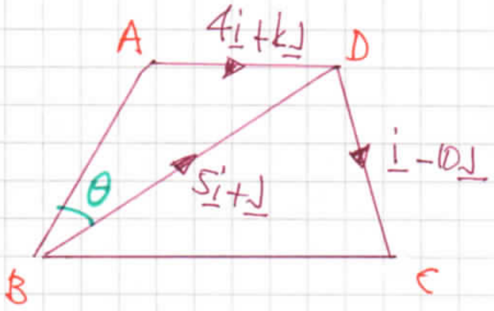
IYGB - MP1 PAPER P - QUESTION 1

a) LOOKING AT THE DIAGRAM



$\Rightarrow \vec{BC} = \vec{BD} + \vec{DC}$

$\Rightarrow \vec{BC} = (5\underline{i}+\underline{j}) + (\underline{i}-10\underline{j})$

$\Rightarrow \vec{BC} = 6\underline{i} - 9\underline{j}$

AS AD IS PARALLEL TO BC, THEIR VECTOR COMPONENTS MUST BE IN PROPORTION

$\Rightarrow \frac{4}{k} = \frac{6}{-9}$

$\Rightarrow 6k = -36$

$\Rightarrow k = -6$ AS REQUIRED

b) FIRST FIND $\vec{AB}$

$\vec{AB} = \vec{AD} + \vec{DB} = (4\underline{i}-6\underline{j}) - (5\underline{i}+\underline{j}) = -\underline{i} - 7\underline{j}$

NEXT THE LENGTH OF $\vec{AB}$

$|\vec{AB}| = |-\underline{i}-7\underline{j}| = \sqrt{(-1)^2+(-7)^2} = \sqrt{50} = 5\sqrt{2}$

c) BY THE COSINE RULE ON $\hat{ABD}$

$|\vec{AD}| = |4\underline{i}-6\underline{j}| = \sqrt{4^2+(-6)^2} = \sqrt{52}$

$|\vec{BD}| = |5\underline{i}+\underline{j}| = \sqrt{5^2+1^2} = \sqrt{26}$

$\cos\theta = \frac{|AB|^2+|BD|^2-|AD|^2}{2|AB||BD|} = \frac{50+26-52}{2\times\sqrt{50}\sqrt{26}} = 0.33282\ldots$

$\therefore \theta \simeq 70.6°$

IYGB - MP1 PAPER P - QUESTION 2

THE EQUATION IS ALREADY FACTORIZED SO WE SOLVE DIRECTLY FOR EACH OF THE TWO FACTORS

$$\Longrightarrow (\sqrt{3}-2\sin 3x)(\sqrt{3}+2\cos 3x)=0$$

$$\Longrightarrow \sqrt{3}-2\sin 3x=0$$
$$\Longrightarrow \sqrt{3}=2\sin 3x$$
$$\Longrightarrow \sin 3x=\frac{\sqrt{3}}{2}$$

$$\arcsin\left(\frac{\sqrt{3}}{2}\right)=60°$$

$$\Longrightarrow \begin{cases} 3x = 60° \pm 360n \\ 3x = 120° \pm 360n \end{cases} \quad n=0,1,2,3,\ldots$$

$$\Longrightarrow \begin{cases} x = 20° \pm 120n \\ x = 40° \pm 120n \end{cases}$$

$$\Longrightarrow \sqrt{3}+2\cos 3x=0$$
$$\Longrightarrow 2\cos 3x=-\sqrt{3}$$
$$\Longrightarrow \cos 3x=-\frac{\sqrt{3}}{2}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right)=150°$$

$$\Longrightarrow \begin{cases} 3x = 150° \pm 360n \\ 3x = 210° \pm 360n \end{cases} \quad n=0,1,2,3,\ldots$$

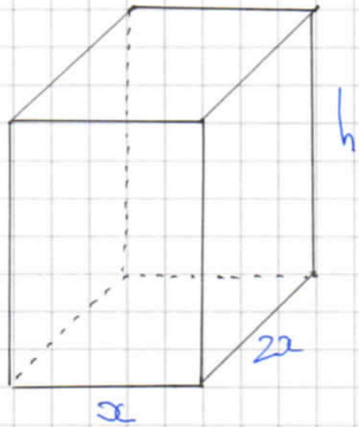
$$\Longrightarrow \begin{cases} x = 50° \pm 120n \\ x = 70° \pm 120n \end{cases}$$

$$x_1 = 20°$$
$$x_2 = 140°$$
$$x_3 = 40°$$
$$x_4 = 160°$$
$$x_5 = 50°$$
$$x_6 = 170°$$
$$x_7 = 70°$$

