IYGB

Special Paper Z

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

The sum of the first n terms of an arithmetic series with first term a and common difference d, is denoted by S_n.

Simplify fully

$$S_n - 2S_{n+1} + S_{n+2}.$$ **(4)**

Question 2

The straight lines L_1 and L_2 have respective equations

$$4x + 2y = a \quad \text{and} \quad 5x + 4y = b.$$

It is given that L_1 and L_2 meet at the point P.

Express a in terms of b, given further that P lies in the second quadrant and is equidistant from the coordinate axes. **(7)**

Question 3

$$h(x) \equiv \frac{1}{\sqrt{x + \sqrt{x^2 - 1}}}, \quad x \in \mathbb{R}, \ x \geq 1.$$

Show that $h(x)$ can be expressed in the form

$$\sqrt{f(x)} - \sqrt{g(x)},$$

where $f(x)$ and $g(x)$ are linear functions to be found. **(8)**

Question 4

A curve C, has equation

$$(x-1)y^2 - 2xy + x = 0, \ x \ge 0 .$$

By completing the square in the above equation, express y in terms of x. **(9)**

Question 5

Two walkers, A and B, start their walk at the point P, at the same time.

They both walk in the same direction along a straight road, each walker with different constant speed.

The points Q and R lies on that road so that $|PQ| = 1$ km and $|QR| = 3$ km.

- Walker B passes through Q 60 s after walker A passed through Q.
- When walker A passes through R, walker B is 400 m behind A.

Determine the speed of each of the two walkers, in km h^{-1}. **(10)**

Question 6

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k} [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

Show by a detailed method that

$$\prod_{r=1}^{\infty} \left[1 + \left(\tfrac{1}{4}\right)^{2^r} \right] = \frac{16}{15} .$$ **(10)**

Question 7

A pole AB, of height h, is standing vertically on level horizontal ground with A on the circumference of a circle of radius a, centred at the point O.

The point C is another point on the circumference of this circle so that $\measuredangle COA = \theta$ and $\measuredangle ACB = \theta$.

Use a detailed method to show that

$$h = \frac{4a\sin\left(\frac{1}{2}\theta\right)\tan\left(\frac{1}{2}\theta\right)}{1-\tan^2\left(\frac{1}{2}\theta\right)}.$$

(9)

Question 8

The variables x and y are such so that

$$ax + by = c,$$

where a, b and c are non zero constants.

Show that the minimum value of $x^2 + y^2$ is

$$\frac{c^2}{a^2+b^2}.$$

(11)

Question 9

$$I=\int_{-1}^{1}(x+3)\sqrt{7-6x-x^2}\ dx$$

a) Use a suitable trigonometric substitution to show that $I=8\sqrt{3}$. **(12)**

b) Verify the answer of part **(a)** by an alternative method. **(4)**

Question 10

Solve the following equation

$$e^{4(x+1)^2}=\ln e^{-e}+\left(1+\frac{1}{e}\right)e^{2x^2+4x+3}.$$ **(10)**

Question 11

The curve C has parametric equations

$$x=t^2+2t,\quad y=2t^2+t,\quad t\in\mathbb{R}.$$

Show that a Cartesian equation of the curve is given by

$$4x^2+y^2-4xy+3x-6y=0.$$ **(8)**

Question 12

The straight lines l_1 and l_2 have the following vector equations

$$\mathbf{r}_1 = 13\mathbf{i} - 5\mathbf{j} + 8\mathbf{k} + \lambda(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{r}_2 = -5\mathbf{i} - 4\mathbf{j} + 8\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}),$$

where λ and μ are scalar parameters.

a) Show that l_1 and l_2 intersect at some point C and find its coordinates. **(4)**

b) Find the cosine of the acute angle between l_1 and l_2. **(2)**

The point A lies on l_1 where $\lambda = -1$ and the point B lies on l_2 where $\mu = 4$.

c) Determine a vector equation of the angle bisector of $\measuredangle ACB$. **(5)**

