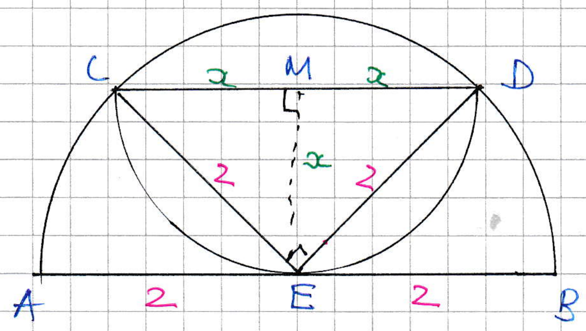
IYGB – MP2 PAPER T – QUESTION 1

LOOKING AT THE DIAGRAM

- TRIANGLE $C\hat{E}D$ IS ISOSCELES AND RIGHT ANGLED



- BY PYTHAGORAS

$$|CM|^2 + |ME|^2 = |CE|^2$$
$$x^2 + x^2 = 2^2$$
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

- AREA OF SHADED SEMICIRCLE CDE

$$\text{"}\tfrac{1}{2}\pi r^2\text{"} = \tfrac{1}{2}\pi \times x^2 = \tfrac{1}{2}\pi(\sqrt{2})^2 = \pi$$

- AREA OF TRIANGLE CDE

$$\tfrac{1}{2}|CD||ME| = \tfrac{1}{2}(2x)x = x^2 = (\sqrt{2})^2 = 2$$

- AREA OF CIRCULAR SECTOR

$$\tfrac{1}{2}r^2\theta^c = \tfrac{1}{2} \times |CE| \times C\hat{E}D = \tfrac{1}{2} \times 2^2 \times \tfrac{\pi}{2} = \pi$$

- REQUIRED AREA IS

$$\underbrace{\pi}_{\text{"SEMICIRCLE"}} + \underbrace{(\pi - 2)}_{\text{"SECTOR – TRIANGLE"}} = 2\pi - 2$$

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METHOD A

$$60^2 - 59^2 + 58^2 - 57^2 + 56^2 - 55^2 + \dots + 22^2 - 21^2$$

$$= (60-59)(60+59) + (58+57)(58-57) + (56-56)(56+55) + \dots + (22+21)(22-21)$$

$$= 119 + 115 + 111 + \dots + 43$$

THIS IS AN ARITHMETIC PROGRESSION WITH $a = 119$, $L = 43$, $d = -4$

$$u_n = a + (n-1)d$$

$$43 = 119 + (n-1)(-4)$$

$$43 = 119 - 4n + 4$$

$$4n = 80$$

$$n = 20$$

THUS USING $S_n = \frac{n}{2}[a+L]$

$$S_{20} = \frac{20}{2}[119+43] = 10 \times 162 = 1620$$

AS REQUIRED

METHOD B

$$60^2 - 59^2 + 58^2 - 57^2 + 56^2 - 55^2 + \dots + 22^2 - 21^2$$

$$= \left[60^2 + 58^2 + 56^2 + \dots + 22^2\right] - \left[59^2 + 57^2 + 55^2 + \dots + 21^2\right]$$

WRITE IN SIGMA NOTATION

$$= \left[\sum_{r=1}^{30}(2r)^2 - \sum_{r=1}^{10}(2r)^2\right] - \left[\sum_{r=1}^{30}(2r-1)^2 - \sum_{r=1}^{10}(2r-1)^2\right]$$

$$= \left[\sum_{r=1}^{30} 4r^2 - \sum_{r=1}^{30}(2r-1)^2\right] - \left[\sum_{r=1}^{10}(2r-1)^2 - \sum_{r=1}^{10} 4r^2\right]$$

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$$= \sum_{r=1}^{30}\left[4r^2-(2r-1)^2\right] - \sum_{r=1}^{10}\left[4r^2-(2r-1)^2\right]$$

$$= \sum_{r=1}^{30}(4r-1) - \sum_{r=1}^{10}(4r-1)$$

USING LINEARITY OF SIGMA OPERATOR

$$= 4\sum_{r=1}^{30} r - \sum_{r=1}^{30} 1 - 4\sum_{r=1}^{10} r + \sum_{r=1}^{10} 1$$

USING THAT $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ & $\sum_{r=1}^{n} 1 = n$

$$= 4\times\frac{1}{2}\times30\times31 - 30 - 4\times\frac{1}{2}\times10\times11 + 10$$

$$= 2\times30\times31 - 30 - 2\times10\times11 + 10$$

$$= 1860 - 30 - 220 + 10$$

$$= 1620$$

AS BEFORE

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EXPAND $f(x)$ UP TO x^3

$$f(x) = (1-5x)^{-2} = 1 + \frac{-2}{1}(-5x) + \frac{-2(-3)}{1\times 2}(-5x)^2 + \frac{-2(-3)(-4)}{1\times 2\times 3}(-5x)^3 + O(x^4)$$

$$= 1 + 10x + 75x^2 + 500x^3 + O(x^4)$$

NOW EXPAND $(8x+3)^3$

$$(8x+3)^3 = 512x^3 + 3(8x)^2(3) + 3(8x)\times 3^2 + 3^3$$

$$= 512x^3 + 576x^2 + 216x + 27$$

LOOKING AT THE EQUATION FOR "SMALL x"

$$f(x) - (8x+3)^3 = -37x^3 - 475x^2 - 157x - 28$$

$$\left.\begin{array}{r} 500x^3 + 75x^2 + 10x + 1 \\ -512x^3 - 576x^2 - 216x - 27 \end{array}\right\} \simeq -37x^3 - 475x^2 - 157x - 28$$

$$-12x^3 - 501x^2 - 206x - 26 \simeq -37x^3 - 475x^2 - 157x - 28$$

$$25x^3 - 26x^2 - 49x + 2 \simeq 0$$

LOOK FOR FACTORS

$$x = 1 \implies 25 - 26 - 49 + 2 \neq 0$$

$$x = -1 \implies -25 - 26 + 49 + 2 = 0$$

ie $(x+1)$ IS A FACTOR

BY LONG DIVISION, MANIPULATIONS OR INSPECTION

$$25x^2(x+1) - 51x(x+1) + 2(x+1) \simeq 0$$

$$(25x^2 - 51x + 2)(x+1) \simeq 0$$

$$(25x - 1)(x - 2)(x+1) \simeq 0$$

$$x = \cancel{-1},\ \cancel{2},\ \frac{1}{25} \qquad |x| < \frac{1}{5}$$

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FORM A RECURRENCE RELATION WHICH GIVES THE AMOUNT OF WASTE AT THE END OF THE DAY

$$u_{n+1} = (u_n + 600) \times 0.6$$

$$u_{n+1} = 360 + 0.6u_n \qquad \text{WITH } u_1 = 600 \times 0.6 = 360$$

(NOTE REMOVING 40% LEAVES 60%)

