IYGB – MP1 PAPER U – QUESTION 1

LET $X = \log_{10} x$ & $Y = \log_{10} y$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{21-3}{8-2} = \frac{18}{6} = 3$$

$$\Rightarrow Y - Y_0 = m(X - X_0)$$

$$Y - 3 = 3(X-2)$$

$$Y - 3 = 3X - 6$$

$$Y = 3X - 3$$

REVERSING THE SUBSTITUTIONS

$$\Rightarrow \log_{10} y = 3\log_{10} x - 3$$

$$\Rightarrow \log_{10} y = \log_{10} x^3 - 3$$

$$\Rightarrow y = 10^{\log_{10} x^3 - 3}$$

$$\Rightarrow y = 10^{\log_{10} x^3} \times 10^{-3}$$

$$\Rightarrow y = x^3 \times \frac{1}{1000}$$

$$\Rightarrow y = \frac{x^3}{1000}$$

IYGB - MP1 PAPER V - QUESTION 2

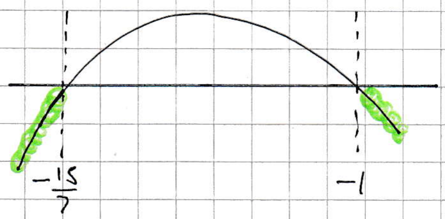
SOLVING THE LINEAR EQUATION FIRST

$$\Rightarrow 5x + 8 \geqslant 4(x+1)$$
$$\Rightarrow 5x + 8 \geqslant 4x + 4$$
$$\Rightarrow 5x - 4x \geqslant 4 - 8$$
$$\Rightarrow x \geqslant -4$$

SOLVING THE QUADRATIC EQUATION NEXT

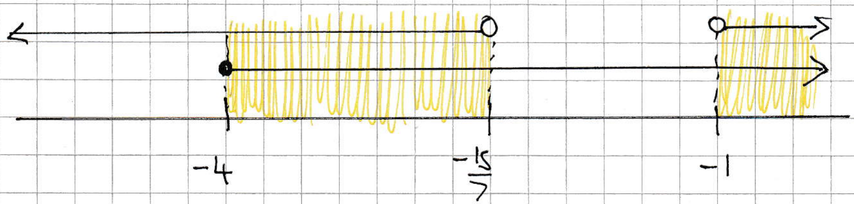
$$\Rightarrow (x+1)^2 - 8(x+1)(x+2) < 0$$
$$\Rightarrow (x+1)\big[(x+1) - 8(x+2)\big] < 0$$
$$\Rightarrow (x+1)\big[x+1-8x-16\big] < 0$$
$$\Rightarrow (x+1)(-7x-15) < 0$$

$$\text{C.V} = \begin{cases} -1 \\ -\frac{15}{7} \end{cases}$$



$x < -\frac{15}{7}$ OR $x > -1$

COMBINING THE SOLUTIONS



$$-4 \leqslant x < -\frac{15}{7} \quad \text{OR} \quad x > -1$$

IYGB - MP1 PAPER V - QUESTION 3

LET $f(x) = \dfrac{1}{2+x^2}$

- $f(x+h) = \dfrac{1}{2+(x+h)^2}$

- $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{2+(x+h)^2} - \frac{1}{2+x^2}}{h} = \frac{\frac{(2+x^2)-[2+(x+h)^2]}{(2+x^2)[2+(x+h)^2]}}{h}$$

$$= \frac{(2+x^2)-[2+(x+h)^2]}{h(2+x^2)[2+(x+h)^2]}$$

$$= \frac{\cancel{2}+\cancel{x^2}-(\cancel{2}+\cancel{x^2}+2xh+h^2)}{h(2+x^2)[2+(x+h)^2]}$$