OR $x = 20°, 40°, 50°, 70°, 140°, 160°, 170°$ //

IYGB - MP1 PAPER P - QUESTION 3

a)



CONSTRAINT $V = 1000\text{ cm}^3$

$\Rightarrow V = x(2x)h$

$\Rightarrow 1000 = 2x^2h$

$\Rightarrow x^2h = 500$

$xh = \frac{500}{x}$

$\boxed{6xh = \frac{3000}{x}}$

SURFACE AREA ($A\text{ cm}^2$)

$\Rightarrow A = 2\left[2x^2 + xh + 2xh\right]$

$\Rightarrow A = 4x^2 + 6xh$

$\Rightarrow A = 4x^2 + \frac{3000}{x}$ //

AS REQUIRED

b) $A = 4x^2 + 3000x^{-1}$

$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

FOR STATIONARY VALUES $\frac{dA}{dx} = 0$

$\Rightarrow 8x - \frac{3000}{x^2} = 0$

$\Rightarrow 8x = \frac{3000}{x^2}$

$\Rightarrow 8x^3 = 3000$

IYGB - MP1 PAPER P - QUESTION 3

$$\Rightarrow \quad x^3 = 375$$

$$\Rightarrow \quad x = \sqrt[3]{375} \approx 7.21 \text{ cm}$$

c) $A = 4x^2 + \dfrac{3000}{x}$

$$\Rightarrow A_{MIN} = 4(7.21\ldots)^2 + \frac{3000}{7.21\ldots}$$

$$\Rightarrow A_{MIN} \approx 624 \text{ cm}^2$$

TO JUSTIFY IT IS A MIN, USE 2ND DERIVATIVE

$$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 8 + 6000x^{-3}$$

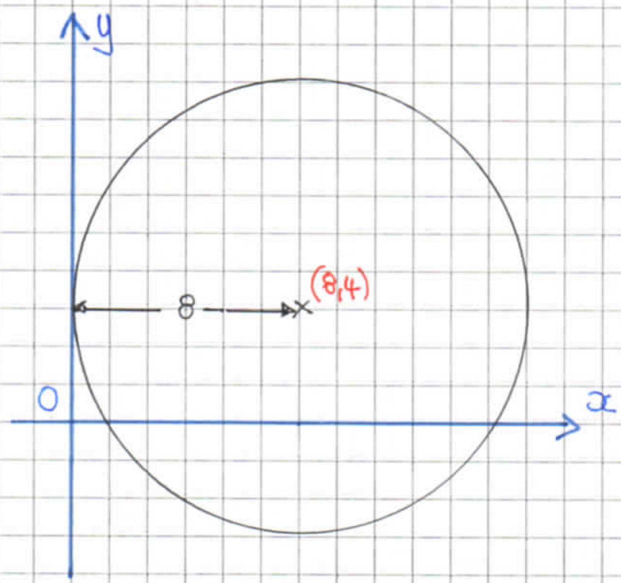
$$\Rightarrow \frac{d^2A}{dx^2} = 8 + \frac{6000}{x^3}$$

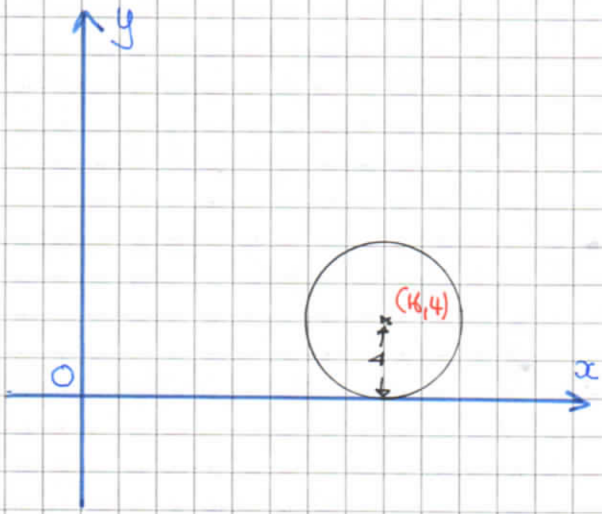
$$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=7.21\ldots} = 8 + \frac{6000}{(7.21\ldots)^3} = 24 > 0$$