LOOK FOR A PATTERN

- $u_1 = 360$
- $u_2 = 360 + 0.6u_1 = 360 + 0.6 \times 360$
- $u_3 = 360 + 0.6u_2 = 360 + 0.6(360 + 0.6 \times 360) = 360 + 0.6 \times 360 + 0.6^2 \times 360$
- $u_4 = 360 + 0.6u_3 = 360 + 0.6(360 + 0.6 \times 360 + 0.6^2 \times 360)$

$$= 360 + 360 \times 0.6 + 360 \times 0.6^2 + 360 \times 0.6^3$$

$$= 360\left[1 + 0.6 + 0.6^2 + 0.6^3\right]$$

GENERALIZING

$$u_n = 360\underbrace{\left[1 + 0.6 + 0.6^2 + 0.6^3 + \dots + 0.6^{n-1}\right]}_{\text{GEOMETRIC PROGRESSION WITH } a=1,\ r=0.6,\ n \text{ TERMS}}$$

$$u_n = 360 \times \frac{1(1-0.6^n)}{1-0.6} \quad \leftarrow S_n = \frac{a(1-r^n)}{1-r}$$

$$u_n = 900(1 - 0.6^n)$$

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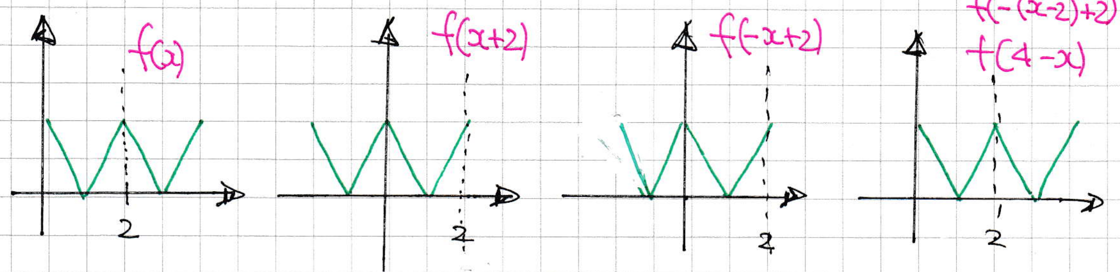
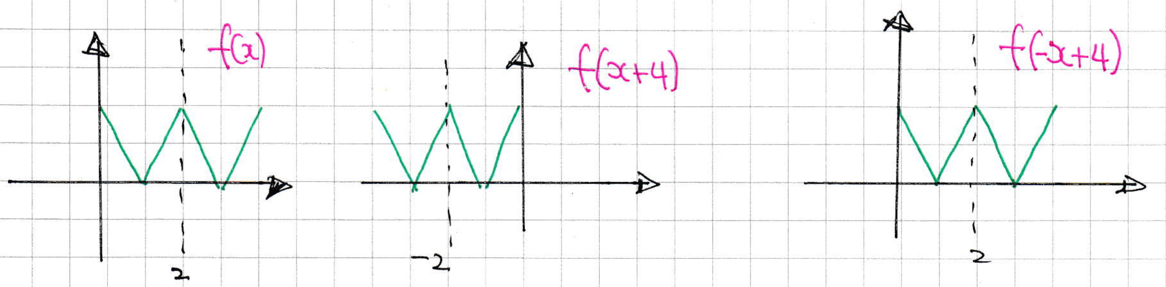
$$y = x(x-4)(x+2)(x-6), \quad x \in \mathbb{R}$$

- IF EVEN ABOUT $x=2$, THEN TRANSLATING "LEFT" BY 2 UNITS, THEN REFLECTING IN THE y AXIS & FINALLY TRANSLATING "RIGHT" BY 2 UNITS WILL LEAVE THE CURVE INVARIANT

E.g.



- OR TRANSLATING BY 4 UNITS TO THE "LEFT" THEN REFLECTING IN THE y AXIS



- HENCE IF $f(x) = f(4-x)$ THEN THE CURVE IS EVEN ABOUT THE LINE $x=2$

$$f(4-x) = (4-x)[(4-x)-4][(4-x)+2][(4-x)-6]$$

$$= (4-x)(-x)(6-x)(-x-2)$$

$$= (-1)^4(x-4)x(x-6)(x+2)$$

$$= x(x-4)(x+2)(x-6) = f(x)$$

ALSO NOTE ON PARTIAL MULTIPLICATION AND $x-2 \mapsto 2-x$

$$f(x) = (x^2-4x)(x^2-4x-12) = [(x-2)^2-4][(x-2)^2-16]$$

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START BY OBTAINING THE GRADIENT FUNCTION IN PARAMETRIC

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta}{-4\sin\theta} = -\frac{3}{4}\cot\theta = -\frac{3}{4\tan\theta}$$

EQUATION OF TANGENT AT $\theta = \alpha$

$$y - 3\sin\alpha = -\frac{3}{4\tan\alpha}(x - 4\cos\alpha)$$

$$y - 3\sin\alpha = -\frac{3\cos\alpha}{4\sin\alpha}(x - 4\cos\alpha)$$

$$4y\sin\alpha - 12\sin^2\alpha = -3x\cos\alpha + 12\cos^2\alpha$$

$$4y\sin\alpha + 3x\cos\alpha = 12(\sin^2\alpha + \cos^2\alpha)$$

$$4y\sin\alpha + 3x\cos\alpha = 12$$

NOW WHEN $x=0$ & $y=0$ YIELDS

$$\left(0, \frac{3}{\sin\alpha}\right) \quad \& \quad \left(\frac{4}{\cos\alpha}, 0\right)$$

THE AREA OF THE TRIANGLE IS GIVEN BY

$$\text{AREA} = \frac{1}{2} \times \frac{3}{\sin\alpha} \times \frac{4}{\cos\alpha}$$

$$\text{AREA} = \frac{12}{\sin 2\alpha}$$

SETTING UP AN INEQUALITY

$$\frac{12}{\sin 2\alpha} < 24$$

$$24\sin 2\alpha > 12 \qquad (\text{AS } \alpha \text{ IS ACUTE}, \sin 2\alpha > 0)$$

$$\sin 2\alpha > \frac{1}{2}$$

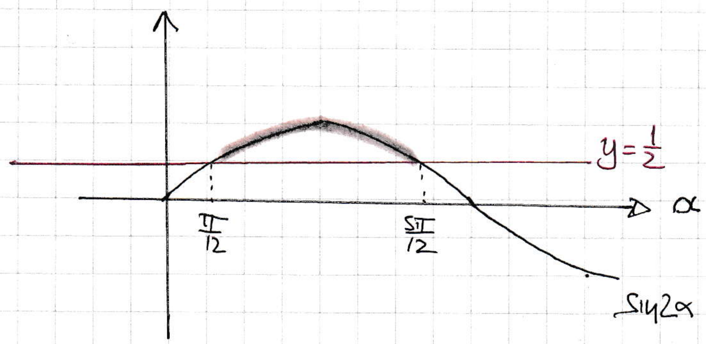
IYGB - MP2 PAPER T - QUESTION 6

OBTAIN THE CRITICAL VALUES FOR THE INEQUALITY

$$\sin 2\alpha = \frac{1}{2}$$

$$\begin{cases} 2\alpha = \pi/6 \pm 2n\pi \\ 2\alpha = 5\pi/6 \pm 2n\pi \end{cases} \qquad n = 0,1,2,3,\ldots$$

$$\begin{cases} \alpha = \pi/12 \pm n\pi \\ \alpha = 5\pi/12 \pm n\pi \end{cases}$$



$$\therefore \quad \frac{\pi}{12} < \alpha < \frac{5\pi}{12}$$

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ASSERTION $\quad \dfrac{x+a}{\sqrt{x^2+a^2}} - \dfrac{x+b}{\sqrt{x^2+b^2}} > 0 \qquad x,a,b>0, \quad a>b, \quad x^2>ab$

