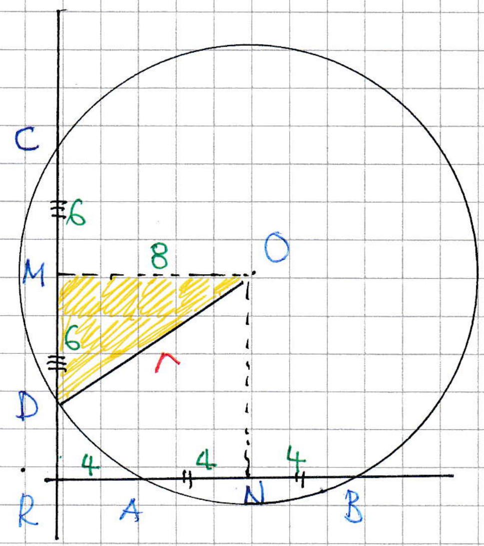
IYGB - MP1 PAPER T - QUESTION 1

BY PYTHAGORAS ON $\hat{OMD}$



$|OM|^2 + |MD|^2 = |OD|^2$

$8^2 + 6^2 = r^2$

$64 + 36 = r^2$

$r^2 = 100$

$r = 10$

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METHOD A

DEFINE A FUNCTION f & MANIPULATE IT AS FOLLOWS

$$\Rightarrow f(x,y) = x^2 - y^2 + y - x$$
$$\Rightarrow f(x,y) = (x^2 - y^2) - (x - y)$$
$$\Rightarrow f(x,y) = (x-y)(x+y) - (x-y)$$
$$\Rightarrow f(x,y) = (x-y)(x+y-1)$$

BUT WE ARE GIVEN THAT $x+y=1$

$$\Rightarrow f(x,y) = (x-y)(1-1)$$
$$\Rightarrow f(x,y) = 0$$

FOR ALL x & y SUCH THAT $x+y=1$

$$\Rightarrow x^2 - y^2 + y - x = 0$$
$$\Rightarrow x^2 + y = y^2 + x$$

AS REQUIRED

METHOD B

FIRSTLY IF $x = y = \frac{1}{2}$

$$x^2 + y = \left(\tfrac{1}{2}\right)^2 + \tfrac{1}{2} = \tfrac{1}{4} + \tfrac{1}{2} = \tfrac{3}{4}$$
$$y^2 + x = \left(\tfrac{1}{2}\right)^2 + \tfrac{1}{2} = \tfrac{1}{4} + \tfrac{1}{2} = \tfrac{3}{4}$$

I.E THE RESULT HOLDS IF $x = y = \frac{1}{2}$

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NEXT SUPPOSE $x \neq y$ SO $x - y \neq 0$

$$\Longrightarrow \quad x + y = 1$$

$$\Longrightarrow \quad (x-y)(x+y) = 1(x-y)$$

$$\Longrightarrow \quad x^2 - y^2 = x - y$$

$$\Longrightarrow \quad x^2 + y^2 = y^2 + x$$

AS REQUIRED FOR ALL x & y

IYGB - MP1 PAPER T - QUESTION 3

- USING THE "DIFFERENCE OF CUBES" IDENTITY

$$a^3 - b^3 \equiv (a-b)(a^2+ab+1)$$

$$\left(\sqrt[3]{4}\right)^3 - 1^3 = \left(\sqrt[3]{4}-1\right)\left[\left(\sqrt[3]{4}\right)^2 + \sqrt[3]{4}+1\right]$$

$$4 - 1 = \left(\sqrt[3]{4}-1\right)\left[\sqrt[3]{16} + \sqrt[3]{4}+1\right]$$

- HENCE WE CAN MANIPULATE THE EXPRESSION AS FOLLOWS

$$\frac{3}{\sqrt[3]{4}-1} = \frac{3\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{\left(\sqrt[3]{4}-1\right)\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}$$

$$= \frac{\cancel{3}\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{\cancel{3}}$$

$$= \sqrt[3]{16} + \sqrt[3]{4} + 1$$

IYGB - MP1 PAPER T - QUESTION 4

$$f(x) = \frac{1}{6}x^2 + 3x + 12$$

FACTORIZING BY COMPLETING THE SQUARE (OR TREAT IT AS THE EQUATION $f(x)=0$ & USE THE QUADRATIC FORMULA)

$$\Rightarrow f(x) = \frac{1}{6}\left[x^2 + 18x + 72\right]$$

$$\Rightarrow f(x) = \frac{1}{6}\left[(x+9)^2 - 81 + 72\right]$$

$$\Rightarrow f(x) = \frac{1}{6}\left[(x+9)^2 - 9\right]$$

$$\Rightarrow f(x) = \frac{1}{6}\left[(x+9)^2 - 3^2\right]$$

$$\Rightarrow f(x) = \frac{1}{6}\left[(x+9+3)(x+9-3)\right]$$

$$\Rightarrow f(x) = \frac{1}{6}(x+12)(x+6)$$

(NATURALLY WE COULD HAVE FACTORIZED DIRECTLY TO THIS IF WE SPOTTED IT.)

