IYGB – MP2 PAPER W – QUESTION 1

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$\frac{d}{dx}\left[\sec x\right] = \lim_{h \to 0}\left[\frac{\sec(x+h) - \sec x}{h}\right]$$

WORK WITH SINES AND COSINES

$$\Rightarrow \frac{d}{dx}\left[\sec x\right] = \lim_{h \to 0}\left[\frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}\right]$$

$$\Rightarrow \frac{d}{dx}\left[\sec x\right] = \lim_{h \to 0}\left[\frac{1}{h}\left[\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}\right]\right]$$

NOW USING THE TRIGONOMETRIC IDENTITY

$$\cos A - \cos B \equiv -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos x - \cos(x+h) = -2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)$$

$$\cos x - \cos(x+h) = -2\sin\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)$$

HENCE WE NOW HAVE

$$\Rightarrow \frac{d}{dx}\left[\sec x\right] = \lim_{h \to 0}\left[\frac{1}{h}\left[\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos(x+h)\cos x}\right]\right]$$

SMALL ANGLE APPROXIMATION

$$= \lim_{h \to 0}\left[\frac{1}{\cancel{h}}\left[\frac{-2\sin\left(x+\frac{h}{2}\right)\times\left[-\frac{\cancel{h}}{2} + O(h^{\cancel{3}\,2})\right]}{\cos(x+h)\cos x}\right]\right]$$

$$= \lim_{h \to 0}\left[\frac{-2\sin\left(x+\frac{h}{2}\right)\left[-\frac{1}{2} + O(h^2)\right]}{\cos(x+h)\cos x}\right]$$

IYGB - MP2 PAPER W - QUESTION 1

$$= \lim_{h \to 0} \left[\frac{\sin\left(x+\frac{h}{2}\right) - 2\,O(h^2)}{\cos(x+h)\cos x} \right]$$

TAKING LIMITS NOW YIELDS

$$= \frac{\sin x}{\cos x \cos x}$$

$$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$$

$$= \tan x \sec x$$

IYGB - MP2 PAPER W - QUESTION 2

CONSIDER $\sqrt{2}^{\sqrt{2}}$

NOW THERE ARE 2 CASES TO CONSIDER

- $\sqrt{2}^{\sqrt{2}}$ = RATIONAL

IF THIS IS TRUE THEN WE FOUND AN IRRATIONAL NUMBER WHICH WHEN RAISED TO THE POWER OF AN IRRATIONAL NUMBER GIVES RATIONAL ...

OR

- $\sqrt{2}^{\sqrt{2}}$ = IRRATIONAL

$$\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = (\text{IRRATIONAL})^{\sqrt{2}}$$

$$\sqrt{2}^{2} = (\text{IRRATIONAL})^{\sqrt{2}}$$

$$2 = (\text{IRRATIONAL})^{\sqrt{2}}$$

AGAIN WE FOUND THAT AN IRRATIONAL NUMBER RAISED TO THE POWER OF AN IRRATIONAL NUMBER ($\sqrt{2}$) CAN GIVE A RATIONAL NUMBER

$\therefore$ AN IRRATIONAL NUMBER RAISED TO THE POWER OF AN IRRATIONAL NUMBER CAN PRODUCE A RATIONAL NUMBER

IYGB - MP2 PAPER W - QUESTION 3

DIFFERENTIATING IMPLICITLY WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}\left[2x\sin y\right] + \frac{d}{dx}\left[2\cos 2y\right] = \frac{d}{dx}(1)$$

$$\Rightarrow 2\sin y + 2x\cos y\frac{dy}{dx} - 4\sin 2y\frac{dy}{dx} = 0$$

$$\Rightarrow 2\sin y = 4\sin 2y\frac{dy}{dx} - 2x\cos y\frac{dy}{dx}$$

$$\Rightarrow \sin y = 2\sin 2y\frac{dy}{dx} - x\cos y\frac{dy}{dx}$$

$$\Rightarrow \sin y = (2\sin 2y - x\cos y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin y}{2\sin 2y - x\cos y}$$