INDEED A MINIMUM

IYGB - MP1 PAPER P - QUESTION 4

a) DRAWING EACH OF THE CIRCLES





$\therefore (x-8)^2+(y-4)^2=8^2$

$(x-8)^2+(y-4)^2=64$

$\therefore (x-16)^2+(y-4)^2=4^2$

$(x-16)^2+(y-4)^2=16$

b) SOLVING THE EQUATIONS SIMULTANEOUSLY

$(x-8)^2+(y-4)^2=64$

$(x-16)^2+(y-4)^2=16$

SUBTRACTING EQUATIONS

$\Rightarrow (x-8)^2-(x-16)^2=48$

$\Rightarrow x^2-16x+64-(x^2-32x+256)=48$

$\Rightarrow \cancel{x^2}-16x+64-\cancel{x^2}+32x-256=48$

$\Rightarrow 16x=240$

$\Rightarrow x=15$

SUBSTITUTING INTO EITHER EQUATION

$\Rightarrow (x-16)^2+(y-4)^2=16$

$\Rightarrow (15-16)^2+(y-4)^2=16$

IYGB - MP1 PAPER P - QUESTION 4

$$\Rightarrow \quad (-1)^2 + (y-4)^2 = 16$$

$$\Rightarrow \quad 1 + (y-4)^2 = 16$$

$$\Rightarrow \quad (y-4)^2 = 15$$

$$\Rightarrow \quad y-4 = \pm\sqrt{15}$$

$$\Rightarrow \quad y = 4 \pm \sqrt{15}$$

THUS THE REQUIRED CO-ORDINATES IN ANY ORDER ARE

$A(15,\ 4+\sqrt{15})$ & $B(15,\ 4-\sqrt{15})$

IYGB - MP1 PAPER P - QUESTION 5

$$(1+kx)^n = 1 + Ax + 264x^2 + 1760x^3 + \dots$$

a) EXPAND $(1+kx)^n$ UP AND INCLUDING THE TERM IN x^3

$$(1+kx)^n = 1 + \frac{n}{1}(kx)^1 + \frac{n(n-1)}{1\times 2}(kx)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(kx)^3 + \dots$$