$$= \frac{-2xh-h^2}{h(2+x^2)[2+(x+h)^2]}$$

$$= \frac{-h(2x+h)}{h(2+x^2)[2+(x+h)^2]}$$

$$= \frac{-(2x+h)}{(2+x^2)[2+(x+h)^2]}$$

TAKING THE LIMIT AS $h \to 0$

$$\frac{d}{dx}\left(\frac{1}{2+x^2}\right) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h\to 0}\left[\frac{-(2x+h)}{(2+x^2)[2+(x+h)^2]}\right]$$

$$= \frac{-2x}{(2+x^2)(2+x^2)} = -\frac{2x}{(2+x^2)^2}$$

IYGB – MP1 PAPER V – QUESTION 4

REARRANGE THE EQUATION

$$\Rightarrow x^2 + (3-m)x + 5 = m^2$$
$$\Rightarrow x^2 + (3-m)x + (5-m^2) = 0$$

AS THE EQUATION HAS REPEATED ROOTS $b^2 - 4ac = 0$

$$\Rightarrow (3-m)^2 - 4 \times 1 \times (5-m^2) = 0$$
$$\Rightarrow 9 - 6m + m^2 - 20 + 4m^2 = 0$$
$$\Rightarrow 5m^2 - 6m - 11 = 0$$
$$\Rightarrow (5m-11)(m+1) = 0$$
$$\Rightarrow m = \begin{cases} -1 \\ 11/5 \end{cases}$$

NOW CONSIDER AND SOLVE EACH CASE SEPARATELY

IF $m = -1$

$$x^2 + [3-(-1)]x + [5-(-1)^2] = 0$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)^2 = 0$$
$$x = -2$$

IF $m = 11/5$

$$x^2 + \left[3 - \tfrac{11}{5}\right]x + \left[5 - \left(\tfrac{11}{5}\right)^2\right] = 0$$
$$x^2 + \tfrac{4}{5}x + \tfrac{4}{25} = 0$$
$$\left(x + \tfrac{2}{5}\right)^2 = 0$$
$$x = -\tfrac{2}{5}$$

IYGB - MP1 PAPER V - QUESTION 5

APPROACH THE TRANSFORMATIONS AS FOLLOWS

- A > REFLECTION ABOUT x THEN REFLECTION ABOUT y > C

$$(x-2)^2 \longmapsto -(x-2)^2 \longmapsto -(-x-2)^2 = -(x^2+4x+4)$$
$$= \underline{\underline{-x^2-4x-4}}$$

- C > TRANSLATION, "UPWARDS", BY 4 UNITS > B

$$-x^2-4x-4 \longmapsto (-x^2-4x-4)+4 = \underline{\underline{-x^2-4x}}$$

- C > TRANSLATION, "RIGHT", BY 4 UNITS > D

$$-x^2-4x-4 \longmapsto -(x-4)^2-4(x-4)-4 = -x^2+8x\cancel{-16}-4x+\cancel{16}-4$$
$$= \underline{\underline{-x^2+4x-4}}$$

SUMMARIZING WE HAVE

- A : $y=(x-2)^2$ $\quad 0 \leqslant x \leqslant 2 \quad 0 \leqslant y \leqslant 4$
- B : $y=-(x-2)^2$
  $y=-x^2-4x-4$ $\quad -2 \leqslant x \leqslant 0 \quad 0 \leqslant y \leqslant 4$
- C : $y=4-(x-2)^2$
  $y=-x^2-4x$ $\quad -2 \leqslant x \leqslant 0 \quad -4 \leqslant y \leqslant 0$
- D : $y=-(2-x)^2$
  $y=-x^2+4x-4$ $\quad 2 \leqslant x \leqslant 4 \quad -4 \leqslant y \leqslant 0$

IYGB - MP1 PAPER U - QUESTION 6

PROCEED AS FOLLOWS

$$x^2 + \frac{240}{x} = \frac{x^3+240}{x} = \frac{\left(120^{\frac{1}{3}}\right)^3 + 240}{120^{\frac{1}{3}}}$$