SUPPOSE THAT

$$\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} \leq 0$$

$$\Rightarrow \frac{x+a}{\sqrt{x^2+a^2}} \leq \frac{x+b}{\sqrt{x^2+b^2}}$$

AS BOTH SIDES ARE POSITIVE WE MAY SQUARE THE INEQUALITY

$$\Rightarrow \frac{x^2+2ax+a^2}{x^2+a^2} \leq \frac{x^2+2bx+b^2}{x^2+b^2}$$

$$\Rightarrow 1+\frac{2ax}{x^2+a^2} \leq 1+\frac{2bx}{x^2+b^2}$$

$$\Rightarrow \frac{2ax}{x^2+a^2} \leq \frac{2bx}{x^2+b^2}$$

AS $2x>0$ WE MAY ALSO DIVIDE IT

$$\Rightarrow \frac{a}{x^2+a^2} \leq \frac{b}{x^2+b^2}$$

AS THE DENOMINATORS ARE POSITIVE WE MAY MULTIPLY ACROSS

$$\Rightarrow a(x^2+b^2) \leq b(x^2+a^2)$$

$$\Rightarrow ax^2+ab^2 \leq bx^2+ba^2$$

IYGB - MP2 PAPER T - QUESTION 7

$$\Rightarrow ax^2 - bx^2 + ab^2 - a^2b \leq 0$$

$$\Rightarrow x^2(a-b) - ab(a-b) \leq 0$$

$$\Rightarrow (a-b)(x^2-ab) \leq 0$$

- BUT $a > b \Rightarrow a-b > 0$
- ALSO $x^2 > ab \Rightarrow x^2 - ab > 0$

$\Rightarrow (a-b)(x^2-ab) > 0$

HENCE BY CONTRADICTION

$$\frac{x+a}{\sqrt{x^2+a^2}} - \frac{x+b}{\sqrt{x^2+b^2}} > 0$$

IYGB – MP2 PAPER T – QUESTION 8

START BY MANIPULATING THE INTEGRAL AS FOLLOWS

$$\int \frac{\left[\ln(x^2+1) - 2\ln x\right]\sqrt{x^2+1}}{x^4}\,dx$$

$$= \int \ln\left(\frac{x^2+1}{x^2}\right) \times \frac{\sqrt{x^2+1}}{x^4}\,dx$$

$$= \int \ln\left(1+\frac{1}{x^2}\right) \times \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \times \frac{1}{x^3}\,dx$$

$$= \int \sqrt{\frac{x^2+1}{x^2}}\ \ln\left(1+\frac{1}{x^2}\right) \times \frac{1}{x^3}\,dx$$

$$= \int \sqrt{1+\frac{1}{x^2}}\ \ln\left(1+\frac{1}{x^2}\right) \times \frac{1}{x^3}\,dx$$

NOW WE HAVE "A SUBSTITUTION"

$$u = \sqrt{1+\frac{1}{x^2}}$$

$$u^2 = 1+\frac{1}{x^2}$$

$$2u\,du = -\frac{2}{x^3}\,dx \qquad \therefore \ \frac{dx}{x^3} = -u\,du$$

TRANSFORM THE INTEGRAL

$$= \int u\ln u^2\,(-u\,du)$$

$$= \int -u^2\ln u^2\,du$$

$$= \int -2u^2\ln u\,du$$

IYGB – MP2 PAPER T – QUESTION 8

PROCEED BY INTEGRATION BY PARTS

$\ln u$	$\frac{1}{u}$
$-\frac{2}{3}u^3$	$-2u^2$

$$\dots = -\frac{2}{3}u^3\ln u - \int \frac{2}{3}u^3\left(\frac{1}{u}\,du\right)$$

$$= -\frac{2}{3}u^3\ln u + \frac{2}{3}\int u^2\,du$$

$$= -\frac{2}{3}u^3\ln u + \frac{2}{9}u^3 + C$$

$$= \frac{2}{9}u^3\left[1 - 6\ln u\right] + C$$

$$= \frac{2}{9}u^3\left[1 - 3\ln u^2\right] + C$$

$$= \frac{2}{9}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[1 - 3\ln\left(1+\frac{1}{x^2}\right)\right] + C$$

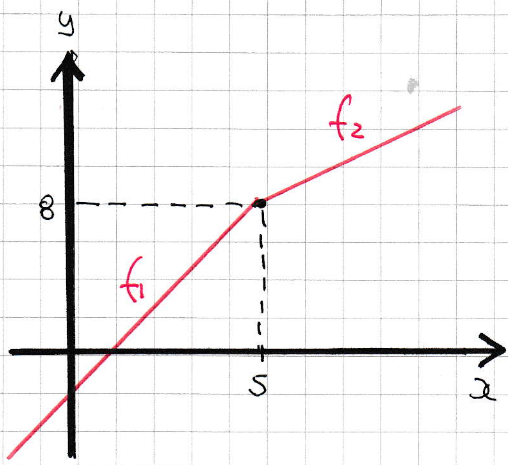
$$= \frac{2}{9}\left(\frac{x^2+1}{x^2}\right)^{\frac{3}{2}}\left[1 - 3\ln\left(\frac{x^2+1}{x^2}\right)\right] + C$$

$$= \frac{2}{9x^3}\sqrt{x^2+1}\left[1 - 3\ln\left(\frac{x^2+1}{x^2}\right)\right] + C$$

IYGB - MP2 PAPER T - QUESTION 9

a)
$$f(x) = \begin{cases} 2x-2 & x \leq 5 \\ x+3 & x > 5 \end{cases}$$

START WITH A SKETCH OF f



- $y = x+3 \quad , \quad y > 8$

  $x = y-3$

  $\therefore f_2^{-1}(x) = x-3 \quad , \quad x > 8$

- $y = 2x-2 \quad , \quad y \leq 8$

  $y+2 = 2x$

  $x = \frac{1}{2}y+1$

  $\therefore f_1^{-1}(x) = \frac{1}{2}x+1 \quad , \quad x \leq 8$

$$\therefore f^{-1}(x) = \begin{cases} \frac{1}{2}x+1 & x \leq 8 \\ x-3 & x > 8 \end{cases}$$

b) THERE ARE 3 CASES TO CONSIDER

IF $x > 5 \longrightarrow f_2 \longrightarrow f_2 \quad \therefore f(f(x)) = f_2(f_2(x))$

IF $\frac{7}{2} < x \leq 5 \longrightarrow f_1 \longrightarrow f_2 \quad \therefore f(f(x)) = f_2(f_1(x))$

IF $x \leq \frac{7}{2} \longrightarrow f_2 \longrightarrow f_2 \quad \therefore f(f(x)) = f_2(f_2(x))$

