IYGB – MP1 – PAPER N – QUESTION 1

AS THE CURVE IS GIVEN IN FACTORIZED FORM, WE HAVE THE INTEGRATION LIMITS, BY INSPECTION

$$\text{AREA} = \int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{3} (3-x)(x+1)\,dx$$

$$= \int_{-1}^{3} 3x + 3 - x^2 - x \,dx$$

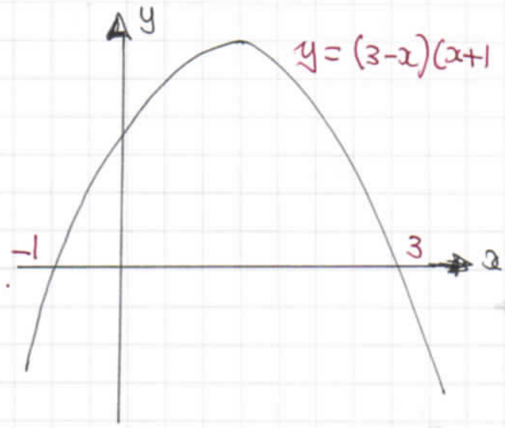
$$= \int_{-1}^{3} -x^2 + 2x + 3 \,dx$$

$$= \left[-\tfrac{1}{3}x^3 + x^2 + 3x \right]_{-1}^{3}$$

$$= (\cancel{-9} + \cancel{9} + 9) - (+\tfrac{1}{3} + 1 - 3)$$

$$= 9 - \left(-\tfrac{5}{3}\right)$$

$$= \frac{32}{3}$$



IYGB – MP1 – PAPER M – QUESTION 2

a) IF $f(x) = 2^{x}$ THEN $f(x-3) = 2^{(x-3)}$

HENCE THIS IS A TRANSLATION, IN THE POSITIVE x DIRECTION BY 3 UNITS

b) REWRITING AS FOLLOWS

$$y = 2^{x-3} = 2^{x} \times 2^{-3} = 2^{x} \times \frac{1}{2^{3}} = \frac{1}{8}(2^{x})$$

HENCE IF $f(x) = 2^{x}$ THEN $\frac{1}{8}f(x) = \frac{1}{8}(2^{x})$

THIS IS ALSO A VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

IYGB - MP1 - PAPER M - QUESTION 3

WE DO NOT ACTUALLY NEED THE EXPANSION AS WE ARE ONLY BEING ASKED FOR A SINGLE TERM - THUS WE HAVE

$$(2+3x)^9 = \dots + \binom{9}{5}(2)^4(3x)^5 + \dots$$

OR

$$\binom{9}{4}(2)^4(3x)^5 \quad \text{SINCE } \binom{9}{4} = \binom{9}{5}$$

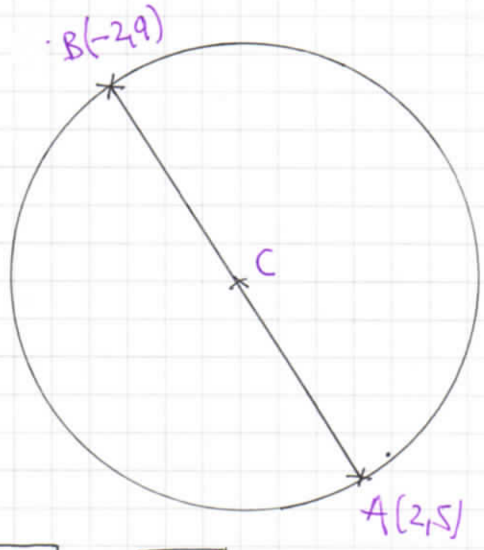
$$= \dots + 126 \times 16 \times 243x^5 + \dots$$

$$= \dots + 489888 + \dots$$

$$\text{i.e. } 489\,888$$

IYGB - MP1 - PAPER M - QUESTION 4

a) THE CENTRE OF THE CIRCLE MUST BE AT THE MIDPOINT OF AB



$$C = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+2}{2}, \frac{9+5}{2}\right)$$

$\therefore$ CENTRE IS AT $C(0,7)$

THE RADIUS WILL BE THE DISTANCE FROM $A(2,5)$ TO $C(0,7)$, OR INDEED FROM B TO C

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(0-2)^2+(7-5)^2} = \sqrt{4+4} = \sqrt{8}$$