NOW THIS CAN BE WRITTEN 4 DIFFERENT WAYS AS A PRODUCT OF TWO LINEAR FACTORS

- $f(x) = \left(\frac{1}{6}x + 2\right)(x+6)$
- $f(x) = (x+12)\left(\frac{1}{6}x+1\right)$
- $f(x) = \left(\frac{1}{2}x+6\right)\left(\frac{1}{3}x+2\right)$
- $f(x) = \left(\frac{1}{3}x+4\right)\left(\frac{1}{2}x+3\right)$

IYGB - MP1 PAPER T - QUESTION 5

- START BY ATTEMPTING TO CREATE AN EQUATION IN A SINGLE TRIGONOMETRIC FUNCTION, AS FOLLOWS

$$\Rightarrow 4\cos^2\theta + \tan^4\theta = 10$$

$$\Rightarrow 4\cos^2\theta + \frac{\sin^4\theta}{\cos^4\theta} = 10$$

$$\Rightarrow 4\cos^6\theta + \sin^4\theta = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta + (1-\cos^2\theta)^2 = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta + 1 - 2\cos^2\theta + \cos^4\theta = 10\cos^4\theta$$

$$\Rightarrow 4\cos^6\theta - 9\cos^4\theta - 2\cos^2\theta + 1 = 0$$

THIS IS A CUBIC IN $\cos^2\theta$ — LET $a = \cos^2\theta$

$$\Rightarrow 4a^3 - 9a^2 - 2a + 1 = 0$$

- WE ARE GIVEN THAT $\theta = \frac{\pi}{3}$ IS A SOLUTION

$$\Rightarrow \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\Rightarrow \cos^2\frac{\pi}{3} = \frac{1}{4}$$

$$\Rightarrow a = \frac{1}{4}$$

$$\Rightarrow 4a - 1 = 0$$

$\therefore (4a-1)$ IS A FACTOR OF THE CUBIC

- BY LONG DIVISION OR MANIPULATIONS

$$\Rightarrow a^2(4a-1) - 2a(4a-1) - (4a-1) = 0$$

$$\Rightarrow (4a-1)(a^2-2a-1) = 0$$

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- NOW LOOKING AT THE QUADRATIC

$$\Rightarrow a^2 - 2a - 1 = 0$$

$$\Rightarrow (a-1)^2 - 2 = 0$$

$$\Rightarrow (a-1)^2 = 2$$

$$\Rightarrow a - 1 = \pm\sqrt{2}$$

$$\Rightarrow a = 1 \pm \sqrt{2}$$

$$\Rightarrow \cos^2\theta = \begin{cases} \cancel{1+\sqrt{2}} > 1 \\ \cancel{1-\sqrt{2}} < 0 \end{cases}$$

$$\left[\cos\theta = \begin{cases} \sqrt{1+\sqrt{2}} > 1 \\ -\sqrt{1+\sqrt{2}} < -1 \end{cases} \right]$$

- THUS ONLY SOLUTION IS $a = \frac{1}{4}$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

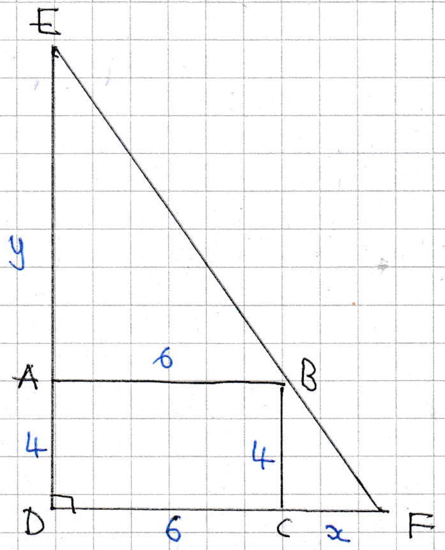
$$\Rightarrow \cos\theta = \begin{cases} 1/2 \\ -1/2 \end{cases}$$

- THUS $\cos\theta = \frac{1}{2}$ & $\cos\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ & $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