$$= 1 + nkx + \tfrac{1}{2}n(n-1)k^2x^2 + \tfrac{1}{6}n(n-1)(n-2)k^3x^3$$

COMPARING COEFFICIENTS IN x^2 & x^3

- $\frac{1}{2}n(n-1)k^2 = 264$

- $\frac{1}{6}n(n-1)(n-2)k^3 = 1760$

$$\frac{1}{3}\left[\tfrac{1}{2}n(n-1)k^2\right](n-2)k = 1760$$

$$\frac{1}{3}\times 264(n-2)k = 1760$$

$$(n-2)k = 20$$

AS REQUIRED

b) NOW WE HAVE

- $\frac{1}{2}n(n-1)k^2 = 264$

$$n(n-1)k^2 = 528$$

- $(n-2)k = 20$

$$(n-2)^2k^2 = 400$$

DIVIDING THE EQUATIONS SIDE BY SIDE

$$\frac{n(n-1)k^2}{(n-2)^2k^2} = \frac{528}{400}$$

IYGB - MP1 PAPER P - QUESTION 5

$$\Longrightarrow \frac{n(n-1)}{(n-2)^2} = \frac{33}{25}$$

$$\Longrightarrow \frac{n^2-n}{n^2-4n+4} = \frac{33}{25}$$

$$\Longrightarrow 25n^2-25n = 33n^2-132n+132$$

$$\Longrightarrow 0 = 8n^2-107n+132$$

$$\Longrightarrow 0 = (8n-11)(n-12)$$

(OR BY THE QUADRATIC FORMULA

$$n = \frac{107 \pm \sqrt{(-107)^2-4\times8\times132}}{2\times8}$$

$$n = \frac{107 \pm \sqrt{7225}}{16} =$$

$$n = \frac{107 \pm 85}{16} = \begin{cases} 12 \\ \frac{11}{8} \end{cases})$$

$$\Longrightarrow n = \begin{cases} 12 \\ \cancel{\frac{11}{8}} \end{cases}$$

HENCE USING $(n-2)k = 20$

$$10k = 20$$
$$k = 2$$

FINALLY $A = nk$

$$A = 12\times2$$

$$\therefore A = 24$$

IYGB - MP1 PAPER P - QUESTION 6

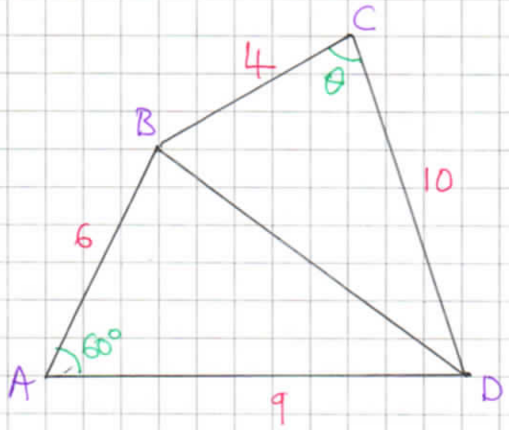
a) BY THE COSINE RULE ON $\hat{ABD}$

$\Rightarrow |BD|^2 = |AB|^2 + |AD|^2 - 2|AB||AD|\cos 60°$

$\Rightarrow |BD|^2 = 36 + 81 - 2\times 6\times 9\times \frac{1}{2}$

$\Rightarrow |BD|^2 = 63$

$\Rightarrow |BD| = \sqrt{63} = 3\sqrt{7}$ // AS REQUIRED



b) BY THE COSINE RULE ON $\hat{BCD}$

$\Rightarrow |BD|^2 = |BC|^2 + |CD|^2 - 2|BC||CD|\cos\theta$

$\Rightarrow 63 = 16 + 100 - 2\times 4\times 10\times \cos\theta$

$\Rightarrow 80\cos\theta = 53$

$\Rightarrow \cos\theta = \frac{53}{80}$

$\Rightarrow \theta \simeq 48.5°$ //

c) FINDING THE AREA OF EACH OF THE TRIANGLES

AREA OF $\hat{ABD} = \frac{1}{2}\times 6\times 9\times \sin 60 \simeq 23.382...$

AREA OF $\hat{BCD} = \frac{1}{2}\times 4\times 10\times \sin(48.5...°) \simeq 14.981...$

$\therefore$ TOTAL AREA $= 38.4 \text{ cm}^2$ // (3 sf)

IYGB - MP1 PAPER P - QUESTION 7

LET THE TWO ROOTS BE α & $\alpha+k$

THEN $(x-\alpha)[x-(\alpha+k)]=0$

$$x^2-(\alpha+k)x-\alpha x+\alpha(\alpha+k)=0$$

$$x^2-(2\alpha+k)x+(\alpha^2+k\alpha)=0$$

THUS $b=-(2\alpha+k)$ & $c=\alpha^2+\alpha k$

$$\Rightarrow b^2-4ac=[-(2\alpha+k)]^2-4\times1\times(\alpha^2+\alpha k)$$

$$=(2\alpha+k)^2-4(\alpha^2+\alpha k)$$

$$=\cancel{4\alpha^2}+\cancel{4\alpha k}+k^2-\cancel{4\alpha^2}-\cancel{4\alpha k}$$

$$=k^2$$

IYGB – MP1 PAPER P – QUESTION 8

a) WRITE IN INDEX NOTATION AND DIFFERENTIATE

$$\Rightarrow \quad y = 4\sqrt{x} - 3x - 3$$

$$\Rightarrow \quad y = 4x^{\frac{1}{2}} - 3x - 3$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x^{-\frac{1}{2}} - 3$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{x=4} = 2\times 4^{-\frac{1}{2}} - 3 = -2 \quad \longleftarrow \text{TANGENT GRADIENT}$$