$\therefore$ THE RADIUS IS $\sqrt{8}$

$\therefore$ THE REQUIRED EQUATION IS

$$(x-0)^2+(y-7)^2 = (\sqrt{8})^2$$
$$x^2+(y-7)^2 = 8$$

b) FIND THE DISTANCE FROM $P(1,5)$ TO THE CENTRE OF THE CIRCLE AT $C(0,7)$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(0-1)^2+(5-7)^2} = \sqrt{1+4} = \sqrt{5}$$

AS $\sqrt{5} < \sqrt{8}$ THE POINT IS INSIDE

IYGB - MP1 - PAPER M - QUESTION 5

$\cos(2y-35) = 0.891 \qquad 0 \leqslant y < 360$

$\arccos(0.891) = 27.000823\ldots \simeq 27.0°$

$\Longrightarrow \begin{cases} 2y-35 = 27.0° \pm 360n \\ 2y-35 = 333.0° \pm 360n \end{cases} \qquad n = 0,1,2,3,\ldots$

$360 - 27$

$\Longrightarrow \begin{cases} 2y = 62 \pm 360n \\ 2y = 368. \pm 360n \end{cases}$

$\Longrightarrow \begin{cases} y = 31 \pm 180n \\ y = 184 \pm 180n \end{cases}$

LOOKING AT THE REQUIRED RANGE

$y_1 = 31°$

$y_2 = 211°$

$y_3 = 184°$

$y_4 = 4°$

IYGB - MP1 - PAPER M - QUESTION 6

$$(x^2-x-3)^2-12(x^2-x-3)+27=0$$

- A SENSIBLE SUBSTITUTION WILL REDUCE THIS INTO A SIMPLE QUADRATIC

LET $y=x^2-x-3$

$$\Rightarrow y^2-12y+27=0$$
$$\Rightarrow (y-9)(y-3)=0$$
$$\Rightarrow y=\begin{cases}9\\3\end{cases}$$
$$\Rightarrow x^2-x-3=\begin{cases}9\\3\end{cases}$$

- SOLVING EACH QUADRATIC SEPARATELY WE OBTAIN

$$\Rightarrow x^2-x-3=9 \qquad\qquad \Rightarrow x^2-x-3=3$$
$$\Rightarrow x^2-x-12=0 \qquad\qquad \Rightarrow x^2-x-6=0$$
$$\Rightarrow (x-4)(x+3)=0 \qquad\qquad \Rightarrow (x+2)(x-3)=0$$
$$\Rightarrow x=\begin{cases}4\\-3\end{cases} \qquad\qquad \Rightarrow x=\begin{cases}3\\-2\end{cases}$$

- HENCE THERE ARE 4 REAL SOLUTIONS

$$x=-3,-2,3,4$$

IYGB - MP1 - PAPER M - QUESTION 7

$$f(x) = x^2 - 3x + 7, \quad x \in \mathbb{R}$$

- FIRST OBTAIN A SIMPLIFIED EXPRESSION FOR $f(x+h)$

$$f(x+h) = (x+h)^2 - 3(x+h) + 7$$
$$= x^2 + 2xh + h^2 - 3x - 3h + 7$$

- USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[\frac{(\cancel{x^2} + 2xh + h^2 \cancel{- 3x} - 3h \cancel{+ 7}) - (\cancel{x^2} \cancel{- 3x} \cancel{+ 7})}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[\frac{2xh + h^2 - 3h}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[2x + \cancel{h} - 3\right]$$