NOW FOR A "VERTICAL" TANGENT WE NEED INFINITE GRADIENT SO THE DENOMINATOR ABOVE MUST BE ZERO

$$\Rightarrow 2\sin 2y - x\cos y = 0$$

$$\Rightarrow 4\sin y\cos y - x\cos y = 0$$

$$\Rightarrow \cos y\left[4\sin y - x\right] = 0$$

EITHER $\cos y = 0$

$$\Rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow \begin{cases} 2x\sin\frac{\pi}{2} + 2\cos\pi = 1 \\ 2x\sin\frac{3\pi}{2} + 2\cos 3\pi = 1 \end{cases}$$

$$\Rightarrow \begin{cases} 2x - 2 = 1 \\ -2x - 2 = 1 \end{cases}$$

$$\Rightarrow x = \begin{cases} 3/2 \\ -3/2 \end{cases}$$

OR $x = 4\sin y$

$$\Rightarrow 2(4\sin y)\sin y + 2\cos 2y = 1$$

$$\Rightarrow 8\sin^2 y + 2(1 - 2\sin^2 y) = 1$$

$$\Rightarrow 4\sin^2 y = -1$$

$$\Rightarrow \sin^2 y = -\frac{1}{4}$$

NO SOLUTIONS HERE

IYGB – MP2 PAPER W – QUESTION 4

PROCEED AS FOLLOWS

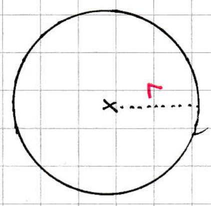
$$\frac{dr}{dt} = \frac{1}{r^2}$$

$$\frac{dr}{dA} \times \frac{dA}{dt} = \frac{1}{r^2}$$

$$\frac{1}{2\pi r} \times \frac{dA}{dt} = \frac{1}{r^2}$$

$$\frac{dA}{dt} = \frac{2\pi r}{r^2}$$

$$\frac{dA}{dt} = \frac{2\pi}{r}$$



$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dr}{dA} = \frac{1}{2\pi r}$$

MANIPULATE FURTHER

$$\frac{dA}{dt} = \frac{2\pi^2}{\pi r}$$

$$\left(\frac{dA}{dt}\right)^2 = \frac{4\pi^4}{\pi^2 r^2}$$

$$\left(\frac{dA}{dt}\right)^2 = \frac{4\pi^3}{\pi r^2}$$

$$\left(\frac{dA}{dt}\right)^2 = \frac{4\pi^3}{A}$$

$$\frac{dA}{dt} = +\sqrt{\frac{4\pi^3}{A}}$$

AS REQUIRED

NOTE AS $r > 0 \Rightarrow \frac{1}{r^2} > 0$

$\Rightarrow \frac{dr}{dt} > 0$

$\Rightarrow \frac{dA}{dt} > 0$

IYGB – MP2 PAPER W – QUESTION 5

a) EXPAND BINOMIALLY UP TO x^3

$$(1-8x)^{\frac{1}{4}} = 1 + \frac{\frac{1}{4}}{1}(-8x) + \frac{\frac{1}{4}(-\frac{3}{4})}{1\times2}(-8x)^2 + \frac{\frac{1}{4}(-\frac{3}{4})(-\frac{7}{4})}{1\times2\times3}(-8x)^3 + O(x^4)$$

$$(1-8x)^{\frac{1}{4}} = 1 - 2x - 6x^2 - 28x^3 + O(x^4)$$

b) PROCEED AS FOLLOWS.

$$(1+ax)(1+bx^2)^5 - (1-8x)^{\frac{1}{4}}$$

$$= (1+ax)\left[1 + \frac{5}{1}(bx^2)^1 + O(x^4)\right] - \left[1 - 2x - 6x^2 - 28x^3 + O(x^4)\right]$$

$$= (1+ax)\left[1 + 5bx^2 + O(x^4)\right] - \left[1 - 2x - 6x^2 - 28x^3 + O(x^4)\right]$$

$$= 1 + ax + 5bx^2 + 5abx^3 + O(x^4) - 1 + 2x + 6x^2 + 28x^3 + O(x^4)$$

$$= \underbrace{(a+2)}_{\text{ZERO}}x + \underbrace{(5b+6)}_{\text{ZERO}}x^2 + (5ab+28)x^3 + O(x^4)$$