Question 13

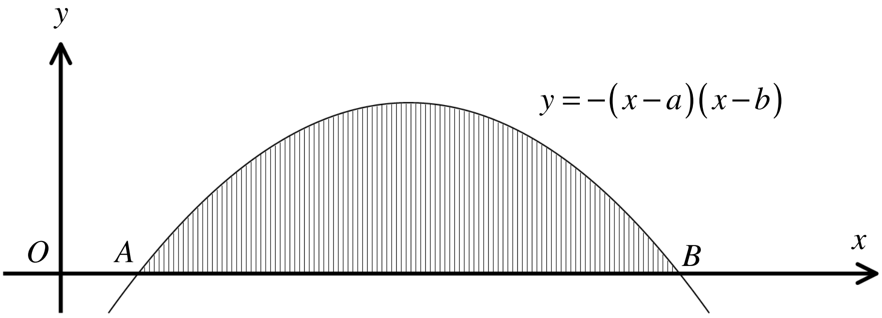
Sketch the graph of

$$y = \frac{|x|}{x+1}, \ x \in \mathbb{R}.$$

The sketch must include the equations of any asymptotes of the curve, and the coordinates of any points where the curve meets the coordinate axes. **(7)**

[*No credit will be given to non analytical sketches based on plotting coordinates*]

Question 14



The figure above shows the parabola with equation

$$y=-(x-a)(x-b),\ b>a>0.$$

The curve meets the x axis at the points A and B.

a) Show that the area of the finite region R, bounded by the parabola and the x axis is

$$\frac{1}{6}(b-a)^3. \qquad \textbf{(7)}$$

The midpoint of AB is N. The point M is the maximum point of the parabola.

b) Show clearly that the area of R is given by

$$k|AB||MN|,$$

where k is a constant to be found. **(5)**

Question 15

Use a trigonometric algebra to solve the following equation

$$(\arctan x)^2+(\operatorname{arccot} x)^2=\frac{5\pi^2}{8}. \qquad \textbf{(8)}$$

You may assume that $y=\operatorname{arccot} x$ is the inverse function of $y=\cot x$, $0\le x\le \pi$

Question 16

In a chemical reaction two substances X and Y bind together to form a third substance Z. In terms of their masses in grams, 1 part of substance X binds with 3 parts of substance Y to form 4 parts of substance Z.

Let z grams be the mass of substance Z formed, t minutes after the reaction started.

The rate at which Z forms is directly proportional to the product of the masses of X and Y, present at that instant.

Initially there were 10 grams of substance X, 10 grams of substance Y and none of substance Z, and the initial rate of formation of Z was 1.6 grams per minute.

a) Show clearly that

$$1000\frac{dz}{dt} = (40-z)(40-3z)\,. \qquad \textbf{(5)}$$

b) Solve the differential equation to show that

$$z = \frac{40\left(1-e^{-0.08t}\right)}{3-e^{-0.08t}}\,. \qquad \textbf{(10)}$$

c) State, with justification, the maximum mass of the substance Z that can ever be produced. **(1)**

Question 17

$$S = \frac{3}{8} + \frac{3\times 9}{8\times 16} + \frac{3\times 9\times 15}{8\times 16\times 24} + \frac{3\times 9\times 15\times 21}{8\times 16\times 24\times 32} + \frac{3\times 9\times 15\times 21\times 27}{8\times 16\times 24\times 32\times 40} \ldots$$

By considering a suitable binomial expansion, show that $S = 1$. **(8)**

Question 18

The function with equation $y = f(x)$ has smooth first and second derivatives.

Show that

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 \frac{d^2x}{dy^2} = 0 .$$ **(8)**

Question 19

$$I = \int_0^1 \left[\prod_{r=1}^{10}(x+r)\right]\left[\sum_{r=1}^{10}\left(\frac{1}{x+r}\right)\right] dx .$$

Show that $I = a \times b!$, where a and b are positive integers to be found. **(12)**

Question 20

Find, in terms of π, the general solution of the equation

$$(x+y)^2 + 4(x+y)\cos(x-y) + 4 = 0 .$$ **(16)**