$\sqrt[3]{120}$

$$= \frac{120+240}{120^{\frac{1}{3}}} = \frac{360}{120^{\frac{1}{3}}} = \frac{360 \times 120^{\frac{2}{3}}}{120^{\frac{1}{3}} \times 120^{\frac{2}{3}}}$$

$$= \frac{360 \times 120^{\frac{2}{3}}}{120^1} = \frac{360 \times 120^{\frac{2}{3}}}{120} = 3 \times 120^{\frac{2}{3}}$$

NOW WE NEED TO REDUCE THE CUBIC SURD

$$= 3 \times \left(\sqrt[3]{120}\right)^2 = 3 \times \left(\sqrt[3]{8}\sqrt[3]{15}\right)^2$$

$$= 3 \times \left(2 \times \sqrt[3]{15}\right)^2 = 3 \times 4 \times \left(\sqrt[3]{15}\right)^2$$

$$= 12 \times \sqrt[3]{15^2} = 12 \times \sqrt[3]{225}$$

AS REQUIRED

IYGB - MP1 PAPER V - QUESTION 7

START BY SOLVING EACH EQUATION NOTING THAT $0° < \alpha, \beta, \gamma < 180°$

- $\tan\alpha = -4.705$

  $\arctan(-4.705) = -78.00°$

  $\alpha = -78 \pm 180n \quad n = 0,1,2,3,\dots$

  $\alpha = 102°$

- $\tan(\beta - \gamma) = 0.404$

  $\arctan(0.404) = 21.9987\dots$

  $\beta - \gamma = 22.0 \pm 180n \quad n = 0,1,2,3,\dots$

  $\boxed{\beta - \gamma = 22°}$

BUT AS α, β, γ ARE ANGLES OF A TRIANGLE

$\Rightarrow \alpha + \beta + \gamma = 180°$

$\Rightarrow 102° + \beta + \gamma = 180°$

$\Rightarrow \boxed{\beta + \gamma = 78°}$

ADDING THE FOLLOWING EQUATIONS

$$\left.\begin{array}{l}\beta - \gamma = 22° \\ \beta + \gamma = 78°\end{array}\right) \Rightarrow 2\beta = 100°$$

$\Rightarrow \beta = 50°$

$\Rightarrow \gamma = 28°$

IYGB - MP1 PAPER V - QUESTION 8

WRITE THE EQUATION IN INDICIAL FORM AND DIFFERENTIATE

$$\Rightarrow y = 6\sqrt[3]{x^5} - 15\sqrt[3]{x^4} - 80x + 16$$

$$\Rightarrow y = 6x^{\frac{5}{3}} - 15x^{\frac{4}{3}} - 80x + 16$$

$$\Rightarrow \frac{dy}{dx} = 10x^{\frac{2}{3}} - 20x^{\frac{1}{3}} - 80$$

FOR STATIONARY POINTS, SOLVING $\frac{dy}{dx} = 0$

$$\Rightarrow 0 = 10x^{\frac{2}{3}} - 20x^{\frac{1}{3}} - 80$$

$$\Rightarrow x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 8 = 0$$

$$\Rightarrow \left(x^{\frac{1}{3}}\right)^2 - 2\left(x^{\frac{1}{3}}\right) - 8 = 0$$

$$\Rightarrow a^2 - 2a - 8 = 0 \quad , \text{ WHERE } a = x^{\frac{1}{3}}$$

$$\Rightarrow (a+2)(a-4) = 0$$

$$\Rightarrow a = \begin{cases} -2 \\ 4 \end{cases}$$

$$\Rightarrow x^{\frac{1}{3}} = \begin{cases} -2 \\ 4 \end{cases}$$

$$\Rightarrow x = \begin{cases} \cancel{-8} \quad (x>0) \\ 64 \end{cases}$$

$$\Rightarrow y = 6\times 64^{\frac{5}{3}} - 15\times 64^{\frac{4}{3}} - 80\times 64 + 16$$