$\therefore\ a = -2$

$b = -\frac{6}{5}$

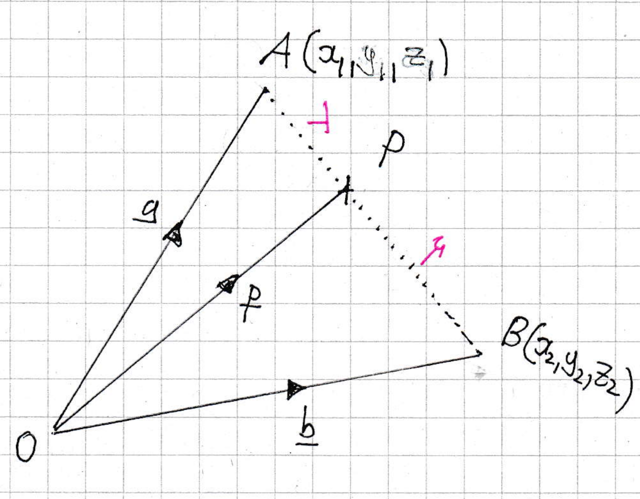
$\therefore$ COEFFICIENT OF x^3 IS $5ab + 28$

$$= 5(-2)\left(-\tfrac{6}{5}\right) + 28$$

$$= 40$$

IYGB – MP2 PAPER W QUESTION 6

STARTING WITH A DIAGRAM



$$\overrightarrow{AP} = \frac{\lambda}{\lambda+\mu}\overrightarrow{AB}$$

$$\overrightarrow{AP} = \frac{\lambda}{\lambda+\mu}\left(\overrightarrow{AO}+\overrightarrow{OB}\right)$$

$$\overrightarrow{AP} = \frac{\lambda}{\lambda+\mu}\left(-\underline{a}+\underline{b}\right)$$

$$\overrightarrow{AP} = \frac{\lambda}{\lambda+\mu}\left(\underline{b}-\underline{a}\right)$$

NOW THE POSITION VECTOR OF P

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \underline{a} + \frac{\lambda}{\lambda+\mu}\left(\underline{b}-\underline{a}\right) = \frac{\underline{a}(\lambda+\mu)+\lambda(\underline{b}-\underline{a})}{\lambda+\mu}$$

$$= \frac{\cancel{\lambda\underline{a}} + \mu\underline{a} + \lambda\underline{b} - \cancel{\lambda\underline{a}}}{\lambda+\mu} = \frac{\mu\underline{a}+\lambda\underline{b}}{\lambda+\mu}$$

SWITCHING INTO COMPONENTS

$$\overrightarrow{OP} = \underline{p} = \frac{\mu(x_1, y_1, z_1) + \lambda(x_2, y_2, z_2)}{\lambda+\mu}$$

$$= \frac{\mu(x_1\underline{i}+y_1\underline{j}+z_1\underline{k}) + \lambda(x_2\underline{i}+y_2\underline{j}+z_2\underline{k})}{\lambda+\mu}$$

$$= \frac{(\mu x_1+\lambda x_2)\underline{i} + (\mu y_1+\lambda y_2)\underline{j} + (\mu z_1+\lambda z_2)\underline{k}}{\lambda+\mu}$$

//

A. 24/01/2020

IYGB - MP2 PAPER W - QUESTION 7

NEGATIVE POWER OF x HAVE THE PROPERTY OF OF REVERSING SIGN ON DIFFERENTIATION, (SO DOES THE NEGATIVE EXPONENTIAL)

E.G. x^{-1}, $-x^{-2}$, $+2x^{-3}$, $-6x^{-4}$, $+24x^{-5}$ ETC

AS WE NEED THE FUNCTION TO HAVE POSITIVE GRADIENT FUNCTION WE MAY START WITH

$$-\frac{1}{x} \qquad +\frac{1}{x^2} \qquad -\frac{2}{x^3}$$

BUT THIS IN NOT DEFINED AT $x=0$, SO WE MAY TRANSLATE BY 2 UNITS TO THE LEFT TO INCLUDE $x=-1$

HENCE $f(x) = -\dfrac{1}{x+2}$

$f'(x) = \dfrac{1}{(x+2)^2}$ WHICH IS POSITIVE $-1 \leqslant x \leqslant 5$

$f''(x) = \dfrac{-2}{(x+2)^3}$ WHICH IS NEGATIVE $-1 \leqslant x \leqslant 5$

IYGB - MP2 PAPER W - QUESTION 8

START BY DIFFERENCING

$$2 \xrightarrow{-3} -1 \xrightarrow{+6} 5 \xrightarrow{-9} -4 \xrightarrow{+12} 8 \xrightarrow{-15} -7$$