IYGB - MP2 PAPER T - QUESTION 9

- If $x > 5$

$$f_2(f_2(x)) = f_2(x+3)$$
$$= (x+3)+3$$
$$= x+6$$

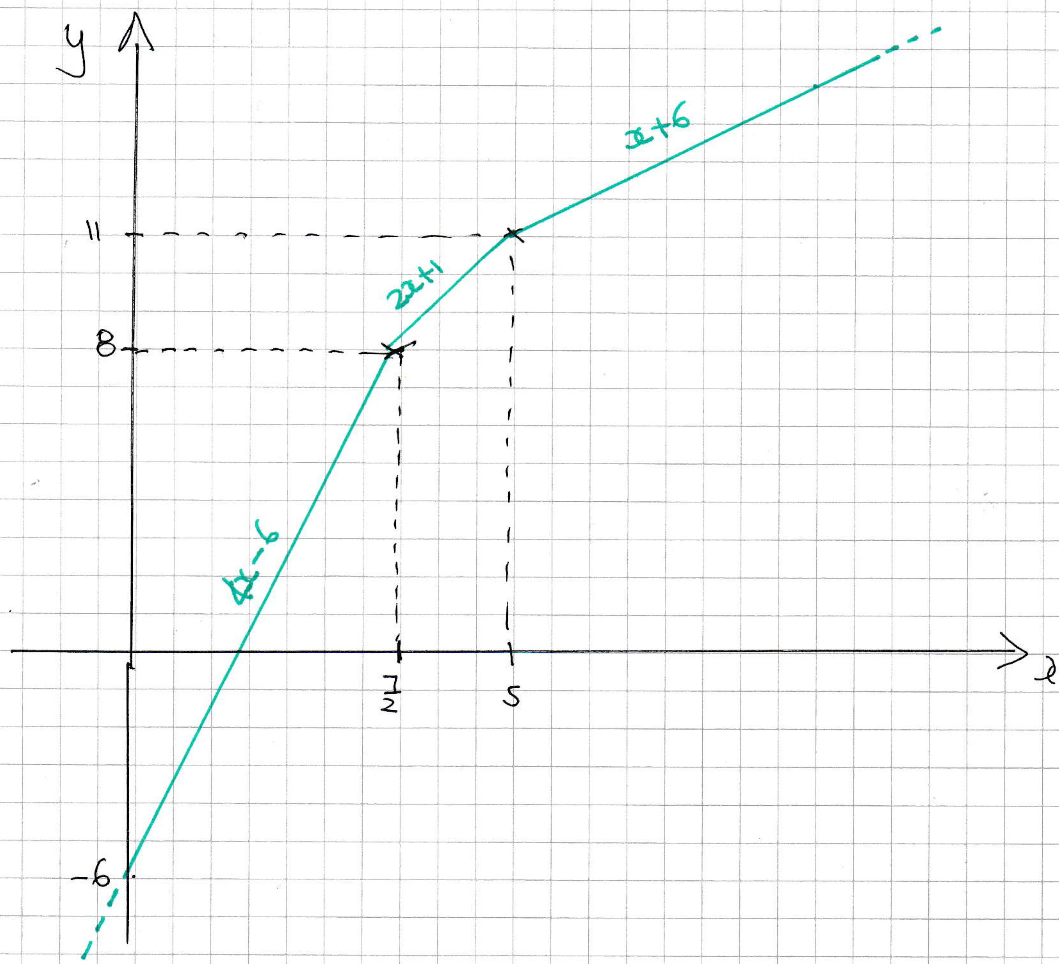
- If $\frac{7}{2} < x \leq 5$

$$f_2(f_1(x)) = f_2(2x-2)$$
$$= (2x-2)+3$$
$$= 2x+1$$

- If $x \leq \frac{7}{2}$

$$f_1(f_1(x)) = f_1(2x-2)$$
$$= 2(2x-2)-2$$
$$= 4x-6$$

SKETCHING THE COMPOSITION FOR ITS ENTIRE DOMAIN



IYGB - MP2 PAPER T - QUESTION 10

$A(10,9,-6)$ $\quad$ $B(6,-3,10)$ $\quad$ $C(2,y,z)$

START BY FINDING $\overrightarrow{AC}$ & $\overrightarrow{BC}$

$$\overrightarrow{AC} = \underline{c} - \underline{a} = (2,y,z) - (10,9,-6) = (-8, y-9, z+6)$$
$$\overrightarrow{BC} = \underline{c} - \underline{b} = (2,y,z) - (6,-3,10) = (-4, y+3, z-10)$$

NEXT SET SIMPLIFIED EXPRESSIONS FOR EACH OF THE MODULI

$$\Rightarrow |-8, y-9, z+6| = 12$$
$$\Rightarrow \sqrt{64 + (y-9)^2 + (z+6)^2} = 12$$
$$\Rightarrow 64 + (y-9)^2 + (z+6)^2 = 144$$
$$\Rightarrow (y-9)^2 + (z+6)^2 = 80$$
$$\Rightarrow y^2 - 18y + 81 + z^2 + 12z + 36 = 80$$
$$\Rightarrow y^2 + z^2 - 18y + 12z = -37$$

$$\Rightarrow |-4, y+3, z-10| = 12$$
$$\Rightarrow \sqrt{16 + (y+3)^2 + (z-10)^2} = 12$$
$$\Rightarrow 16 + (y+3)^2 + (z-10)^2 = 144$$
$$\Rightarrow (y+3)^2 + (z-10)^2 = 128$$
$$\Rightarrow y^2 + 6y + 9 + z^2 - 20z + 100 = 128$$
$$\Rightarrow y^2 + z^2 + 6y - 20z = 19$$

SOLVING SIMULTANEOUSLY BY SUBTRACTING THE EQUATIONS

$$\Rightarrow \left\{ \begin{array}{l} \cancel{y^2} + \cancel{z^2} + 6y - 20z = 19 \\ \cancel{y^2} + \cancel{z^2} - 18y + 12z = -37 \end{array} \right\} \Rightarrow 24y - 32z = 56$$
$$\Rightarrow 3y - 4z = 7$$
$$\Rightarrow 3y = 4z + 7$$

IYGB – MP2 PAPER T – QUESTION 10

TAKE ONE OF THE EQUATIONS SUCH AS

$$\Rightarrow y^2 + z^2 + 6y - 20z = 19$$

$\times 9$

$$\Rightarrow 9y^2 + 9z^2 + 54y - 180z = 171$$

$$\Rightarrow (3y)^2 + 9z^2 + 18(3y) - 180z = 171$$

$$\Rightarrow (4z+7)^2 + 9z^2 + 18(4z+7) - 180z = 171$$

$$\Rightarrow 16z^2 + 56z + 49 + 9z^2 + 72z + 126 - 180z - 171 = 0$$

$$\Rightarrow 25z^2 - 52z + 4 = 0$$

$$\Rightarrow (z-2)(25z-2) = 0$$

$$\Rightarrow z = \begin{cases} 2 \\ \frac{2}{25} \end{cases}$$

FINALLY USING $3y = 4z + 7$

- IF $z = 2$

  $3y = 15$

  $\underline{\underline{y = 5}}$

- IF $z = \frac{2}{25}$

  $3y = \frac{8}{25} + 7$

  $3y = \frac{183}{25}$

  $\underline{\underline{y = \frac{61}{25}}}$

$\therefore$ $(2, 5, 2)$ & $\left(2, \frac{61}{25}, \frac{2}{25}\right)$

IYGB - MP2 PAPER T - QUESTION 11

SOLVE THE O.D.E BY SEPARATION OF VARIABLES

$$\frac{dy}{dx} = \sqrt{\frac{y^4-y^2}{x^4-x^2}} = \frac{|y|\sqrt{y^2-1}}{|x|\sqrt{x^2-1}} = \frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}} \quad \text{AS } x,y>0$$

$$\Longrightarrow \int \frac{1}{y\sqrt{y^2-1}}\,dy = \int \frac{1}{x\sqrt{x^2-1}}\,dx$$