FIND THE y CO-ORDINATE OF P & THEN THE TANGENT

$$y\Big|_{x=4} = 4\sqrt{4} - 3\times 4 - 3 = -7$$

$$\therefore \quad y - y_0 = m(x - x_0)$$

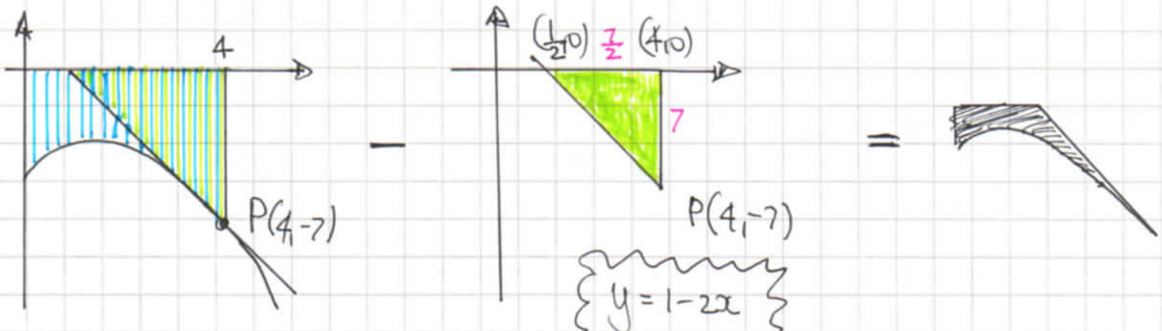
$$y + 7 = -2(x - 4)$$

$$y + 7 = -2x + 8$$

$$y = 1 - 2x$$

AS REQUIRED

b) LOOKING AT THE PICTORIAL EQUATION BELOW



$$y = 1 - 2x$$
$$0 = 1 - 2x$$
$$x = \frac{1}{2}$$

AREA ABOVE IS GIVEN BY

$$-\int_0^4 4x^{\frac{1}{2}} - 3x - 3 \, dx$$

AREA OF TRIANGLE $= \frac{1}{2}\times 7\times\frac{7}{2} = \frac{49}{4}$

IYGB - MP1 PAPER P - QUESTION 8

$$= -\int_0^4 4x^{\frac{1}{2}} - 3x - 3 \, dx = \int_4^0 4x^{\frac{1}{2}} - 3x - 3 \, dx$$

$$= \left[\frac{8}{3}x^{\frac{3}{2}} - \frac{3}{2}x^2 - 3x \right]_4^0 = 0 - \left(\frac{64}{3} - 24 - 12 \right)$$

$$= \frac{44}{3}$$

HENCE THE REQUIRED AREA CAN BE FOUND

$$\frac{44}{3} - \frac{49}{4} = \frac{176}{12} - \frac{147}{12} = \frac{29}{12}$$

IYGB - MP1 PAPER P - QUESTION 9

USING THE RULES OF LOGARITHMS

$$\Rightarrow 2\log_2 x + \log_2(x-1) - \log_2(5x+4) = 1$$

$$\Rightarrow \log_2 x^2 + \log_2(x-1) - \log_2(5x+4) = \log_2 2$$

$$\Rightarrow \log_2\left[\frac{x^2(x-1)}{5x+4}\right] = \log_2 2$$

$$\Rightarrow \frac{x^2(x-1)}{5x+4} = 2$$

$$\Rightarrow \frac{x^3-x^2}{5x+4} = 2$$

$$\Rightarrow x^3 - x^2 = 10x + 8$$

$$\Rightarrow x^3 - x^2 - 10x - 8 = 0$$

LOOK FOR "OBVIOUS" SOLUTIONS $\pm 1, \pm 2, \pm 4, \pm 8$

- $x = 1$ $\quad 1 - 1 - 10 - 8 \neq 0$
- $x = -1$ $\quad -1 - 1 + 10 - 8 = 0$!!

$\therefore (x+1)$ IS A FACTOR

BY LONG DIVISION OR MANIPULATIONS

$$\Rightarrow x^2(x+1) - 2x(x+1) - 8(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 2x - 8) = 0$$

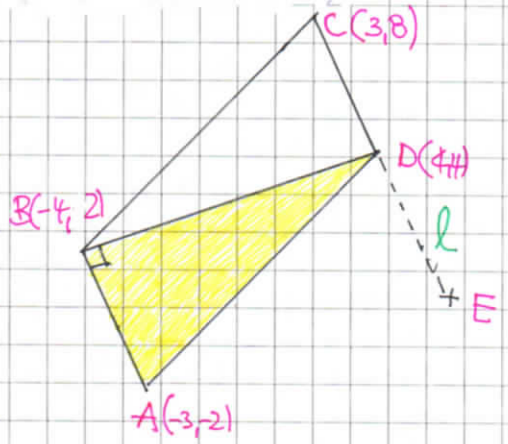
$$\Rightarrow (x+1)(x+2)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} -1 \\ -2 \\ 4 \end{cases}$$

ONLY $x = 4$ IS ACCEPTABLE FOR THE ARGUMENT OF $\log_2 x$

IYGB - MP1 PAPER P - QUESTION 10

a) AS WE REQUIRED TO FIND THE AREA WE SHALL WORK WITH LENGTHS AND PYTHAGORAS TO SHOW $A\hat{B}D = 90°$ (RATHER THAN GRADIENTS)



$$\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

- $|AB| = \sqrt{(-4+3)^2 + (2+2)^2}$
$= \sqrt{1+16}$
$= \sqrt{17}$

- $|BD| = \sqrt{(-4-4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$
- $|AD| = \sqrt{(-3-4)^2 + (-2-4)^2} = \sqrt{49+36} = \sqrt{85}$