$$f'(x) = 2x - 3$$

AS REQUIRED

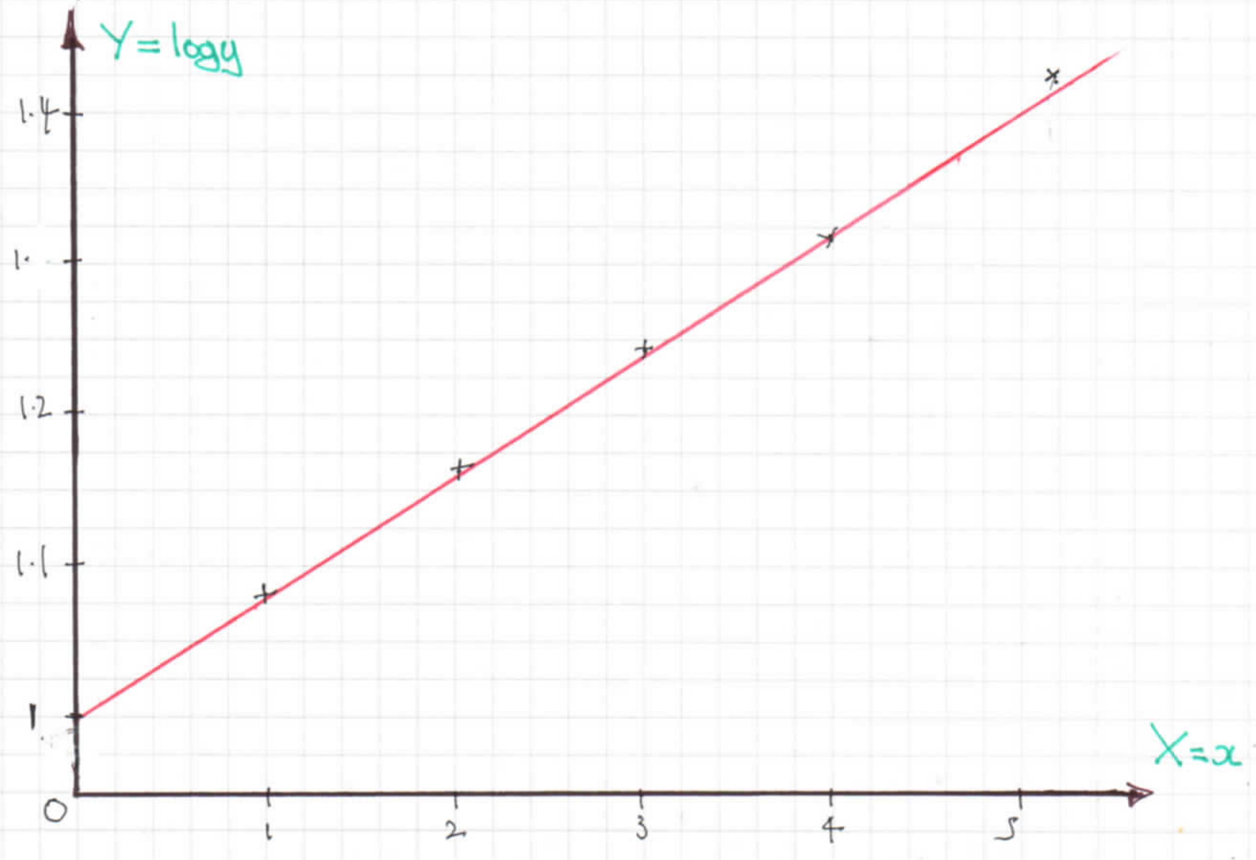
IYGB - MP1 - PAPER M - QUESTION 8

a) START BY TAKING LOGS (BASE 10) TO BOTH SIDES OF THE EQUATION

$$\Longrightarrow \quad y = ab^x$$
$$\Longrightarrow \quad \log_{10} y = \log_{10}(ab^x)$$
$$\Longrightarrow \quad \log_{10} y = \log_{10} a + \log_{10} b^x$$
$$\Longrightarrow \quad \log_{10} y = \log_{10} a + x\log_{10} b$$
$$\Longrightarrow \quad \log_{10} y = (\log_{10} b)x + (\log_{10} a)$$
$$Y = mX + c$$

b) WE REQUIRE TO PLOT $Y = \log_{10} y$ AGAINST $X = x$

$X = x$	1	2	3	4	5
$Y = \log_{10} y$	1.08	1.16	1.24	1.32	1.43



IYGB - MP1 - PAPER M - QUESTION 8

AS WE HAVE OBTAINED A STRAIGHT LINE THE ASSUMPTION THAT THE LAW IS THIS FORM IS SUPPORTED

c) THE y INTERCEPT OF THE LINE IS APPROX 0.99

$$c = \log_{10} a$$

$$0.99 = \log_{10} a$$

$$10^{0.99} = a$$

$$a \simeq 9.8$$

(2 sf)