IYGB - MP1 PAPER T - QUESTION 6

STARTING WITH A DIAGRAM



$\overset{\triangle}{ABE} \sim \overset{\triangle}{BCF}$

$$\frac{y}{6} = \frac{4}{x}$$

$$xy = 24$$

GET AN EXPRESSION FOR THE AREA OF THE TRIANGLE DEF

$$A = \frac{1}{2}(x+6)(y+4)$$

$$A = \frac{1}{2}(x+6)\left(\frac{24}{x}+4\right)$$

$$A = \frac{1}{2}\left(24 + 4x + \frac{144}{x} + 24\right)$$

$$A = 24 + 2x + \frac{72}{x}$$

DIFFERENTIATE "A" W.R.T x & SOLVE FOR ZERO

$$\frac{dA}{dx} = 2 - \frac{72}{x^2}$$

$$0 = 2 - \frac{72}{x^2}$$

$$2x^2 = 72$$

$$x^2 = 36$$

$$x = +6$$

$$\therefore A\Big|_{x=6} = 24 + 2\times6 + \frac{72}{6} = 24 + 12 + 12 = 48$$

JUSTIFYING IT IS MINIMUM $\quad \frac{d^2A}{dx^2} = \frac{216}{x^3}$

$$\frac{d^2A}{dx^3}\Big|_{x=6} = \frac{216}{6^3} > 0$$ INDEED MINIMUM AREA

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EXPAND IN GENERAL FORM UP TO x^2

$$(a+bx)^n = \binom{n}{0}(a)^n(bx)^0 + \binom{n}{1}(a)^{n-1}(bx)^1 + \binom{n}{2}(a)^{n-2}(bx)^2 + \dots$$

$$= \left[1 \times a^n \times 1\right] + \left[\frac{n}{1} \times a^{n-1} \times bx\right] + \left[\frac{n(n-1)}{1\times 2} \times a^{n-2} \times b^2x^2\right] + \dots$$

$$= a^n + \left[na^{n-1}b\right]x + \left[\tfrac{1}{2}n(n-1)a^{n-2}b^2\right]x^2$$

8192 6656 2496

COLLECTING THE EQUATIONS

(I) – $a^n = 8192$

(II) – $na^{n-1}b = 6656$ ← SQUARE

(III) – $\frac{1}{2}n(n-1)a^{n-2}b^2 = 2496$ ← DOUBLE

$$\Longrightarrow \left\{ \begin{array}{l} n^2a^{2n-2}b^2 = 6656^2 \\ n(n-1)a^{n-2}b^2 = 4992 \end{array} \right.$$

$$\Longrightarrow \frac{n^2 \times a^{2n} \times \cancel{a^{-2}} \times \cancel{b^2}}{n(n-1) \times a^n \times \cancel{a^{-2}} \times \cancel{b^2}} = \frac{6656^2}{4992}$$

$$\Longrightarrow \frac{n}{n-1} \times a^n = \frac{26624}{3}$$

(I)

$$\Longrightarrow \frac{n}{n-1} \times 8192 = \frac{26624}{3}$$

$$\Longrightarrow \frac{n}{n-1} = \frac{13}{12}$$

IYGB - MP1 PAPER T - QUESTION 7

$$\Rightarrow 12n = 13n - 13$$

$$\Rightarrow 13 = n$$

$$\therefore n = 13$$

NOW USING $a^n = 8192$ (I)

$$\Rightarrow a^{13} = 8192$$

$$\Rightarrow a = \sqrt[13]{8192}$$

$$\Rightarrow a = 2$$

FINALLY USING $na^{n-1}b = 6656$

$$\Rightarrow 13 \times 2^{12} \times b = 6656$$

$$\Rightarrow b = \frac{6656}{13 \times 2^{12}}$$

$$\Rightarrow b = \frac{1}{8}$$

IYGB - MP1 PAPER T - QUESTION 8

USING CALCULUS, NOTING k IS A CONSTANT

$$f(x) = x^2+6x+20+k(x^2-3x-12)$$
$$f'(x) = 2x+6+k(2x-3)$$

NOW $f'(-2)=0$, SO WE HAVE

$$0 = -4+6+k(-4-3)$$
$$-2 = -7k$$
$$k = \frac{2}{7}$$

FINALLY $f(-2)=p$, SO WE HAVE

$$p = 4-12+20-\frac{2}{7}(4+6-12)$$
$$p = 12+\frac{2}{7}(-2)$$
$$p = \frac{80}{7}$$

ALTERNATIVE BY COMPLETING THE SQUARE AS $f(x)$ IS A QUADRATIC

$$f(x) = (k+1)x^2+(6-3k)x+(20-12k)$$
$$f(x) = (k+1)\left[x^2+\frac{6-3k}{k+1}x+\frac{20-12k}{k+1}\right]$$
$$f(x) = (k+1)\left[\left[x+\frac{6-3k}{2(k+1)}\right]^2-g(k)\right]$$