- FORMULA MUST BE A RECURRENCE
- THERE MUST BE A TERM $(-1)^n$ AS THE TERMS ALTERNATE
- THERE MUST BE A MULTIPLE OF 3 TERM

TRY THE RECURRENCE

$$u_{n+1} = u_n + (-1)^n(3n)$$

$$u_1 = 2$$

$$u_2 = u_1 + (-1)^1(3\times1) = 2 - 3 = -1$$

$$u_3 = u_2 + (-1)^2(3\times2) = -1 + 6 = 5$$

$$u_4 = u_3 + (-1)^3(3\times3) = 5 - 9 = -4$$

$$u_5 = u_4 + (-1)^4(3\times4) = -4 + 12 = 8$$

$$u_6 = u_5 + (-1)^5(3\times5) = 8 - 15 = -7$$

ETC

$$\therefore \underline{u_{n+1} = u_n + (-1)^n(3n)}$$

IYGB - MP2 PAPER W - QUESTION 9

USING THE FACT THAT $P(0,5)$ LIES ON BOTH OBJECTS

$$y = |3x+a| + b$$
$$5 = |0+a| + b$$
$$5 = |a| + b$$
$$a + b = 5$$

$a > 0$

$$b = 5 - a$$

NOW SOLVING SIMULTANEOUSLY

$$|3x+a| + b = 2x+5$$
$$|3x+a| + \cancel{5} - a = 2x + \cancel{5}$$
$$|3x+a| = 2x+a$$

SOLVING WE OBTAIN

$$\begin{cases} 3x+a = 2x+a \\ 3x+a = -(2x+a) \end{cases}$$

$$\begin{cases} \cancel{x=0} \text{ (ALREADY KNOWN)} \\ 5x = -2a \end{cases}$$

$$x = -\tfrac{2}{5}a$$

FINALLY USING $y = 2x+5$

$$y = 2\left(-\tfrac{2}{5}a\right) + 5$$
$$y = -\tfrac{4}{5}a + 5$$

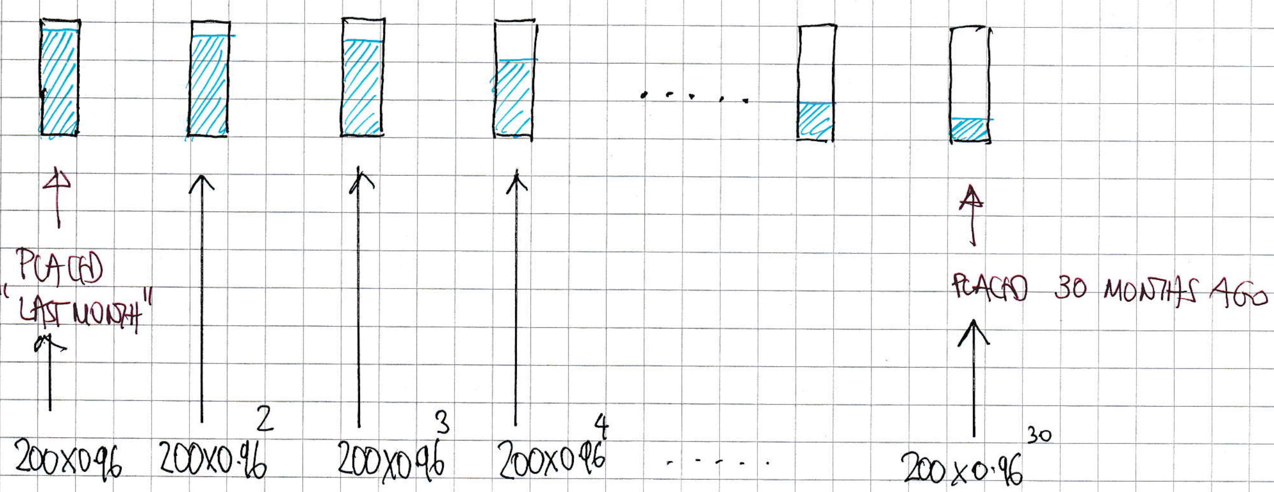
$$\therefore \; Q\left(-\tfrac{2}{5}a,\; 5-\tfrac{4}{5}a\right)$$