$$= 6\times 1024 - 15\times 256 - 80\times 64 + 16$$

$$= 6144 - 3840 - 5120 + 16$$

$$= -2800$$

IYGB – MP1 PAPER V – QUESTION 8

DETERMINING THE NATURE BY THE SECOND DERIVATIVE TEST

$$\frac{dy}{dx} = 10x^{\frac{2}{3}} - 20x^{\frac{1}{3}} - 80$$

$$\frac{d^2y}{dx^2} = \frac{20}{3}x^{-\frac{1}{3}} - \frac{20}{3}x^{-\frac{2}{3}}$$

$$\frac{d^2y}{dx^2} = \frac{20}{3}\left[\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt[3]{x^2}}\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=64} = \frac{20}{3}\left[\frac{1}{\sqrt[3]{64}} - \frac{1}{\sqrt[3]{64^2}}\right] = \frac{5}{4} > 0$$

$\therefore$ $(16, -2800)$ IS A LOCAL MINIMUM

IYGB - MP1 PAPER V - QUESTION 9

DEFINE A FUNCTION AS

$$f(k) = \frac{2k+2}{2k+3} - \frac{2k}{2k+1} = \frac{(2k+2)(2k+1) - 2k(2k+3)}{(2k+1)(2k+3)}$$

$$= \frac{4k^2+6k+2-4k^2-6k}{(2k+1)(2k+3)} = \frac{2}{(2k+1)(2k+3)}$$

NOW AS $k \in \mathbb{N}$,

$$2k+1 > 0$$

$$2k+3 > 0$$

$$(2k+1)(2k+3) > 0$$

$$\frac{1}{(2k+1)(2k+3)} > 0$$

$$\frac{2}{(2k+1)(2k+3)} > 0$$

$$f(k) > 0$$

$$\frac{2k+2}{2k+3} - \frac{2k}{2k+1} > 0$$

$$\frac{2k+2}{2k+3} > \frac{2k}{2k+1} \quad //$$

ALTERNATIVE APPROACH

$$\frac{2k+2}{2k+3} = \frac{2k+3-1}{2k+3} = 1 - \frac{1}{2k+3}$$

$$\frac{2k}{2k+1} = \frac{2k+1-1}{2k+1} = 1 - \frac{1}{2k+1}$$