NOW WE HAVE

$$\frac{6-3k}{2(k+1)} = 2$$

IYGB MP1 PAPER T - QUESTION 8

$$\Rightarrow 6 - 3k = 2 \times 2(k+1)$$

$$\Rightarrow 6 - 3k = 4k + 4$$

$$\Rightarrow 2 = 7k$$

$$\Rightarrow k = \frac{2}{7}$$

AND THE VALUE OF p FOLLOWS

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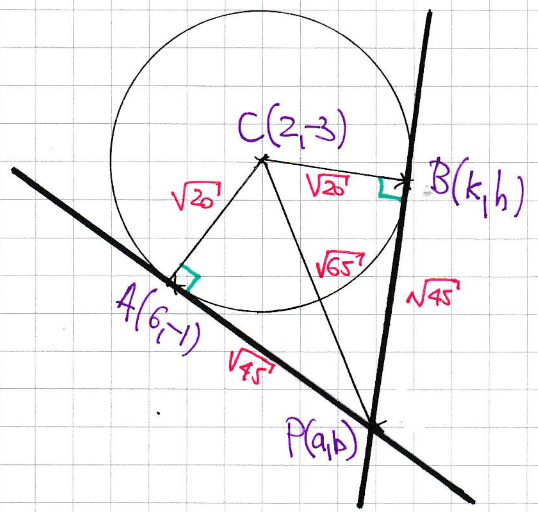
START WITH A DIAGRAM — THEN OBTAIN SOME STANDARD INFO



$$\Rightarrow x^2+y^2-4x+6y=7$$
$$\Rightarrow x^2-4x+y^2+6y=7$$
$$\Rightarrow (x-2)^2-4+(y+3)^2-9=7$$
$$\Rightarrow (x-2)^2+(y+3)^2=20$$

$C(2,-3)$, $r=\sqrt{20}$

TRIVIALLY BY PYTHAGORAS AND CIRCLE GEOMETRY $|AP|=|BP|=\sqrt{45}$

LET $P(a,b)$ & $B(k,h)$, AND MARK ALL KNOWN INFORMATION IN THE DIAGRAM

$$\Rightarrow \text{"GRAD AC"} \times \text{"GRAD AP"} = -1$$
$$\Rightarrow \frac{-1-(-3)}{6-2} \times \frac{b-(-1)}{a-6} = -1$$
$$\Rightarrow \frac{2}{4} \times \frac{b+1}{a-6} = -1$$
$$\Rightarrow \frac{b+1}{a-6} = -2$$
$$\Rightarrow b+1 = -2a+12$$
$$\Rightarrow b = 11-2a$$

NOW WE HAVE DISTANCE CONSTRAINTS, IE $|PA|=\sqrt{45}$ & $|PC|=\sqrt{65}$

$$\left.\begin{aligned}(a-6)^2+(b+1)^2&=45\\(a-2)^2+(b+3)^2&=65\end{aligned}\right\} \Rightarrow \text{USING EITHER EQUATION WITH } b=11-2a$$

IYGB – MP1 PAPER T – QUESTION 9

$$\Rightarrow (a-6)^2 + \left[(11-2a)+1\right]^2 = 45$$

$$\Rightarrow a^2 - 12a + 36 + (12-2a)^2 = 45$$

$$\Rightarrow a^2 - 12a - 9 + (144 - 48a + 4a^2) = 0$$

$$\Rightarrow 5a^2 - 60a + 135 = 0$$

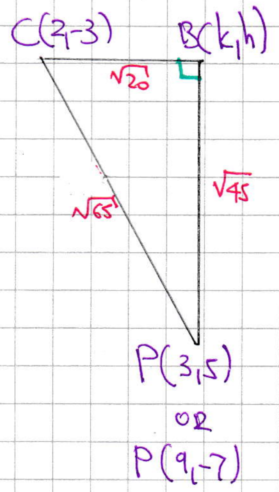
$$\Rightarrow a^2 - 12a + 27 = 0$$

$$\Rightarrow (a-9)(a-3) = 0$$

$$\Rightarrow a = \left\langle \begin{matrix} 3 \\ 9 \end{matrix} \right. \qquad b = \left\langle \begin{matrix} 5 \\ -7 \end{matrix} \right.$$

$\therefore$ $P(3,5)$ OR $P(9,-7)$

NOW LOOKING AT THE TRIANGLE $\hat{BCP}$



USE $P(3,5)$ FIRST

$|BP| = \sqrt{45}$ & $|BC| = \sqrt{20}$

$$\Rightarrow \left.\begin{matrix} (k-3)^2 + (h-5)^2 = 45 \\ (k-2)^2 + (h+3)^2 = 20 \end{matrix}\right\}$$

$$\Rightarrow \begin{matrix} k^2 - 6k + 9 + h^2 - 10h + 25 = 45 \\ k^2 - 4k + 4 + h^2 + 6h + 9 = 20 \\ \hline -2k + 5 \quad -16h \quad +16 = 25 \end{matrix}$$