IYGB – MP2 PAPER W – QUESTION 10

STARTING THE MODELLING BY LOOKING AT A SINGLE CONTAINER

MONTH	START	END
1	200	200×0.96
2	200×0.96	200×0.96^2
3	200×0.96^2	200×0.96^3
4	200×0.96^3	200×0.96^4
$\vdots$		
k	$200 \times 0.96^{k-1}$	200×0.96^k

LOOKING AT ALL THE 30 CONTAINERS, PLACED OVER THE ENTIRE 30 MONTH PERIOD



ADDING ALL THESE AMOUNTS

$$\Rightarrow \text{TOTAL} = 200 \times 0.96 + 200 \times 0.96^2 + 200 \times 0.96^3 + \dots + 200 \times 0.96^{30}$$

IYGB – MP2 PAPER W – QUESTION 10

$$\Longrightarrow \text{TOTAL} = 200\Big[\underbrace{0.96^1 + 0.96^2 + 0.96^3 + \dots + 0.96^{30}}\Big]$$

THIS IS A G.P

$a = 0.96$

$r = 0.96$

$n = 30$

$$\Longrightarrow \text{TOTAL} = 200 \times \frac{0.96\left[1 - 0.96^{30}\right]}{1 - 0.96}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

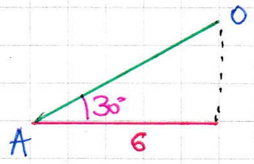
$$\Longrightarrow \text{TOTAL} = 4800\left(1 - 0.96^{30}\right)$$

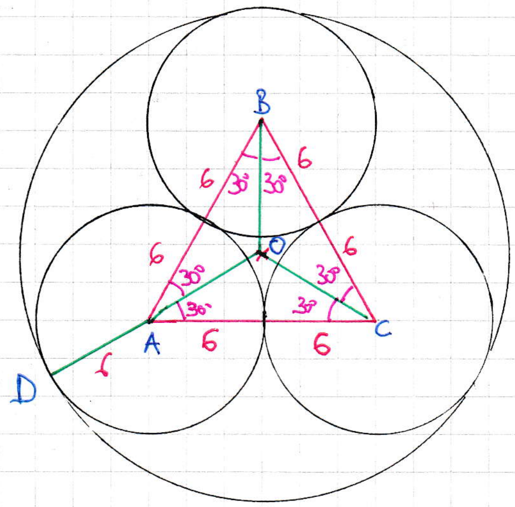
$$\Longrightarrow \text{TOTAL} = 3389$$

(NEAREST LITRE)