THE GRADIENT OF THE LINE IS APPROXIMATELY

$$\frac{1.43 - 0.99}{5 - 0} = 0.088$$

$$m = \log_{10} b$$

$$0.088 = \log_{10} b$$

$$b = 10^{0.088}$$

$$b \simeq 1.2$$

2 s.f

d) THE FORMULA NOW READS (APPROXIMATELY)

$$y \simeq 9.8 \times 1.2^{x}$$

WHEN $x = 2.5$

$$y = 9.8 \times 1.2^{2.5}$$

$$y \simeq 15$$

(2 sf)

IYGB - MP1 - PAPER M - QUESTION 9

SOLVING BY SUBSTITUTION ② INTO ①

① $2x + 2y - z = 2$
② $z = x^2 + y^2$

$$\Rightarrow 2x + 2y - (x^2 + y^2) = 2$$

$$\Rightarrow 2x + 2y - x^2 - y^2 = 2$$

$$\Rightarrow 0 = x^2 - 2x + y^2 - 2y + 2$$

$$\Rightarrow 0 = (x-1)^2 - 1 + (y-1)^2 - 1 + 2$$

$$\Rightarrow 0 = (x-1)^2 + (y-1)^2$$

ONLY SOLUTION IS $x = 1$ & $y = 1$

AND USING $z = x^2 + y^2$, $z = 2$

$$\therefore (x, y, z) = (1, 1, 2)$$

IYGB, MP1, PAPER M – QUESTION 10

$T = 22 + 50e^{-\frac{1}{8}t}$, $t > 0$

T = TEMPERATURE OF DRINK

t = TIME (IN MINUTES)

a) WHEN $t=0$ $\Rightarrow T = 22 + 50e^{0}$

$\Rightarrow T = 22 + 50$

$\Rightarrow T = 72\ °C$

b) I) WHEN $T = 40$

$\Rightarrow 40 = 22 + 50e^{-\frac{1}{8}t}$

$\Rightarrow 18 = 50e^{-\frac{1}{8}t}$

$\Rightarrow \frac{9}{25} = e^{-\frac{1}{8}t}$

$\Rightarrow \frac{25}{9} = e^{+\frac{1}{8}t}$

$\Rightarrow \frac{1}{8}t = \ln\frac{25}{9}$

$\Rightarrow t = 8\ln\frac{25}{9}$

$\Rightarrow t = 8.1732\ldots$

$\Rightarrow t \simeq 8$ minutes

II) DIFFERENTIATE FIRST

$\Rightarrow T = 22 + 50e^{-\frac{1}{8}t}$

$\Rightarrow \frac{dT}{dt} = -\frac{25}{4}e^{-\frac{1}{8}t}$

WE REQUIRE $\frac{dT}{dt} = -2.5$

$\Rightarrow -2.5 = -\frac{25}{4}e^{-\frac{1}{8}t}$

$\Rightarrow \frac{2}{5} = e^{-\frac{1}{8}t}$

WE DO NOT ACTUALLY NEED TO EVALUATE THIS FULLY AS THIS "LUMP" APPEARS IN THE FORMULA — THUS WE HAVE

$\Rightarrow T = 22 + 50\left(\frac{2}{5}\right)$

$\Rightarrow T = 42\ °C$

IYGB - MP1 - PAPER M - QUESTION 11

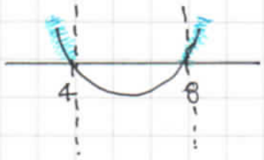
$$x^2 + (m+2)x + 4m - 7 = 0 \qquad x \in \mathbb{R}$$

FOR DISTINCT REAL ROOTS $b^2 - 4ac > 0$

$a = 1$
$b = (m+2)$
$c = (4m-7$

$$\Rightarrow (m+2)^2 - 4 \times 1 \times (4m-7) > 0$$
$$\Rightarrow m^2 + 4m + 4 - 16m + 28 > 0$$
$$\Rightarrow m^2 - 12m + 32 > 0$$
$$\Rightarrow (m-4)(m-8) > 0$$

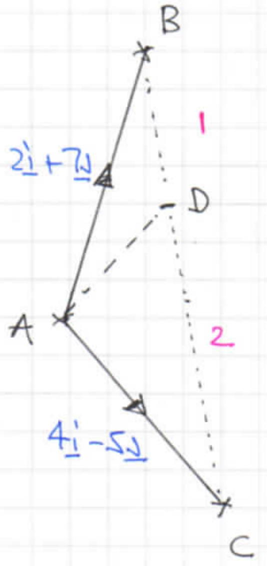
CRITICAL VALUES $<$ 4, 8



$m < 4$ OR $m > 8$

IYGB – MP1 – PAPER M – QUESTION 12

a) START WITH A DIAGRAM (NOT TO SCALE)