IDENTICAL SUBSTITUTION OR DIRECTLY RECOGNISING THE DERIVATIVE OF "arcsec"

LET $x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \int \frac{1}{\sec\theta\sqrt{\sec^2\theta-1}}(\sec\theta\tan\theta)\,d\theta = \int \cancel{\frac{\sec\theta\tan\theta}{\sec\theta\tan\theta}}\,d\theta$$

$$= \int 1\,d\theta = \theta + C = \operatorname{arcsec}x + C$$

RETURNING TO THE O.D.E.

$$\Longrightarrow \operatorname{arcsec}y = \operatorname{arcsec}x + C$$

APPLY CONDITION $\left(2, \frac{2}{\sqrt{3}}\right)$

$$\operatorname{arcsec}\frac{2}{\sqrt{3}} = \operatorname{arcsec}2 + C$$

$$\frac{\pi}{6} = \frac{\pi}{3} + C$$

$$C = -\frac{\pi}{6}$$

$$\Longrightarrow \operatorname{arcsec}y = \operatorname{arcsec}x - \frac{\pi}{6}$$

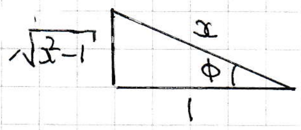
$$\Longrightarrow \cos(\operatorname{arcsec}y) = \cos\left(\operatorname{arcsec}x - \frac{\pi}{6}\right)$$

$$\Longrightarrow \cos(\operatorname{arcsec}y) = \cos(\operatorname{arcsec}x)\cos\frac{\pi}{6} + \sin(\operatorname{arcsec}x)\sin\frac{\pi}{6}$$

IYGB – MP2 PAPER T – QUESTION 11

NEXT REMOVE THE "arcsec"

$$\operatorname{arcsec} x = \phi$$
$$\sec\phi = x$$
$$\cos\phi = \frac{1}{x} \quad \& \quad \sin\phi = \frac{\sqrt{x^2-1}}{x}$$



$$\therefore \cos(\operatorname{arcsec} x) = \frac{1}{x} \qquad \sin(\operatorname{arcsec} x) = \frac{\sqrt{x^2-1}}{x}$$
$$\cos(\operatorname{arcsec} y) = \frac{1}{y}$$

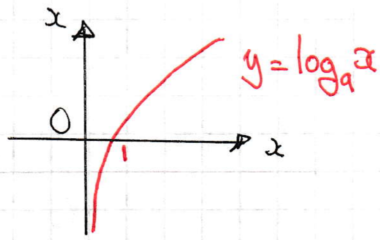
RETURNING TO THE O.D.E

$$\frac{1}{y} = \frac{1}{x} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{x^2-1}}{x} \times \frac{1}{2}$$
$$\frac{1}{y} = \frac{\sqrt{3}}{2x} + \frac{\sqrt{x^2-1}}{2x}$$
$$\frac{1}{y} = \frac{\sqrt{3} + \sqrt{x^2-1}}{2x}$$
$$y = \frac{2x}{\sqrt{3} + \sqrt{x^2-1}} \quad //$$

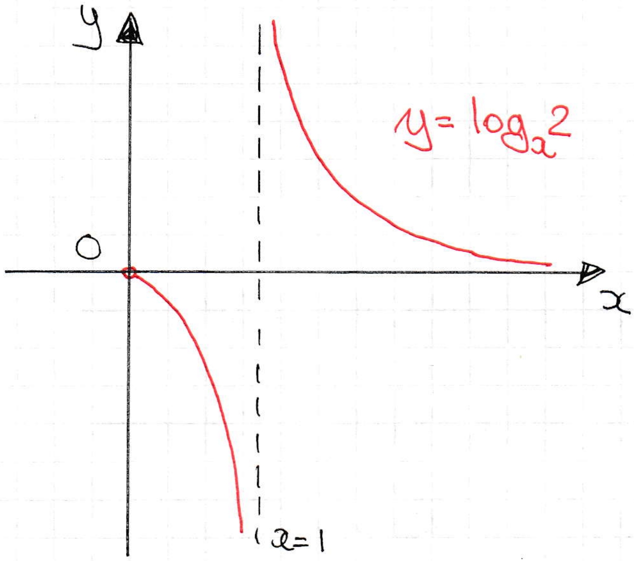
IYGB – MP2 PAPER T – QUESTION 12

a) TO SKETCH, WE EMPLOY THE RULES OF LOGARITHMS

$$y = \log_x 2 = \frac{1}{\log_2 x}$$



"RECIPROCATING" THE GRAPH WE OBTAIN



b) DIFFERENTIATING USING MORE LOGARITHM RULES

$$y = \log_x 2 = \frac{\log_e 2}{\log_e x} = \frac{\ln 2}{\ln x} = (\ln 2)(\ln x)^{-1}$$

$$\frac{dy}{dx} = -(\ln 2)(\ln x)^{-2} \times \frac{1}{x} = -\frac{\ln 2}{x(\ln x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = -\frac{\ln 2}{2(\ln 2)^2} = -\frac{1}{2\ln 2} = -\frac{1}{\ln 4}$$

NORMAL GRADIENT IS $\ln 4$ & THE POINT HAS FULL CO-ORDINATES $(2,1)$

IYGB - MP2 PAPER T - QUESTION 12

EQUATION OF THE NORMAL IS GIVEN BY

$$y - 1 = (\ln 4)(x - 2)$$

SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE USING THE FORM

$$y = \frac{\ln 2}{\ln x}$$

$$\Rightarrow \frac{\ln 2}{\ln x} - 1 = x\ln 4 - 2\ln 4$$

$$\Rightarrow \ln 2 - \ln x = x\ln x\ln 4 - 2\ln x\ln 4$$

$$\Rightarrow \ln 2 = \ln x + x\ln x\ln 4 - 2\ln x\ln 4$$

$$\Rightarrow \ln 2 = \ln x\left[1 + x\ln 4 - 2\ln 4\right]$$

$$\Rightarrow \left[1 + x\ln 4 - \ln 16\right]\ln x = \ln 2$$

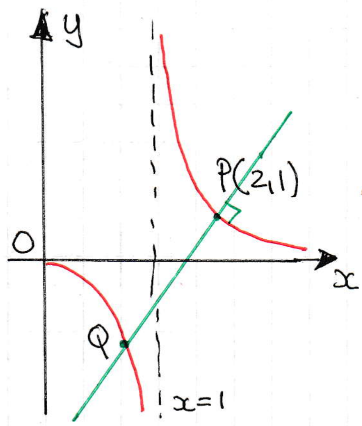
c) "EXPONENTIATING" GIVES

$$\Rightarrow \ln x = \frac{\ln 2}{1 + x\ln 4 - \ln 16}$$

$$\Rightarrow x = e^{\frac{\ln 2}{1 + x\ln 4 - \ln 16}}$$

$$\Rightarrow x_{n+1} = e^{\frac{\ln 2}{1 + x_n\ln 4 - \ln 16}}$$

STARTING SAY WITH $x_1 = 0.5$



IYGB - MP2 PAPER T - QUESTION 12

$x_1 = 0.5$

$x_2 = 0.526168$

$x_3 = 0.514549$

$x_4 = 0.519774$

$x_5 = 0.517437$

$x_6 = 0.518485$

$x_7 = 0.518016$

$x_8 = 0.518226$

$\vdots$

$\therefore x \simeq 0.518$

AND USING

$$y = \frac{\ln 2}{\ln(0.518)}$$

$y \simeq -1.054$

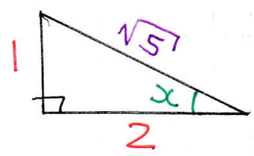
$\therefore Q(0.518, -1.054)$

IYGB - MP2 PAPER T - QUESTION 13

- STARTING WITH x

$$\Rightarrow 2\tan x = 1$$

$$\Rightarrow \tan x = \frac{1}{2}$$



$\Rightarrow$ AS x IS ACUTE

$$\boxed{\sin x = +\frac{1}{\sqrt{5}}} \quad \& \quad \boxed{\cos x = +\frac{2}{\sqrt{5}}}$$