IYGB - MP2 PAPER W - QUESTION 11

START WITH A DIAGRAM





$$\frac{6}{|AO|} = \cos 30$$

$$\frac{6}{|AO|} = \frac{\sqrt{3}}{2}$$

$$\sqrt{3}|AO| = 12$$

$$3|AO| = 12\sqrt{3}$$

$$|AO| = 4\sqrt{3}$$

HENCE $|OD| = 6 + 4\sqrt{3}$

IYGB – MP2 PAPER N – QUESTION 12

SUBSTITUTE THE PARAMETRICS INTO $xy = 3$

$$\frac{4tp}{t+p} \times \frac{4}{t+p} = 3$$

$$\frac{16tp}{(t+p)^2} = 3$$

$$16tp = 3(t+p)^2$$

$$16tp = 3(t^2 + 2tp + p^2)$$

$$16tp = 3t^2 + 6tp + 3p^2$$

$$0 = 3t^2 - 10tp + 3p^2$$

$$0 = (3t - p)(t - 3p)$$

THUS WE HAVE

$$3t - p = 0$$

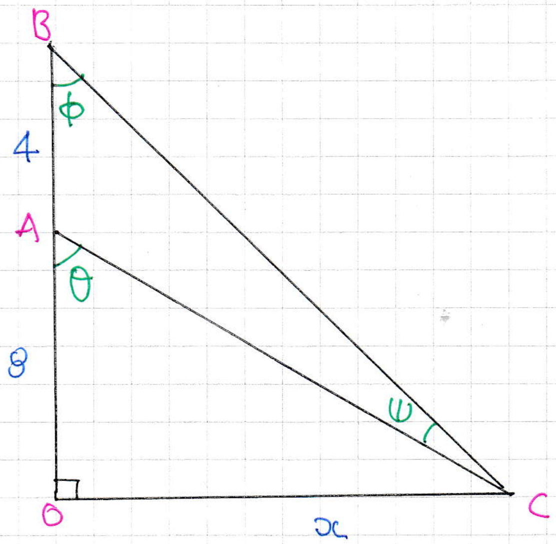
$$p = 3t$$

$$t - 3p = 0$$

$$3p = t$$

$$p = \frac{1}{3}t$$

IYGB - MP2 PAPER W - QUESTION 13



LOOKING AT THE DIAGRAM

$$\hat{BAC} = 180 - \theta$$

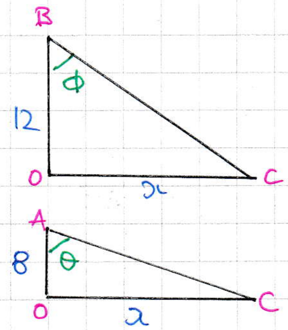
LOOKING AT $\overset{\triangle}{ABC}$

$$\phi + (180 - \theta) + \psi = 180$$

$$\phi - \theta + \psi = 0$$

$$\psi = \theta - \phi$$

TAKING TANGENTS BOTH SIDES



$$\tan\psi = \tan(\theta - \phi)$$

$$\tan\psi = \frac{\tan\theta - \tan\phi}{1 - \tan\theta\tan\phi}$$

$$\tan\psi = \frac{\frac{x}{8} - \frac{x}{12}}{1 - \frac{x}{8} \times \frac{x}{12}}$$

$$\tan\psi = \frac{\frac{x}{8} - \frac{x}{12}}{1 - \frac{x^2}{96}}$$

$$\tan\psi = \frac{96\left(\frac{x}{8}\right) - 96\left(\frac{x}{12}\right)}{96 \times 1 - 96\,\frac{x^2}{96}}$$

$$\tan\psi = \frac{12x - 8x}{96 - x^2}$$

$$\therefore \tan\psi = \frac{4x}{96 - x^2}$$

AS REQUIRED

IYGB – MP2 PAPER W – QUESTION 14

DIFFERENTIATING WITH RESPECT TO x, NOTING THAT $\frac{d}{dx}(a^x) = a^x \ln a$

$$y = 4 \times 8^{x+1} - 2^{x+1}$$

$$\frac{dy}{dx} = 4 \times 8^{x+1} \times \ln 8 - 2^{x+1} \ln 2$$

NEXT FIND THE x INTERCEPT IE $y=0$

$$0 = 4 \times 8^{x+1} - 2^{x+1}$$

$$0 = 4 \times 8 \times 8^x - 2 \times 2^x$$

$$0 = 32 \times 8^x - 2 \times 2^x$$

$$2 \times 2^x = 32 \times 8^x$$

$$\frac{1}{16} = \frac{8^x}{2^x}$$

$$\frac{1}{16} = 4^x$$

$$x = -2 \qquad \text{IE} \quad (-2, 0)$$

FIND THE GRADIENT AT $x=-2$

$$\left.\frac{dy}{dx}\right|_{x=-2} = 4 \times 8^{-1} \times \ln 8 - 2^{-1} \times \ln 2$$

$$= \frac{1}{2}\ln 8 - \frac{1}{2}\ln 2$$

$$= \frac{1}{2}(\ln 8 - \ln 2)$$

$$= \frac{1}{2}\ln 4$$

$$= \ln 2$$

IYGB - MP2 PAPER W - QUESTION 14

FINALLY THE EQUATION OF THE TANGENT, $m = \ln 2$ THROUGH $(-2, 0)$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = \ln 2 (x + 2)$$