SUBTRACT

$$\Rightarrow -2k - 16h - 4 = 0$$

IYGB - MP1 PAPER T - QUESTION 9

$\Rightarrow \quad -k - 8h - 2 = 0$

$\Rightarrow \quad k = -2 - 8h$

SUBSTITUTE INTO $(k-2)^2 + (h-3)^2 = 20$

$\Rightarrow \quad [(-2-8h)-2]^2 + (h+3)^2 = 20$

$\Rightarrow \quad [-4-8h]^2 + (h+3)^2 = 20$

$\Rightarrow \quad (8h+4)^2 + (h+3)^2 = 20$

$\Rightarrow \quad 64h^2 + 64h + 16 + h^2 + 6h + 9 = 20$

$\Rightarrow \quad 65h^2 + 70h + 5 = 0$

$\Rightarrow \quad 13h + 14h + 1 = 0$

$\Rightarrow \quad (13h+1)(h+1) = 0$

$h = \begin{cases} -1 \\ -\frac{1}{13} \end{cases} \qquad k = \begin{cases} 6 \\ -\frac{18}{13} \end{cases}$

$\therefore \quad B\left(-\frac{1}{13}, -\frac{18}{13}\right)$ & $A(-1,6)$ AS EXPECTED

USE $P(9,-7)$ NEXT

USING $|BP| = \sqrt{45}$ & $|BC| = \sqrt{20}$ AND SAME DIAGRAM

$$\Rightarrow \left.\begin{array}{l} (k-9)^2 + (h+7)^2 = 45 \\ (k-2)^2 + (h+3)^2 = 20 \end{array}\right\}$$

$$\Rightarrow \begin{array}{l} k^2 - 18k + 81 + h^2 + 14h + 49 = 45 \\ k^2 - 4k + 4 + h^2 + 6h + 9 = 20 \\ \hline -14k + 77 \qquad + 8h + 40 = 25 \end{array}$$

SUBTRACT

IYGB – MP1 PAPER T – QUESTION 9

$$\Rightarrow -14k + 8h + 92 = 0$$

$$\Rightarrow -7k + 4h + 46 = 0$$

$$\Rightarrow 4h = 7k - 46$$

PROCEED BY THE SUBSTITUTION INTO $(k-2)^2 + (h+3)^2 = 20$

$$\Rightarrow (k-2)^2 + (h+3)^2 = 20$$

$$\Rightarrow 16(k-2)^2 + 16(h+3)^2 = 320$$

$$\Rightarrow 16(k^2 - 4k + 4) + (4h+12)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + (7k - 46 + 12)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + (7k - 34)^2 = 320$$

$$\Rightarrow 16k^2 - 64k + 64 + 49k^2 - 476k + 1156 = 320$$

$$\Rightarrow 65k^2 - 540k + 900 = 0$$

$$\Rightarrow 13k^2 - 108k + 180 = 0$$

$$\Rightarrow (13k - 30)(k - 6) = 0$$

FROM POINT A

$$\Rightarrow k = \begin{cases} 6 \\ \frac{30}{13} \end{cases} \qquad h = \begin{cases} -1 & \leftarrow \text{POINT A} \\ -\frac{97}{13} & \leftarrow \text{POINT B} \end{cases}$$

HENCE THE REQUIRED ANSWERS ARE

EITHER $P(3,5)$ $B\left(-\frac{1}{13}, -\frac{18}{13}\right)$

OR $P(9,-7)$ $B\left(\frac{30}{13}, -\frac{97}{13}\right)$

IYGB - MP1 PAPER T - QUESTION 10

$$(1+e^{-2})(e^{2})^{x^2-4x+5} = e^{2} + e^{4(x-2)^2}$$

TIDY THE EQUATION AS FOLLOWS

$$\Rightarrow (1+e^{-2})\, e^{2(x^2-4x+5)} = e^{2} + e^{4(x-2)^2}$$

$$\Rightarrow (1+e^{-2})\, e^{2[(x-2)^2+1]} = e^{2} + e^{4(x-2)^2}$$

$$\Rightarrow (1+e^{-2})\, e^{2(x-2)^2+2} = e^{2} + e^{2\times 2(x-2)^2}$$

$$\Rightarrow (1+e^{-2})\, e^{2(x-2)^2} \times e^{2} = e^{2} + \left[e^{2(x-2)^2}\right]^2$$

$$\Rightarrow (e^{2}+1)\, e^{2(x-2)^2} = e^{2} + \left[e^{2(x-2)^2}\right]^2$$

$$\Rightarrow (e^{2}+1)A = e^{2} + A^2 \qquad A = e^{2(x-2)^2}$$

REGROUP & FACTORIZE THE QUADRATIC

$$\Rightarrow 0 = A^2 - (e^2+1)A + e^2$$

$$\Rightarrow 0 = (A - e^2)(A-1)$$

$$\Rightarrow A = \begin{cases} 1 \\ e^2 \end{cases} \Rightarrow e^{2(x-2)^2} = \begin{cases} 1 \\ e^2 \end{cases}$$