IYGB - MP1 PAPER V - QUESTION 9

NOW PROCEED AS FOLLOWS

IF $k \in \mathbb{N}$

$$2k+3 > 2k+1$$

$$\frac{1}{2k+3} < \frac{1}{2k+1}$$

$$-\frac{1}{2k+3} > -\frac{1}{2k+1}$$

$$1-\frac{1}{2k+3} > 1-\frac{1}{2k+1}$$

$$\frac{2k+2}{2k+3} > \frac{2k}{2k+1}$$

// AS REQUIRED

IYGB – MP1 PAPER V – QUESTION 10

a) APPROACH THE PROBLEM GEOMETRICALLY

$$x^2-8x+y^2-2y+13=0$$
$$(x-4)^2-16+(y-1)^2-1+13=0$$
$$(x-4)^2+(y-1)^2=4$$

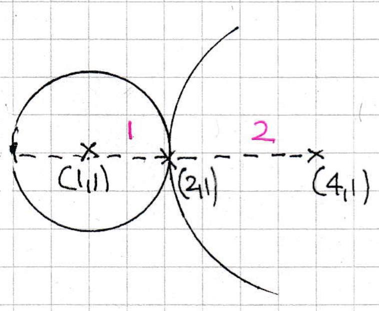
CENTRE $(4,1)$, RADIUS 2

$$x^2-2x+y^2-2y+1=k$$
$$(x-1)^2-1+(y-1)^2-1+1=k$$
$$(x-1)^2+(y-1)^2=k+1$$

CENTRE $(1,1)$, RADIUS $\sqrt{k+1}$

THE DISTANCE BETWEEN THEIR CENTRES $(4,1)$ & $(1,1)$ IS 3 UNITS

EXTERNALLY

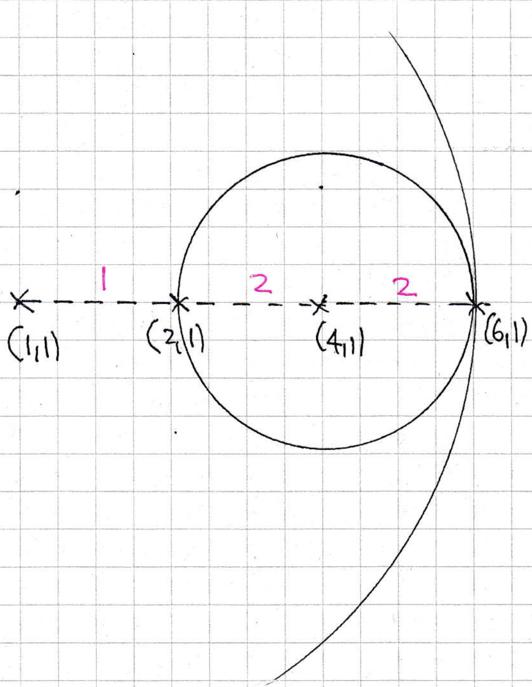


- $\sqrt{k+1}=1$

  $k+1=1$

  $k=0$

INTERNALLY



- $\sqrt{k+1}=5$

  $k+1=25$

  $k=24$

b) BY INSPECTION $0<k<24$

(NO INTERSECTIONS FOR $-1<k<0$ OR $k>24$)

IYGB - MP1 PAPER V - QUESTION 11

EXPAND & COMPARE COEFFICIENTS

$$(1+kx)^n = 1 + \frac{n}{1}(kx)^1 + \frac{n(n-1)}{1\times 2}(kx)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(kx)^3 + \dots$$

$$= 1 + \underbrace{nk}_{\frac{7}{2}}x + \underbrace{\tfrac{1}{2}n(n-1)k^2}_{B}x^2 + \underbrace{\tfrac{1}{6}n(n-1)(n-2)k^3}_{B}x^3 + \dots$$