// AS REQUIRED

IYGB - MP2 PAPER W - QUESTION 15

a) I) TRANSPOSING AS FOLLOWS

$$u^2 = \frac{1-x^2}{(1-x)^2} = \frac{(1-x)(1+x)}{(1-x)^2} = \frac{1+x}{1-x}$$

$$\Rightarrow u^2(1-x) = 1+x$$

$$\Rightarrow u^2 - xu^2 = 1+x$$

$$\Rightarrow u^2 - 1 = xu^2 + x$$

$$\Rightarrow u^2 - 1 = x(u^2+1)$$

$$\Rightarrow x = \frac{u^2-1}{u^2+1}$$

AS REQUIRED

II) USING THE ABOVE RESULT

$$1 - x^2 = 1 - \left(\frac{u^2-1}{u^2+1}\right)^2 = 1 - \frac{u^4-2u^2+1}{u^4+2u^2+1}$$

$$= \frac{u^4+2u^2+1-(u^4-2u^2+1)}{u^4+2u^2+1}$$

$$= \frac{\cancel{u^4}+2u^2\cancel{+1}\cancel{-u^4}+2u^2\cancel{-1}}{(u^2+1)^2}$$

$$= \frac{4u^2}{(u^2+1)^2}$$

AS REQUIRED

(III) DIFFERENTIATE (I) WITH RESPECT TO u

$$x = \frac{u^2-1}{u^2+1} = \frac{(u^2+1)-2}{u^2+1} = 1 - \frac{2}{u^2+1} = 1-2(u^2+1)^{-1}$$

$$\frac{dx}{du} = 0 + 2(u^2+1)^{-2} \times (2u)$$

$$\frac{dx}{du} = \frac{4u}{(u^2+1)^2}$$

AS REQUIRED

IYGB - MP2 PAPER W - QUESTION 15

b) USING THE RESULTS FROM PART (a)

$$\int \frac{3}{(4x+5)\sqrt{1-x^2} - 3(1-x^2)}\,dx$$

$$u^2 = \frac{1-x^2}{(1-x)^2}$$

$$x = \frac{u^2-1}{u^2+1}$$

$$dx = \frac{4u\,du}{(u^2+1)^2}$$

$$\sqrt{1-x^2} = \frac{2u}{u^2+1}$$

$$= \int \frac{3}{\left[4\left(\frac{u^2-1}{u^2+1}\right)+5\right]\left(\frac{2u}{u^2+1}\right) - 3\left(\frac{4u^2}{(u^2+1)^2}\right)} \times \frac{4u}{(u^2+1)^2}\,du$$

$$= \int \frac{12u}{\left[4\left(\frac{u^2-1}{u^2+1}\right)+5\right]\times(2u)(u^2+1) - 3\times 4u^2}\,du$$

$$= \int \frac{12u}{\left(\frac{4u^2-4+5u^2+5}{\cancel{u^2+1}}\right)(2u)\cancel{(u^2+1)} - 12u^2}\,du$$

$$= \int \frac{12u}{(9u^2+1)2u - 12u^2}\,du = \int \frac{6}{(9u^2+1) - 6u}\,du$$

$$= \int \frac{6}{9u^2-6u+1}\,du = \int \frac{6}{(3u-1)^2}\,du = \int 6(3u-1)^{-2}\,du$$

$$= -2(3u-1)^{-1} + C = \frac{-2}{3u-1} + C = \frac{2}{1-3u} + C$$

$$= \frac{2}{1 - 3\frac{\sqrt{1-x^2}}{1-x}} + C = \frac{2(1-x)}{1-x-3\sqrt{1-x^2}} + C$$

$$= \frac{2(1-x)}{(1-x) - 3(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}} + C = \frac{2\sqrt{1-x}}{\sqrt{1-x} - 3\sqrt{1+x}} + C$$