$\Rightarrow$ EITHER $x = 2$

OR $2(x-2)^2 = 2$

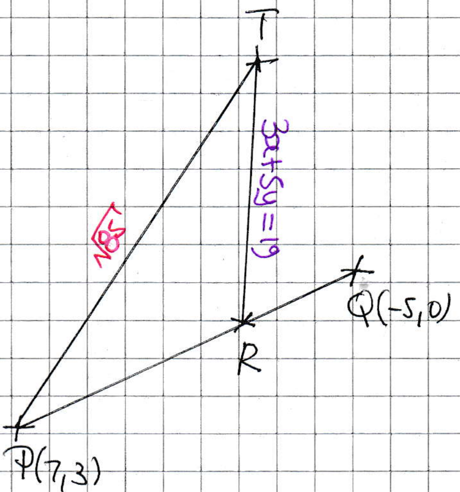
$(x-2)^2 = 1$

$x-2 = \begin{cases} -1 \\ +1 \end{cases}$

$x = \begin{cases} 1 \\ 3 \end{cases}$

IYGB - MP1 PAPER T - QUESTION 11

STARTING WITH A DIAGRAM - NOT TO ANY SCALE



OBTAIN THE CO-ORDINATES OF R

- GRADIENT PQ $= \dfrac{0-3}{-5-7} = \dfrac{-3}{-12} = \dfrac{1}{4}$

- EQUATION PQ: $y - y_0 = m(x - x_0)$

$$y - 0 = \tfrac{1}{4}(x+5)$$
$$y = \tfrac{1}{4}x + \tfrac{5}{4}$$
$$4y = x + 5$$

- SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} 4y = x+5 \\ 3x+5y = 19 \end{array}\right\} \Rightarrow x = 4y - 5$$
$$\Rightarrow 3(4y-5) + 5y = 19$$
$$\Rightarrow 17y - 15 = 19$$
$$\Rightarrow 17y = 34$$
$$\Rightarrow y = 2 \quad \& \quad x = 3$$

$$\therefore R(3,2)$$

NOW PROCEED AS FOLLOWS

- LET $T(a,b)$

- $|PT| = \sqrt{45} \Rightarrow \sqrt{(a-7)^2+(b-3)^2} = \sqrt{85}$

$$\Rightarrow (a-7)^2 + (b-3)^2 = 85$$

- BUT AS T LIES ON $3x+5y=19 \Rightarrow 3a+5b=19$

COMBINING RESULTS

$$\left.\begin{array}{l} 3a+5b=19 \\ (a-7)^2+(b-3)=45 \end{array}\right\} \begin{array}{l} \Rightarrow 3a = 19-5b \\ \Rightarrow 9(a-7)^2+9(b-3)^2 = 85\times 9 \end{array} \Big\} \Rightarrow (3a-21)^2+9(b-3)^2 = 765$$

$$\Rightarrow (19-5b-21)^2 + 9(b-3)^2 = 765$$
$$\Rightarrow (-5b-2)^2 + 9(b-3)^2 = 765$$

IYGB - MP1 PAPER T - QUESTION 11

$\Rightarrow (5b+2)^2 + 9(b-3)^2 = 765$

$\Rightarrow \begin{matrix} 25b^2 + 20b + 4 \\ 9b^2 - 54b + 81 \end{matrix} = 765$

$\Rightarrow 34b^2 - 34b - 680 = 0$

$\Rightarrow b^2 - b - 20 = 0$

$\Rightarrow (b+4)(b-5) = 0$

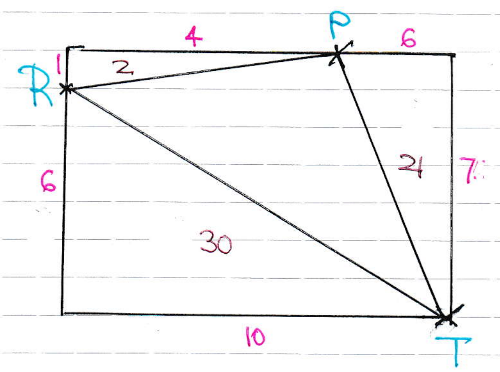
$\boxed{a = \frac{19-5b}{3}}$

$\Rightarrow b = \begin{cases} -4 \\ 5 \end{cases} \qquad a = \begin{cases} 13 \\ -2 \end{cases}$

$T(13,-4)$ OR $T(-2,5)$

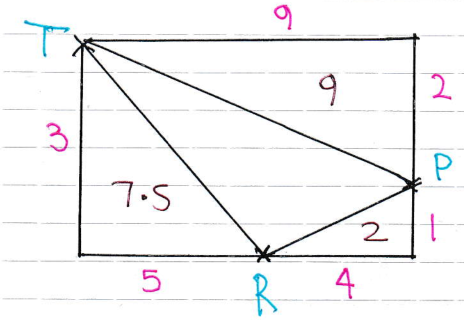
BOTH POSITIONS ARE POSSIBLE

$P(7,3)$ $R(3,2)$ $T(13,-4)$



AREA OF $\hat{PRT} = (7 \times 10) - 30 - 21 - 2$
$= 17$

$P(7,3)$ $R(3,2)$ $T(-2,5)$



AREA OF $\hat{PRT} = (9 \times 3) - 9 - 2 - 7.5$
$= 27 - 11 - 7.5$
$= 16 - 7.5$
$= 8.5$