- NOW USING THE COMPOUND ANGLE IDENTITY FOR $\sin(x+y)$

$$\Rightarrow \sin(x+y) = \frac{7}{\sqrt{50}}$$

$$\Rightarrow \sin x\cos y + \cos x\sin y = \frac{7}{\sqrt{50}}$$

$$\Rightarrow \frac{1}{\sqrt{5}}\cos y + \frac{2}{\sqrt{5}}\sin y = \frac{7}{\sqrt{50}}$$

$$\Rightarrow \sqrt{10}\cos y + 2\sqrt{10}\sin y = 7$$

$$\Rightarrow \sqrt{10}\cos y + 2\sqrt{10}\left(+\sqrt{1-\cos^2 y}\right) = 7$$

$$\Rightarrow \boxed{2\sqrt{10}\sqrt{1-\cos^2 y} = 7 - \sqrt{10}\cos y}$$

$$\Rightarrow 4\times 10\times(1-\cos^2 y) = (7-\sqrt{10}\cos y)^2$$

$$\Rightarrow 40 - 40\cos^2 y = 49 - 14\sqrt{10}\cos y + 10\cos^2 y$$

$$\Rightarrow 0 = 50\cos^2 y - 14\sqrt{10}\cos y + 9$$

$$\Rightarrow \cos^2 y - \frac{14}{50}\sqrt{10}\cos y + \frac{9}{50}$$

$$\Rightarrow \left[\cos y - \frac{7}{50}\sqrt{10}\right]^2 - \left(\frac{7}{50}\sqrt{10}\right)^2 + \frac{9}{50} = 0$$

IYGB - MP2 PAPER T - QUESTION 13

$$\Rightarrow \left[\cos y - \frac{7}{50}\sqrt{10}\right]^2 = \frac{490}{2500} - \frac{9}{50}$$

$$\Rightarrow \left[\cos y - \frac{7}{50}\sqrt{10}\right]^2 = \frac{490}{2500} - \frac{450}{2500}$$

$$\Rightarrow \left[\cos y - \frac{7}{50}\sqrt{10}\right]^2 = \frac{40}{2500} = \frac{4}{2500}\times 10$$

$$\Rightarrow \cos y - \frac{7}{50}\sqrt{10} = \begin{cases} \frac{2}{50}\sqrt{10} \\ -\frac{2}{50}\sqrt{10} \end{cases}$$

$$\Rightarrow \cos y = \begin{cases} \frac{2}{50}\sqrt{10} + \frac{7}{50}\sqrt{10} = \frac{9}{50}\sqrt{10} \\ -\frac{2}{50}\sqrt{10} + \frac{7}{50}\sqrt{10} = \frac{5}{50}\sqrt{10} = \frac{1}{10}\sqrt{10} \end{cases}$$

- CHECK THE SOLUTIONS AGAINST THE EQUATION BEFORE SQUARING

$$\boxed{\sqrt{10}\cos y + 2\sqrt{10}\sqrt{1-\cos^2 y} = 7}$$

If $\cos y = \frac{9}{50}\sqrt{10}$ $\Rightarrow$ $\sqrt{10}\times\frac{9}{50}\sqrt{10} + 2\sqrt{10}\sqrt{1-\frac{81}{2500}\times 10}$

$$= \frac{90}{50} + 2\sqrt{10}\sqrt{\frac{2500-810}{2500}}$$

$$= \frac{9}{5} + 2\sqrt{10}\sqrt{\frac{1690}{2500}}$$

$$= \frac{9}{5} + 2\sqrt{10}\sqrt{\frac{169}{2500}\times 10}$$

$$= \frac{9}{5} + 2\sqrt{10}\times\frac{13}{50}\times\sqrt{10}$$

$$= \frac{9}{5} + \frac{260}{50}$$

$$= \frac{9}{5} + \frac{26}{5}$$

$$= 7$$

IYGB - MP2 PAPER T - QUESTION 13

If $\cos y = \frac{1}{10}\sqrt{10}$ $\Longrightarrow$ $\sqrt{10} \times \frac{1}{10}\sqrt{10} + 2\sqrt{10}\sqrt{1 - \frac{10}{100}}$

$$= 1 + 2\sqrt{10}\sqrt{\frac{90}{100}}$$

$$= 1 + 2\sqrt{10}\sqrt{\frac{9}{100} \times 10}$$

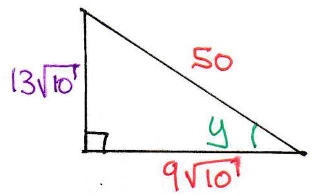
$$= 1 + 2\sqrt{10}\ \frac{3}{10}\sqrt{10}$$

$$= 1 + 6$$

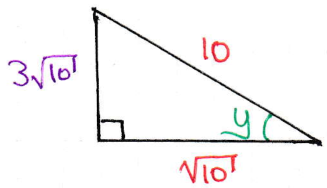
$$= 7$$

$\therefore$ BOTH SOLUTIONS ARE FINE

- FINALLY WE CAN OBTAIN THE TWO POSSIBLE VALUES OF $\tan y$, NOTING FURTHER THAT y IS ACUTE



- $\sqrt{50^2 - (9\sqrt{10})^2} = \sqrt{2500 - 810}$
$= \sqrt{1690}$
$= \sqrt{169 \times 10}$
$= 13\sqrt{10}$

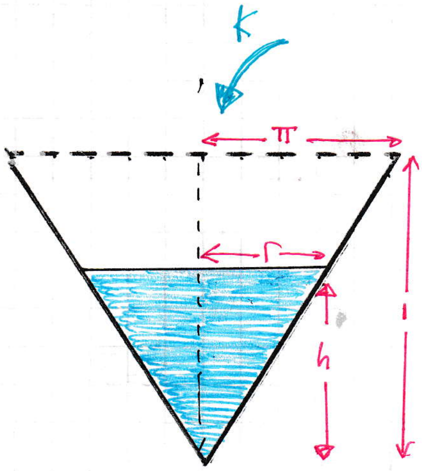


- $\sqrt{10^2 - (\sqrt{10})^2} = \sqrt{100 - 10}$
$= \sqrt{90}$
$= 3\sqrt{10}$

$$\therefore \tan y = \begin{cases} \dfrac{13\sqrt{10}}{9\sqrt{10}} = \dfrac{13}{9} \\[2ex] \dfrac{3\sqrt{10}}{\sqrt{10}} = 3 \end{cases}$$