IYGB - MP2 PAPER W - QUESTION 16

USING A SUBSTITUTION

$$u = \frac{dy}{dx} \qquad \frac{du}{dx} = \frac{d^2y}{dx^2}$$

HENCE THE O.D.E. TRANSFORMS TO

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{2dy}{dx} = 1$$

$$\Rightarrow \frac{du}{dx} + 2u = 1$$

$$\Rightarrow \frac{du}{dx} = 1 - 2u$$

$$\Rightarrow \frac{1}{1-2u}\,du = 1\,dx$$

$$\Rightarrow \int \frac{1}{1-2u}\,du = \int 1\,dx$$

$$\Rightarrow -\frac{1}{2}\ln|1-2u| = x + C$$

$$\Rightarrow \ln|1-2u| = -2x + A$$

$$\Rightarrow 1 - 2u = e^{A-2x}$$

$$\Rightarrow 1 - 2u = Ce^{-2x}$$

$$\Rightarrow 1 + Ce^{-2x} = 2u$$

$$\Rightarrow u = \frac{1}{2} + Ce^{-2x}$$

IYGB – MP2 PAPER W – QUESTION 16

REVERSING THE TRANSFORMATION

$$\frac{dy}{dx} = \frac{1}{2} + Ce^{-2x}$$

$x = 0 \quad \frac{dy}{dx} = 1$

$1 = \frac{1}{2} + C$

$C = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{2}e^{-2x}$$

$$y = \frac{1}{2}x - \frac{1}{4}e^{-2x} + k$$

APPLY CONDITION $x = 0$, $y = -\frac{1}{4}$

$$-\frac{1}{4} = 0 - \frac{1}{4} + k$$

$$k = 0$$

$$\therefore \ y = \frac{1}{2}x - \frac{1}{4}e^{-2x}$$

IYGB – MP2 PAPER W – QUESTION 17

FIND THE x CO-ORDINATES OF THE POINTS OF INTERSECTION A & B

$$\left.\begin{array}{l} y = x \\ y = x + \cos x \end{array}\right\} \Rightarrow x = x + \cos x$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = \begin{cases} -\pi/2 \\ \pi/2 \end{cases} \quad \text{(BY INSPECTION)}$$

HENCE THE REQUIRED AREA IS GIVEN BY THE INTEGRAL

$$\int_{-\pi/2}^{\pi/2} (x + \cos x) - x \, dx = \int_{-\pi/2}^{\pi/2} \cos x \, dx$$

$$= 2\int_0^{\pi/2} \cos x \, dx$$

$$= \Big[2\sin x\Big]_0^{\pi/2}$$

$$= 2\sin\tfrac{\pi}{2} - \cancel{2\sin 0}$$

$$= 2$$

AS REQUIRED

ALTERNATIVE METHOD

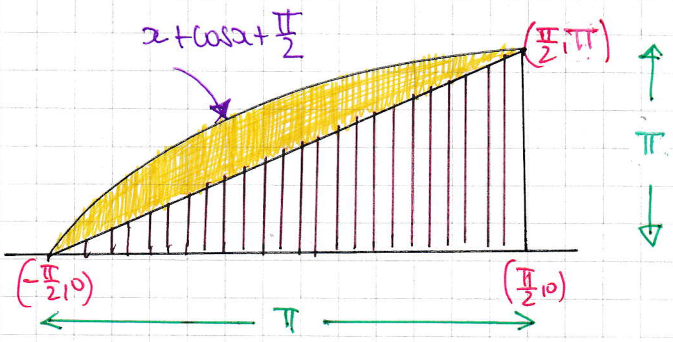
AFTER OBTAINING THE CO-ORDINATES OF A & B AS

$$A\left(-\tfrac{\pi}{2}, -\tfrac{\pi}{2}\right) \;\&\; B\left(\tfrac{\pi}{2}, \tfrac{\pi}{2}\right)$$

TRANSLATE BOTH GRAPHS UPWARDS BY $\pi/2$ IE

$$y = x + \cos x + \tfrac{\pi}{2}$$
$$y = x + \tfrac{\pi}{2}$$

IYGB − MP2 PAPER W − QUESTION 17



THE AREA UNDER THE CURVE $x+\cos x+\frac{\pi}{2}$ IS

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cancel{x}+\cos x+\frac{\pi}{2}\,dx = \int_{0}^{\pi} 2\cos x+\pi \quad dx$$

ODD EVEN

$$\Big[2\sin x+\pi x\Big]_{0}^{\pi/2} = \left(2+\tfrac{1}{2}\pi^2\right)-(0) = 2+\frac{\pi^2}{2}$$

AREA OF THE TRIANGLE

$$\tfrac{1}{2}\times\pi\times\pi = \tfrac{1}{2}\pi^2$$

HENCE THE REQUIRED AREA IS

$$2+\frac{\pi^2}{2}-\tfrac{1}{2}\pi^2 = 2$$

AS BEFORE