INDEED TWO POSSIBLE AREAS OF WHICH ONE IS TWICE AS LARGE AS THE OTHER

NOTE THAT THE AREA OF A TRIANGLE WITH VERTICES AT (x_1,y_1), (x_2,y_2), (x_3,y_3) CAN ALSO BE FOUND BY THE FORMULA

$$\text{AREA} = \frac{1}{2} \left\| \begin{matrix} 1 & 1 & 1 \\ x_1 & x_2 & y_2 \\ y_1 & y_2 & y_3 \end{matrix} \right\|$$

IYGB - MP1 PAPER T - QUESTION 12

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = \frac{3}{4}x^2 - 4\sqrt{x} + 7 \\ y = 5x - 9 \end{array}\right\} \Rightarrow \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 = 5x - 9$$

$$\Rightarrow 3x^2 - 16x^{\frac{1}{2}} + 28 = 20x - 36$$

$$\Rightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 = 0$$

USING A SUBSTITUTION, $t = x^{\frac{1}{2}}$

$$\Rightarrow 3t^4 - 20t^2 - 16t + 64 = 0$$

$$\Rightarrow f(t) = 3t^4 - 20t^2 - 16t + 64$$

$$\Rightarrow f'(t) = 12t^3 - 40t - 16 = 4[3t^3 - 10t - 4]$$

AS WE ARE LOOKING FOR A POINT OF TANGENCY WE ARE LOOKING FOR A SOLUTION FOR BOTH

$$3t^4 - 20t^2 - 16t + 64 = 0 \quad \& \quad 3t^3 - 10t - 4 = 0$$

- $t = 1$ $\quad 3 - 20 - 16 + 64 \neq 0 \quad\quad 3 - 10 - 4 \neq 0$
- $t = -1$ $\quad 3 - 20 + 16 + 64 \neq 0 \quad\quad -3 + 10 - 4 \neq 0$
- $t = 2$ $\quad 48 - 80 - 32 + 64 = 0 \quad\quad 24 - 20 - 4 = 0$

$\therefore$ $t = 2$ YIELDS A REPEATED ROOT

BY LONG DIVISION OR EQUIVALENT METHOD

$$\begin{array}{r|l} & 3t^2 + 12t + 16 \\ \hline t^2 - 4t + 4 & 3t^4 + 0t^3 - 20t^2 - 16t + 64 \\ & -3t^4 + 12t^3 - 12t^2 \\ \hline & \quad 12t^3 - 32t^2 - 16t + 64 \\ & \quad -12t^3 + 48t^2 - 48t \\ \hline & \quad\quad 16t^2 - 64t + 64 \\ & \quad\quad -16t^2 + 64t - 64 \\ \hline & \quad\quad\quad 0 \end{array}$$

$(t-2)^2$

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COLLECTING THE RESULTS

$\Rightarrow 3t^4 - 20t^2 - 16t + 64 = 0$

$\Rightarrow (t-2)^2(3t^2 + 12t + 16) = 0$

$b^2 - 4ac = 12^2 - 4 \times 3 \times 16$
$= 144 - 192$
$= -48 < 0$

$\therefore$ ONLY SOLUTION IS THE POINT OF TANGENCY

$t = 2 \Rightarrow \sqrt{x} = 2$
$\Rightarrow x = 4$
$\Rightarrow y = 11$

$\therefore$ $y = 5x - 9$ IS A TANGENT AT $(4,11)$ & DOES NOT MEET THE CURVE AGAIN

ALTERNATIVE METHOD

START BY DIFFERENTIATION, TO FIND WHERE $\frac{dy}{dx} = 5$

GRADIENT OF THE GIVEN LINE

$\Rightarrow y = \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2}x - 2x^{-\frac{1}{2}}$

$\Rightarrow 5 = \frac{3}{2}x - \frac{2}{\sqrt{x}}$ $\quad \times 2$

$\Rightarrow 10 = 3x - \frac{4}{\sqrt{x}}$ $\quad \times \sqrt{x} = x^{\frac{1}{2}}$