$\overrightarrow{AB} = 2\underline{i} + 7\underline{j}$

$\overrightarrow{AC} = 4\underline{i} - 5\underline{j}$

IF $|BD| : |DC| = 1 : 2$, THEN

$\Rightarrow \overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC}$

$\Rightarrow \overrightarrow{BD} = \frac{1}{3}\left(-\overrightarrow{BA} + \overrightarrow{AC}\right)$

$\Rightarrow \overrightarrow{BD} = \frac{1}{3}\left(-2\underline{i} - 7\underline{j} + 4\underline{i} - 5\underline{j}\right)$

$\Rightarrow \overrightarrow{BD} = \frac{2}{3}\underline{i} - 4\underline{j}$

b) HENCE WE HAVE, LOOKING AT THE DIAGRAM

$\Rightarrow \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$\Rightarrow \overrightarrow{AD} = (2\underline{i} + 7\underline{j}) + \left(\frac{2}{3}\underline{i} - 4\underline{j}\right)$

$\Rightarrow \overrightarrow{AD} = \frac{8}{3}\underline{i} + 3\underline{j}$

$\Rightarrow |\overrightarrow{AD}| = \sqrt{\left(\frac{8}{3}\right)^2 + 3^2} = \sqrt{\frac{64}{9} + 9} = \sqrt{\frac{145}{9}} = 4.0138\ldots$

i.e. APPROX 4

IYGB - MP1 - PAPER M - QUESTION 13

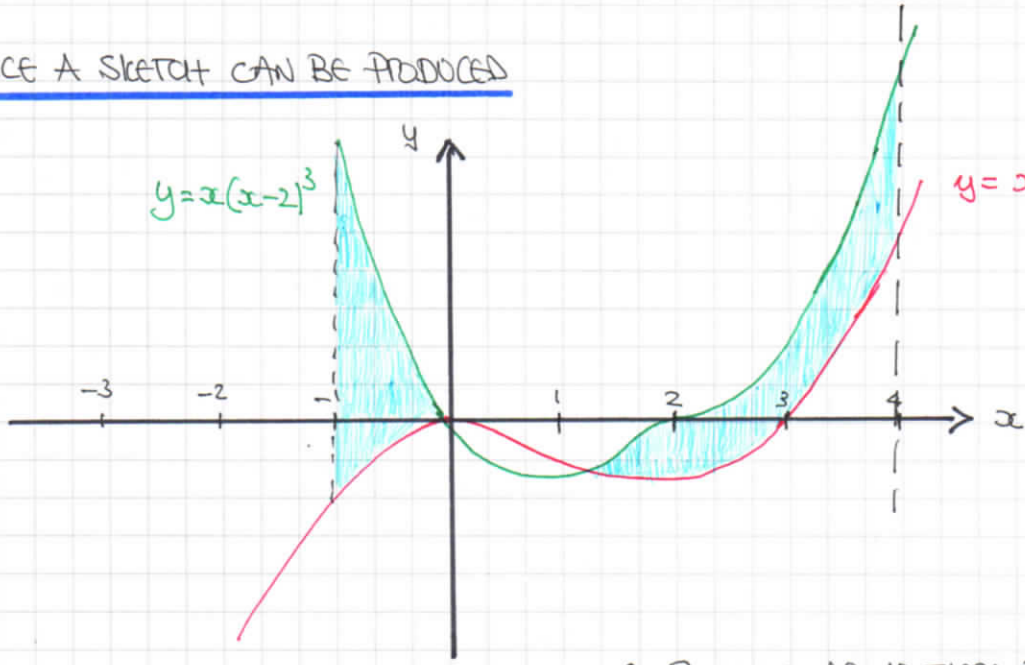
a) START WITH THE SKETCH, OF EACH OF THE CURVE