EXTRACTING TWO EQUATIONS

- $nk = \frac{7}{2}$
- $\frac{1}{2}n(n-1)k^2 = \frac{1}{6}n(n-1)(n-2)k^3$

$n>2$ & $k \neq 0$

$\Rightarrow \frac{1}{2} = \frac{1}{6}(n-2)k$

$\Rightarrow 3 = (n-2)k$

$\Rightarrow 3 = nk - 2k$

$\Rightarrow 2k+3 = nk$ //

AS REQUIRED

SOLVING SIMULTANEOUSLY YIELDS

$\Rightarrow 2k+3 = nk$

$\Rightarrow 2k+3 = \frac{7}{2}$

$\Rightarrow 2k = \frac{1}{2}$

$\Rightarrow k = \frac{1}{4}$ //

&

$nk = \frac{7}{2}$

$n \times \frac{1}{4} = \frac{7}{2}$

$n = 14$ //

IYGB - MP1 PAPER Y - QUESTION 12

MANIPULATE TO A POLYNOMIAL EQUATION IN $\ln x$

$$\Rightarrow \frac{2 - \ln x^7}{7 - \ln x^2} + (\ln x)^2 = 0$$

$$\Rightarrow \frac{2 - 7\ln x}{7 - 2\ln x} + (\ln x)^2 = 0$$

$$\Rightarrow \frac{2 - 7a}{7 - 2a} + a^2 = 0$$

MULTIPLY THE EQUATION THROUGH BY $7-2a$

$$\Rightarrow 2 - 7a + a^2(7-2a) = 0$$

$$\Rightarrow 2 - 7a + 7a^2 - 2a^3 = 0$$

$$\Rightarrow 2a^3 - 7a^2 + 7a - 2 = 0$$

LOOK FOR FACTORS BY INSPECTION

$\rightarrow a = 1 \quad 2\times1^3 - 7\times1 + 7\times1 - 2 = 0$

BY LONG DIVISION, $(a-1)$ IS A FACTOR, OR MANIPULATION

$$\Rightarrow 2a^2(a-1) - 5a(a-1) + 2(a-1) = 0$$

$$\Rightarrow (a-1)(2a^2 - 5a + 2) = 0$$

$$\Rightarrow (a-1)(2a-1)(a-2) = 0$$

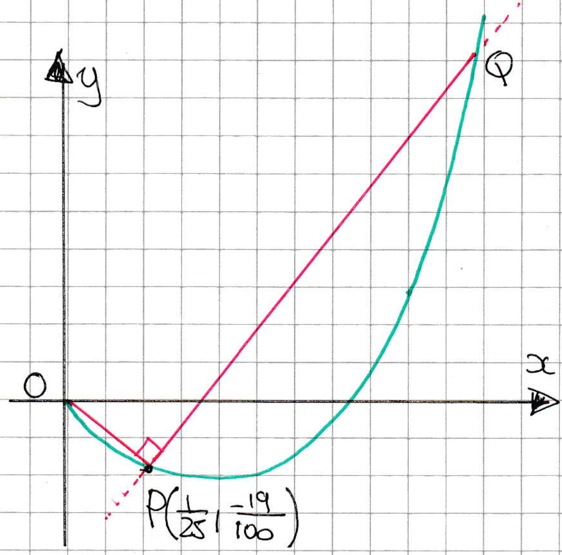
$$\Rightarrow a = \ln x = \begin{cases} 1 \\ 2 \\ \frac{1}{2} \end{cases}$$

$$\therefore x = \begin{cases} e \\ e^2 \\ e^{\frac{1}{2}} = \sqrt{e} \end{cases}$$

IYGB – MP1 PAPER V – QUESTION 13

$y = \frac{1}{4}x - \sqrt{x}\ ,\ x \geqslant 0 \qquad P\left(\frac{1}{25}, -\frac{19}{100}\right)$

LOOKING AT THE DIAGRAM



- GRADIENT OP $= \dfrac{-19/100 - 0}{1/25 - 0} = -\dfrac{19}{4}$
- GRADIENT PQ $= +\dfrac{4}{19}$
- EQUATION OF LINE THROUGH P & Q

$$y + \frac{19}{100} = \frac{4}{19}\left(x - \frac{1}{25}\right)$$

SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$$\Longrightarrow \frac{1}{4}x - \sqrt{x} + \frac{19}{100} = \frac{4}{19}\left(x - \frac{1}{25}\right)$$

$$\Longrightarrow \frac{1}{4}x - \sqrt{x} + \frac{19}{100} = \frac{4}{19}x - \frac{4}{475}$$

$$\Longrightarrow 475x - 1900\sqrt{x} + 361 = 400x - 16$$

$$\Longrightarrow 75x - 1900\sqrt{x} + 377 = 0$$

NOW $x = \frac{1}{25}$ IS A SOLUTION DUE THE POINT P

$$\Longrightarrow (5\sqrt{x} - 1)(15\sqrt{x} - 377) = 0$$

POINT P

$\sqrt{x} = \frac{1}{5}$

$x = \frac{1}{25}$

POINT Q

IYGB - MP1 PAPER V - QUESTION 13

HENCE WE OBTAIN

$$\sqrt{x} = \frac{377}{15} \implies x = \frac{142129}{225} \approx 631.68444\ldots$$

$$\implies \frac{1}{4}x = \frac{142129}{900}$$

$$\implies \frac{1}{4}x - \sqrt{x} = \frac{142129}{900} - \frac{377}{15}$$

$$\implies y = \frac{142129}{900} - \frac{377 \times 60}{15 \times 60}$$

$$\implies y = \frac{142129 - 22620}{900}$$

$$\implies y = \frac{119509}{900} \approx 132.78777\ldots$$