AS REQUIRED

d) **FIND T, I.E THE VALUE OF t WHEN h=0**

$$T = 5(2+\sqrt{2})(1-\sqrt{0})$$

$$T = 5(2+\sqrt{2})$$

$$\therefore \frac{1}{2}T = \frac{5}{2}(2+\sqrt{2})$$

FINALLY USING

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{10}(2-\sqrt{2})t$$

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{10}(2-\sqrt{2}) \times \frac{5}{2}(2+\sqrt{2})$$

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{4}(2-\sqrt{2})(2+\sqrt{2})$$

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{4}(4-2)$$

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{4} \times 2$$

$$\Rightarrow \sqrt{h} = 1 - \frac{1}{2}$$

$$\Rightarrow \sqrt{h} = \frac{1}{2}$$

$$\Rightarrow h = \frac{1}{4}$$

IYGB - SYNOPTIC PAPER O - QUESTION 17

LET $f(x) = \dfrac{x}{x+1}$ AND $f(x+h) = \dfrac{x+h}{x+h+1}$

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{\cancel{x^2} + \cancel{x} + \cancel{xh} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{(x+h+1)(x+1)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\frac{h}{(x+h+1)(x+1)}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{h}{(x+h+1)(x+1)} \div h\right]$$

$$= \lim_{h \to 0}\left[\frac{\cancel{h}}{(x+h+1)(x+1)} \times \frac{1}{\cancel{h}}\right]$$

$$= \lim_{h \to 0}\left[\frac{1}{(x+h+1)(x+1)}\right]$$

$$= \frac{1}{(x+1)(x+1)}$$

$$= \frac{1}{(x+1)^2}$$

IYGB - SYNOPTIC PAPER O - QUESTION 18

a) WRITE IN FUNCTION NOTATION

$$\Rightarrow 2x\tan x = 1$$

$$\Rightarrow 2x\tan x - 1 = 0$$

$$\Rightarrow f(x) = 2x\tan x - 1$$

$$f(0.6) = -0.17903\ldots < 0$$

$$f(0.7) = +0.179203\ldots > 0$$

AS $f(x)$ IS CONTINUOUS IN THE INTERVAL $(0.6, 0.7)$ THERE MUST BE AT LEAST ONE SOLUTION IN THIS INTERVAL

b) PREPARING TO USE THE NEWTON-RAPHSON METHOD WITH $x_1 = 0.65$

$$\Rightarrow f'(x) = 2\tan x + 2x\sec^2 x$$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x\tan x_n - 1}{2\tan x_n + 2x_n\sec^2 x_n}$$

$$= x_n - \frac{2x\sin x_n\cos x_n - \cos^2 x_n}{2\sin x_n\cos x_n + 2x_n}$$

$$= x_n - \frac{x\sin 2x_n - \cos^2 x_n}{\sin 2x_n + 2x_n}$$

- $x_1 = 0.65$
- $x_2 = 0.6532853557\ldots$
- $x_3 = 0.6532711874\ldots$
- $x_4 = 0.6532711871\ldots$

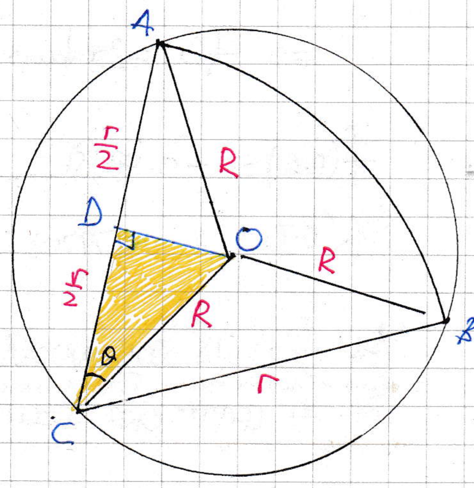
ETC

$\therefore\ x = 0.65327$ (5 d.p)

IYGB - SYNOPTIC PAPER O - QUESTION 18

c) LOOKING AT THE DIAGRAM ON TRIANGLE DOC

- LET THE RADIUS OF THE CIRCLE BE "R"
- LET THE RADIUS OF THE SECTOR BE "r"



$$\frac{\frac{r}{2}}{R} = \cos\theta$$

$$\frac{r}{2} = R\cos\theta$$

$$r = 2R\cos\theta$$

- AREA OF THE CIRCLE IS

$$\pi R^2$$

- AREA OF THE SECTOR

$$\frac{1}{2}r^2(2\theta) = r^2\theta = (2R\cos\theta)^2\theta = 4R^2\theta\cos^2\theta$$

- THE PROPORTION COVERED BY THE SECTOR IS

$$\frac{4\cancel{R^2}\theta\cos^2\theta}{\pi\cancel{R^2}} = \frac{4}{\pi}\theta\cos^2\theta$$

USING CALCULUS

$$V(\theta) = \frac{4}{\pi}\theta\cos^2\theta$$

$$V'(\theta) = \frac{4}{\pi}\left[1\times\cos^2\theta + \theta\times 2\cos\theta(-\sin\theta)\right]$$

$$V'(\theta) = \frac{4}{\pi}\left[\cos^2\theta - 2\theta\cos\theta\sin\theta\right]$$

$$V'(\theta) = \frac{4}{\pi}\cos\theta\left[\cos\theta - 2\theta\sin\theta\right]$$

IYGB – SYNOPTIC PAPER O – QUESTION 18

SOLVING FOR ZERO

$$\Rightarrow \frac{4}{\pi}\cos\theta\left[\cos\theta - 2\theta\sin\theta\right] = 0$$

$$\Rightarrow \cos\theta - 2\theta\sin\theta = 0$$

$$\Rightarrow 2\theta\sin\theta = \cos\theta$$

$$\Rightarrow \frac{2\theta\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$$

$$\Rightarrow 2\theta\tan\theta = 1$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

$0 < \theta < \frac{\pi}{2}$

THIS EQUATION HAS SOLUTION $\theta = 0.65327$

$$V(\theta) = \frac{4}{\pi}\theta\cos^2\theta$$

$$V(0.65327) = \frac{4}{\pi}(0.65327)\cos^2(0.65327) = 0.52451\ldots$$

$\therefore$ MAX PERCENTAGE IS 52.45 %