IYGB - MP2 PAPER T - QUESTION 14

- START WITH A DIAGRAM TO OBTAIN THE VOLUME OF THE WATER IN THE CONTAINER



$$\frac{r}{h} = \frac{\pi}{1} \quad \text{(SIMILAR TRIANGLES)}$$

$$\underline{\underline{r = \pi h}}$$

$$\Rightarrow V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow V = \frac{1}{3}\pi r^2 \left(\frac{r}{\pi}\right)$$

$$\Rightarrow \underline{\underline{V = \frac{1}{3}r^3}}$$

- NOW BY THE CHAIN RULE WE CONNECT DERIVATIVES

$$\Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \times \frac{1}{r^2} \times k$$

$$\Rightarrow \frac{dA}{dt} = \frac{2\pi k}{r}$$

- $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

- $V = \frac{1}{3}r^3$

$$\frac{dV}{dr} = r^2$$

$$\frac{dr}{dV} = \frac{1}{r^2}$$

- NEXT WE NEED TO RELATE THE TIME TO THE RADIUS r

$$\frac{dV}{dt} = k \quad \Rightarrow \quad V = kT$$

$$\Rightarrow \frac{1}{3}r^3 = kT$$

$$\Rightarrow r^3 = 3kT$$

IYGB – MP2 PAPER T – QUESTION 14

$$\Longrightarrow \quad r = (3kT)^{\frac{1}{3}}$$

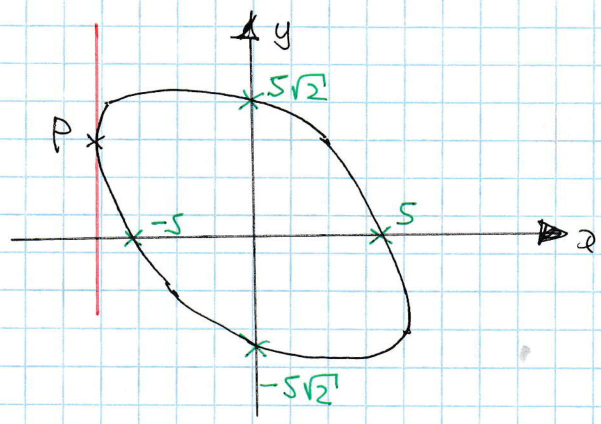
- FINALLY WE OBTAIN

$$\left.\frac{dA}{dt}\right|_{t=T} = \left.\frac{dA}{dt}\right|_{r=(3kT)^{\frac{1}{3}}} = \frac{2\pi k}{(3kT)^{\frac{1}{3}}} = 2\pi \frac{(k^3)^{\frac{1}{3}}}{(3kT)^{\frac{1}{3}}}$$

$$= 2\pi\left(\frac{k^3}{3kT}\right)^{\frac{1}{3}} = 2\pi\sqrt[3]{\frac{k^2}{3T}}$$

IYGB - MP2 PAPER T - QUESTION 15

- FIRSTLY PRODUCE A SKETCH WITH x & y INTERCEPTS



- NEXT FIND THE x CO-ORDINATE OF THE POINT P (VERTICAL TANGENT)

$$2x^2 + 2xy + y^2 = 50$$

$$\frac{d}{dx}\left(2x^2 + 2xy + y^2\right) = 0$$

$$4x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2(y+2x) = -2(x+y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{y+2x}{x+y}$$ ← INFINITE GRADIENT ⟹ DENOMINATOR ZERO

- SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE WE OBTAIN

$$y + x = 0$$
$$y = -x$$

$$2x^2 + 2x(-x) + (-x)^2 = 50$$
$$x^2 = 50$$
$$x = \pm 5\sqrt{2}$$
$$\therefore P(-5\sqrt{2}, 5\sqrt{2})$$

- NEXT REARRANGE THE EQUATION OF THE CURVE, IN THE FORM $y = f(x)$

$$\Rightarrow y^2 + 2xy + 2x^2 = 50$$

$$\Rightarrow (y+x)^2 - x^2 + 2x^2 = 50$$

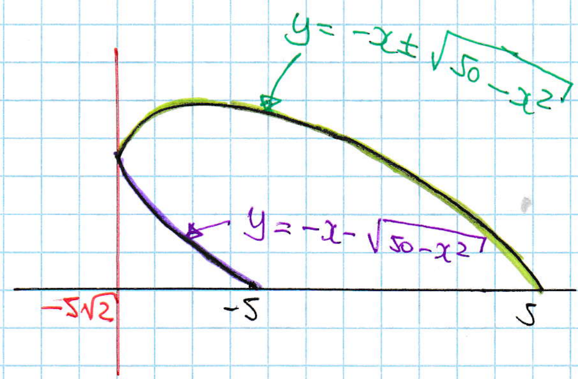
$$\Rightarrow (y+x)^2 = 50 - x^2$$

$$\Rightarrow y + x = \pm\sqrt{50 - x^2}$$

IYGB - MP2 PAPER T - QUESTION 15

$$\Rightarrow y = -x \pm \sqrt{50 - x^2}$$

- THUS WE NOW HAVE THE "PICTURE" OPPOSITE - IN BOTH CASES WE HAVE TO INTEGRATE $\sqrt{50-x^2}$, SO PREPARE THIS PART FIRST



$$\int_{x_1}^{x_2} \sqrt{50-x^2}\, dx = \ldots \text{ BY SUBSTITUTION}$$

$$x = \sqrt{50}\sin\theta$$
$$dx = \sqrt{50}\cos\theta$$

$$= \int_{\theta_1}^{\theta_2} \sqrt{50 - 50\sin^2\theta}\,(\sqrt{50}\cos\theta\, d\theta)$$

$$= \int_{\theta_1}^{\theta_2} \sqrt{50(1-\sin^2\theta)}\,\sqrt{50}\cos\theta\, d\theta$$

$$= \int_{\theta_1}^{\theta_2} 50\cos^2\theta\, d\theta = \int_{\theta_1}^{\theta_2} 25 + 25\cos 2\theta\, d\theta$$

$$= \left[25\theta + \frac{25}{2}\sin 2\theta\right]_{\theta_1}^{\theta_2}$$

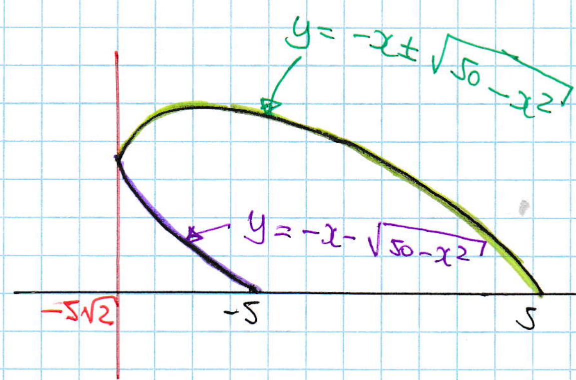
- HENCE THE REQUIRED AREA CAN BE FOUND

$$\text{⌓} - \text{◺} = \int_{-5\sqrt{2}}^{5} -x + \sqrt{50-x^2}\, dx - \int_{-5\sqrt{2}}^{-5} -x - \sqrt{50-x^2}\, dx$$