$\Rightarrow 10x^{\frac{1}{2}} = 3x^{\frac{3}{2}} - 4$

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$$\Rightarrow 3x^{\frac{3}{2}} - 10x^{\frac{1}{2}} - 4 = 0$$

$$\Rightarrow 3t^3 - 10t - 4 = 0 \qquad (t = x^{\frac{1}{2}})$$

$$\Rightarrow f(t) \equiv 3t^3 - 10t - 4$$

LOOK FOR SOLUTIONS BY INSPECTION

$$f(1) = 3 - 10 - 4 \neq 0$$
$$f(-1) = -3 + 10 - 4 \neq 0$$
$$f(2) = 24 - 20 - 4 = 0$$

$$\Rightarrow (t-2)(3t^2 + At + 2) = 0$$

$-6t^2$

At^2

$$\therefore At^2 - 6t^2 = 0$$
$$t^2(A-6) = 0$$
$$A = 6$$

$$\Rightarrow (t-2)(3t^2 - 6t + 2) = 0$$

$$b^2 - 4ac = 36 - 4 \times 3 \times 2 = 12 > 0$$

$$t = \frac{6 \pm \sqrt{12}}{6} = 1 \pm \frac{1}{6}\sqrt{12}$$

$$t = 1 \pm \frac{1}{3}\sqrt{3}$$

$$\Rightarrow t = x^{\frac{1}{2}} = \begin{cases} 2 \\ 1 + \frac{1}{2}\sqrt{3} \\ 1 - \frac{1}{2}\sqrt{3} \end{cases}$$

$y = 5x - 9$

$$\Rightarrow x = \begin{cases} 4 \\ 1 + \frac{3}{4} + \sqrt{3} = \frac{7}{4} + \sqrt{3} \\ 1 + \frac{3}{4} - \sqrt{3} = \frac{7}{4} - \sqrt{3} \end{cases} \qquad y = \begin{cases} 11 \\ \frac{35}{4} + 5\sqrt{3} - 9 \\ \frac{35}{4} - 5\sqrt{3} - 9 \end{cases}$$

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$$x^2 = \begin{cases} 16 \\ \frac{49}{16} + \frac{7}{2}\sqrt{3} + 3 = \frac{97}{16} + \frac{7}{2}\sqrt{3} \\ \frac{49}{16} - \frac{7}{2}\sqrt{3} + 3 = \frac{97}{16} - \frac{7}{2}\sqrt{3} \end{cases}$$

$$y = \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 = \begin{cases} \frac{3}{4} \times 16 - 4 \times 2 + 7 = 11 \\ \frac{3}{4}\left(\frac{97}{16} + \frac{7}{2}\sqrt{3}\right) - 4\left(1 + \frac{1}{2}\sqrt{3}\right) + 7 = \frac{483}{64} + \frac{5}{8}\sqrt{3} \\ \frac{3}{4}\left(\frac{97}{16} - \frac{7}{2}\sqrt{3}\right) - 4\left(1 - \frac{1}{2}\sqrt{3}\right) + 7 = \frac{483}{64} - \frac{5}{8}\sqrt{3} \end{cases}$$

ONLY THE POINT $(4,11)$ LIES ON BOTH CURVE & LINE & THE GRADIENT AT $(4,11)$ IS 5

$\therefore$ $y = 5x - 11$ IS A TANGENT TO THE CURVE AT $(4,11)$

WE STILL NEED TO SHOW THAT THERE IS NO OTHER INTERSECTION

$$\left.\begin{array}{l} y = 5x - 11 \\ y = \frac{3}{4}x^2 - 4\sqrt{x} + 7 \end{array}\right\} \Rightarrow \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 = 5x - 11$$

$$\Rightarrow 3x^2 - 16x^{\frac{1}{2}} + 28 = 20x - 44$$

$$\Rightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 = 0$$

AS $x = 4$ IS A REPEATED ROOT

$$\Rightarrow (x-4)^2 \times f(x) \equiv 3x^2 - 20x - 16x^{\frac{1}{2}} + 64$$

$$\Rightarrow (x-4)^2 \times f(x) \equiv 3\left[x^2 - 8x + 16\right] + 4x - 16x^{\frac{1}{2}} + 16$$

$$\Rightarrow (x-4)^2 \times f(x) \equiv 3(x-4)^2 + 4\left(x - 4x^{\frac{1}{2}} + 4\right)$$

$$\Rightarrow (x-4)^2 \times f(x) \equiv 3(x-4)^2 + 4\left(\sqrt{x} - 2\right)^2$$

$$\Rightarrow f(x) \equiv 3 + \frac{4(\sqrt{x} - 2)}{(x-4)^2}$$

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$$\Longrightarrow 3x^2 - 20x - 16x^{\frac{1}{2}} + 64 = 0$$

$$\Longrightarrow (x-4)^2\left[3 + \frac{4(\sqrt{x}-2)^2}{(x-4)^2}\right] = 0$$

$$\Longrightarrow (x-4)^2\left[3 + \left(\frac{2(\sqrt{x}-2)}{x-4}\right)^2\right] = 0$$

$\therefore$ EITHER $x=4$ (REPEATED)

OR $\cancel{\left(\frac{2(\sqrt{x}-2)}{x-4}\right)^2 = -3}$

$\therefore$ TANGENT DOES NOT RE-INTERSECT