$y = x^2(x-3)$ $\qquad$ $y = x(x-2)^3$

(0,0) TOUCHES $\qquad$ (0,0) CROSSES

(3,0) CROSSES $\qquad$ (2,0) STATIONARY POINT OF INFLEXION

HENCE A SKETCH CAN BE PRODUCED



b) ∴ 2 SOLUTIONS AS THERE 2 INTERSECTIONS

$$\begin{pmatrix} x^3 - 3x^2 = x(x-2)^2 \\ x^2(x-3) = x(x-2) \end{pmatrix}$$

c) $y \geqslant x^3 - 3x^2$ IS THE REGION "ABOVE" THE CUBIC IN RED

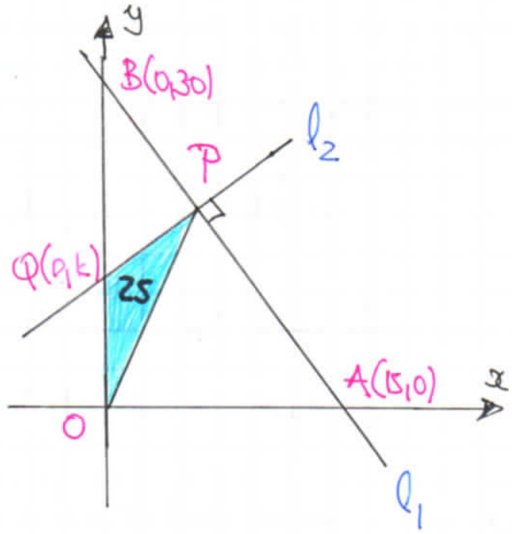
$y \leq x(x-2)^3$ IS THE REGION "BELOW" THE QUARTIC IN GREEN

COMBINING WITH $-1 < x < 4$ WE OBTAIN THE REGION ABOVE

(SHADED IN BLUE)

IYGB - MP1 - PAPER M - QUESTION 14

a) START BY FINDING THE GRADIENT OF l_1



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-30}{15-0} = -2$$

EQUATION OF l_1 USING $(0,30)$ IS

$$y = 30 - 2x$$

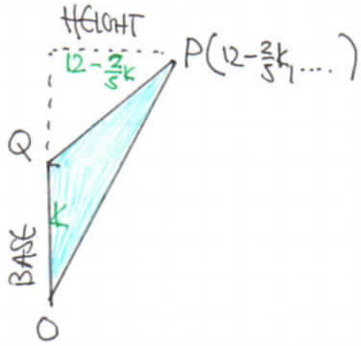
b) EQUATION OF l_2, WITH GRADIENT $+\frac{1}{2}$ PASSING THROUGH $Q(0,k)$

$$y = \frac{1}{2}x + k$$

SOLVING SIMULTANEOUSLY WITH l_1 BY SUBSTITUTION

$$\Rightarrow 30 - 2x = \frac{1}{2}x + k$$
$$\Rightarrow 60 - 4x = x + 2k$$
$$\Rightarrow 60 - 2k = 5x$$
$$\Rightarrow x = 12 - \frac{2}{5}k$$

c) AREA OF $O\hat{Q}P = 25$



$$\Rightarrow \frac{1}{2} \times k \times \left(12 - \frac{2}{5}k\right) = 25$$
$$\Rightarrow k\left(12 - \frac{2}{5}k\right) = 50$$
$$\Rightarrow 12k - \frac{2}{5}k^2 = 50$$
$$\Rightarrow 6k - \frac{1}{5}k^2 = 25$$
$$\Rightarrow 30k - k^2 = 125$$
$$\Rightarrow 0 = k^2 - 30k + 125$$
$$\Rightarrow (k-5)(k-25) = 0$$

$$\therefore k = \begin{cases} 5 \\ 25 \end{cases}$$

IYGB - MP1 - PAPER M - QUESTION 14

NOW IF $k=5$

$y=\frac{1}{2}x+5$ & THE x COORDINATE OF P IS $12-\frac{2}{5}\times5=10$

$\therefore P(10,10)$

$\therefore$ AREA OF $\overset{\triangle}{OPA}=\frac{1}{2}\times15\times10=75$

$\therefore$ AREA OF $OQPA=75+25=100$

AND IF $k=25$

$y=\frac{1}{2}x+25$ & THE x CO-ORDINATE OF P IS $12-\frac{2}{5}\times25=2$

$\therefore P(2,26)$

$\therefore$ AREA OF $\overset{\triangle}{OPA}=\frac{1}{2}\times15\times26=195$

$\therefore$ AREA OF $OQPA=195+25=220$

IYGB - MP1 - PAPER M - QUESTION 15

a) START BE REWRITING THE EQUATION IN INDICIAL FORM, THEN DIFFERENTIATE

$$y = \frac{x^3(5x\sqrt{x} - 128)}{\sqrt{x}} = \frac{5x^4\sqrt{x} - 128x^3}{\sqrt{x}} = \frac{5x^4\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} - \frac{128x^3}{\sqrt{x}}$$