IYGB - MP2 PAPER T - QUESTION 15

$$\Longrightarrow y = -x \pm \sqrt{50 - x^2}$$

- THUS WE NOW HAVE THE "PICTURE" OPPOSITE - IN BOTH CASES WE HAVE TO INTEGRATE $\sqrt{50-x^2}$, SO PREPARE THIS PART FIRST



$$\int_{x_1}^{x_2} \sqrt{50 - x^2}\, dx = \ldots \text{ BY SUBSTITUTION}$$

$$\boxed{\begin{aligned} x &= \sqrt{50}\sin\theta \\ dx &= \sqrt{50}\cos\theta \end{aligned}}$$

$$= \int_{\theta_1}^{\theta_2} \sqrt{50 - 50\sin^2\theta}\,\left(\sqrt{50}\cos\theta\, d\theta\right)$$

$$= \int_{\theta_1}^{\theta_2} \sqrt{50(1-\sin^2\theta)}\,\sqrt{50}\cos\theta\, d\theta$$

$$= \int_{\theta_1}^{\theta_2} 50\cos^2\theta\, d\theta = \int_{\theta_1}^{\theta_2} 25 + 25\cos 2\theta\, d\theta$$

$$= \left[25\theta + \frac{25}{2}\sin 2\theta\right]_{\theta_1}^{\theta_2}$$

- HENCE THE REQUIRED AREA CAN BE FOUND

$$\text{[upper region]} - \text{[lower region]} = \int_{-5\sqrt{2}}^{5} -x + \sqrt{50-x^2}\, dx - \int_{-5\sqrt{2}}^{-5} -x - \sqrt{50-x^2}\, dx$$

IYGB - MP2 PAPER T - QUESTION 15

$$= \int_{-5\sqrt{2}}^{5} -x \, dx + \int_{-5\sqrt{2}}^{-5} x \, dx + \int_{-5\sqrt{2}}^{5} \sqrt{50-x^2} \, dx + \int_{-5\sqrt{2}}^{-5} \sqrt{50-x^2} \, dx$$

- CHANGING THE LIMITS IN THE SUBSTITUTION INTEGRALS

$x = 5\sqrt{2}\sin\theta$

$x = 5 \mapsto \theta = \frac{\pi}{4}$

$x = -5 \mapsto \theta = -\frac{\pi}{4}$

$x = -5\sqrt{2} \mapsto \theta = -\frac{\pi}{2}$

$$= \left[-\frac{1}{2}x^2\right]_{-5\sqrt{2}}^{5} + \left[\frac{1}{2}x^2\right]_{-5\sqrt{2}}^{-5} + \left[25\theta + \frac{25}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{4}} + \left[25\theta + \frac{25}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{-\frac{\pi}{4}}$$

$$= \cancel{\left[-\frac{25}{2} + \frac{50}{2}\right]} + \cancel{\left[\frac{25}{2} - \frac{50}{2}\right]} + \left[\left(\frac{25\pi}{4} + \cancel{\frac{25}{2}}\right) - \left(-\frac{25\pi}{2} + 0\right)\right]$$

$$+ \left[\left(-\cancel{\frac{25\pi}{4}} - \cancel{\frac{25}{2}}\right) - \left(-\frac{25\pi}{2} + 0\right)\right]$$

$$= 25\pi$$

IYGB - MP2 PAPER T - QUESTION 16

- LET A GENERAL POINT P LIE ON THE PART OF THE CURVE FOR WHICH $y>0$, IE $P\left(p, \frac{p}{(p-1)^{\frac{1}{2}}}\right)$

- DIFFERENTIATING

$$\Rightarrow y = +\frac{x}{(x-1)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x-1)^{\frac{1}{2}}\times 1 - x\times\frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1} = \frac{\frac{1}{2}(x-1)^{-\frac{1}{2}}\left[2(x-1)-x\right]}{x-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-2}{2(x-1)^{3/2}}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=p} = \frac{p-2}{2(p-1)^{3/2}}$$

- FORMING THE EQUATION OF A TANGENT AT P

$$\Rightarrow \boxed{y - \frac{p}{(p-1)^{\frac{1}{2}}} = \frac{p-2}{2(p-1)^{3/2}}(x-p)}$$

- TANGENT PASSES THROUGH $(1,2)$

$$\Rightarrow 2 - \frac{p}{(p-1)^{\frac{1}{2}}} = \frac{p-2}{2(p-1)^{\frac{3}{2}}}(1-p)$$

$$\Rightarrow 2 - \frac{p}{(p-1)^{\frac{1}{2}}} = \frac{(2-p)}{2(p-1)^{\frac{3}{2}}}(p-1)$$

$$\Rightarrow 2 - \frac{p}{(p-1)^{\frac{1}{2}}} = \frac{2-p}{2(p-1)^{\frac{1}{2}}}$$

$\times\, 2(p-1)^{\frac{1}{2}}, \quad p\neq 1$

$$\Rightarrow 4(p-1)^{\frac{1}{2}} - 2p = 2-p$$

$$\Rightarrow 4(p-1)^{\frac{1}{2}} = p+2$$

IYGB - MP2 PAPER T - QUESTION 16

$$\Rightarrow 16(p-1) = p^2+4p+4$$

$$\Rightarrow 16p-16 = p^2+4p+4$$

$$\Rightarrow 0 = p^2-12p+20$$

$$\Rightarrow (p-2)(p-10)=0$$

$$p = \begin{cases} 2 \\ 10 \end{cases}$$ BOTH WORK

- NEXT CHECK THE PART OF THE CURVE FOR WHICH $y<0$

I.E $P\left(p, -\frac{p}{(p-1)^{\frac{1}{2}}}\right)$ & $\left.\frac{dy}{dx}\right|_{x=p} = -\frac{p-2}{2(p-1)^{\frac{3}{2}}} = \frac{2-p}{2(p-1)^{\frac{3}{2}}}$

- HENCE THE EQUATION OF THE TANGENT AT P, THROUGH (1,2)

$$\Rightarrow \boxed{2+\frac{p}{(p-1)^{\frac{1}{2}}} = \frac{2-p}{2(p-1)^{3/2}}(1-p)}$$

$$\Rightarrow 2+\frac{p}{(p-1)^{\frac{1}{2}}} = \frac{p-2}{2(p-1)^{\frac{3}{2}}}(p-1)$$

$$\Rightarrow 2+\frac{p}{(p-1)^{\frac{1}{2}}} = \frac{p-2}{2(p-1)^{\frac{1}{2}}} \quad \times 2(p-1)^{\frac{1}{2}},\ p\neq 1$$

$$\Rightarrow 4(p-1)^{\frac{1}{2}}+2p = p-2$$

$$\Rightarrow 4(p-1)^{\frac{1}{2}} = -p-2$$

$$\Rightarrow 16(p-1) = p^2+4p+4$$

AS BEFORE $p = \cancel{\begin{cases} 2 \\ 10 \end{cases}}$ NEITHER WORKS

IYGB - MP2 PAPER T - QUESTION 16

- HENCE THE EQUATIONS OF THE TWO TANGENTS CAN NOW BE FOUND

$$y - \frac{p}{(p-1)^{\frac{1}{2}}} = \frac{p-2}{2(p-1)^{\frac{3}{2}}}(x-p)$$

IF $p=2$

$$y - 2 = 0$$

$$y = 2$$

IF $p=10$

$$y - \frac{10}{3} = \frac{8}{2\times 27}(x-10)$$

$$y - \frac{10}{3} = \frac{4}{27}(x-10)$$

$$27y - 90 = 4x - 40$$

$$27y = 4x + 50$$