$\Rightarrow |AB|^2 + |BD|^2 = (\sqrt{17})^2 + (\sqrt{68})^2 = 17 + 68 = 85 = (\sqrt{85})^2 = |AD|^2$

THE PYTHAGOREAN RELATIONSHIP IS SATISFIED FOR A RIGHT ANGLE AT $A\hat{B}D$

FINALLY THE AREA OF THE PARALLELOGRAM IS THAT OF 2 IDENTICAL TRIANGLES

$\Rightarrow$ AREA $= 2 \times$ AREA OF $A\hat{B}D$
$= 2 \times \frac{1}{2} \times |AB||BD|$
$= \sqrt{17} \times \sqrt{68}$
$= \sqrt{17} \times 2\sqrt{17}$
$= 34$

b) START WITH THE GRADIENT OF CD

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-8}{4-3} = \frac{-4}{1} = -4$$

EQUATION OF l IS GIVEN BY

$y - y_0 = m(x - x_0)$
$y - 4 = -4(x-4)$ (USED D(4,4))

IYGB - MP1 PAPER P - QUESTION 10

WHEN $y=0$ THE EQUATION REDUCES TO

$$0-4=-4(x-4)$$
$$1=x-4$$
$$x=5$$

$\therefore E(5,0)$

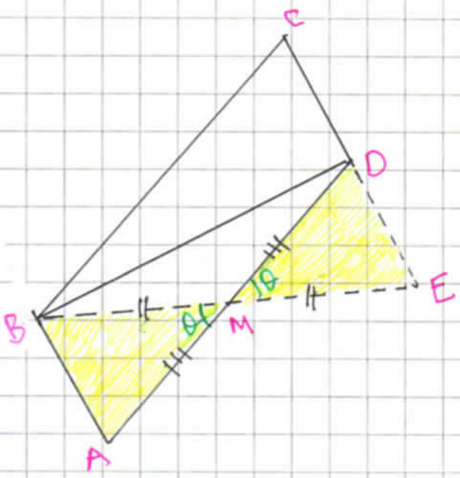
c) CONSIDERING MIDPOINTS

MIDPOINT OF $AD = \left(\frac{-3+4}{2}, \frac{-2+4}{2}\right) = \left(\frac{1}{2}, 1\right)$

MIDPOINT OF $BE = \left(\frac{-4+5}{2}, \frac{2+0}{2}\right) = \left(\frac{1}{2}, 1\right)$

INDEED THEY BISECT EACH OTHER

d) LOOKING AT A DETAILED DIAGRAM



$|AM| = |MD|$ (part c)

$|BM| = |ME|$ (part c)

$B\hat{M}A = D\hat{M}E$ (vertically opposite)

$\therefore \overset{\Delta}{ABM}$ IS CONGRUENT TO $\overset{\Delta}{DME}$

$\therefore$ AREA OF $\overset{\Delta}{EBC}$ = AREA OF $\overset{\square}{ABCD}$

REMOVE $\overset{\Delta}{MDE}$ FROM $\overset{\Delta}{EBC}$ & PLACE IN THE POSITION OF $\overset{\Delta}{ABM}$, TO FORM PARALLELOGRAM ABCD

IYGB - MP1 PAPER P - QUESTION 11

a) $\boxed{\text{LET } y = f(x) = 8^x}$

I) THEN

$$\left(\frac{1}{8}\right)^x = \left(8^{-1}\right)^x = 8^{-x} = f(-x)$$

$\therefore$ REFLECTION ABOUT THE y AXIS

II) THEN

$$2^x = \left(8^{\frac{1}{3}}\right)^x = 8^{\frac{1}{3}x} = f\left(\tfrac{1}{3}x\right)$$

$\therefore$ HORIZONTAL STRETCH, BY SCALE FACTOR 3

(OR ENLARGEMENT PARALLEL TO THE x AXIS BY SCALE FACTOR 3)

b) PROCEED AS FOLLOWS

$$8^{x-1} = 8^x \times 8^{-1} = \frac{1}{8}\left(8^x\right) = \frac{1}{8}f(x)$$

$\therefore$ VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

(OR ENLARGEMENT PARALLEL TO THE y AXIS BY SCALE FACTOR $\frac{1}{8}$)