- $y = 5x^4 - 128x^{\frac{5}{2}}$
- $\frac{dy}{dx} = 20x^3 - 320x^{\frac{3}{2}}$
- $\frac{d^2y}{dx^2} = 60x^2 - 480x^{\frac{1}{2}}$
- $\frac{d^3y}{dx^3} = 120x - 240x^{-\frac{1}{2}}$

b) FOR STATIONARY POINTS $\frac{dy}{dx} = 0$

$$\Rightarrow 20x^3 - 320x^{\frac{3}{2}} = 0$$
$$\Rightarrow x^3 - 16x^{\frac{3}{2}} = 0$$
$$\Rightarrow x^3 = 16x^{\frac{3}{2}}$$
$(x \neq 0)$
$$\Rightarrow \frac{x^3}{x^{3/2}} = 16$$
$$\Rightarrow x^{\frac{3}{2}} = 16.$$
$$\Rightarrow \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = 16^{\frac{2}{3}}$$
$$\Rightarrow x^1 = \left(2^4\right)^{\frac{2}{3}}$$
$$\Rightarrow x = 2^{\frac{8}{3}}$$

SUBSTITUTE INTO y & TIDY

$$\Rightarrow y = 5x^4 - 128x^{\frac{5}{2}}$$
$$\Rightarrow y = x^{\frac{5}{2}}\left[5x^{\frac{3}{2}} - 128\right]$$
$$\Rightarrow y = \left(2^{\frac{8}{3}}\right)^{\frac{5}{2}}\left[5 \times 16 - 128\right]$$
$$\Rightarrow y = 2^{\frac{20}{3}}\left[80 - 128\right]$$
$$\Rightarrow y = 2^{6\frac{2}{3}}(-48)$$
$$\Rightarrow y = 2^6 \times 2^{\frac{2}{3}} \times (-48)$$
$$\Rightarrow y = -48 \times 64 \times \left(2^2\right)^{\frac{1}{3}}$$
$$\Rightarrow y = -3072 \times \sqrt[3]{4}$$

IYGB - MP1 - PAPER M - QUESTION 15

c) $\frac{d^2y}{dx^2} = 60x^2 - 480x^{\frac{1}{2}}$

$\Rightarrow \frac{d^2y}{dx^2} = 60x^{\frac{1}{2}}\left(x^{\frac{3}{2}} - 8\right)$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x^{\frac{3}{2}}=16} = 60\left(2^{\frac{8}{3}}\right)^{\frac{1}{2}}(16-8) = 60 \times 2^{\frac{4}{3}} \times 8 = 60 \times 2^1 \times 2^{\frac{1}{3}} \times 8$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x^{\frac{3}{2}}=16} = 960\sqrt[3]{2}$

d) FIRSTLY $\frac{d^2y}{dx^2} = 0$

$\Rightarrow 60x^2 - 480x^{\frac{1}{2}} = 0$

$\Rightarrow x^2 - 8x^{\frac{1}{2}} = 0$

$\Rightarrow x^2 = 8x^{\frac{1}{2}}$

$\Rightarrow \frac{x^2}{x^{\frac{1}{2}}} = 8$ $\quad x \neq 0$

$\Rightarrow x^{\frac{3}{2}} = 8$

$\Rightarrow \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$

$\Rightarrow x^1 = \left(\sqrt[3]{8}\right)^2$

$\Rightarrow x = 4$

FINALLY

$\Rightarrow \frac{d^3y}{dx^3} = 120x - 240x^{-\frac{1}{2}}$

$\Rightarrow \left.\frac{d^3y}{dx^3}\right|_{x=4} = 120 \times 4 - 240 \times 4^{-\frac{1}{2}}$

$= 480 - 240 \times \frac{1}{2}$

$= 480 - 120 = 360$