IYGB - MMS PAPER U - QUESTION 1

- TRYING TO FORM SOME EQUATIONS FROM THE INFORMATION GIVEN

$$P(A) = 0.4 \qquad P(A|B) = 0.6 \qquad P(A \cup B) = 2P(A \cap B)$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$\Rightarrow 2P(A \cap B) = 0.4 + P(B) - P(A \cap B)$$

$$\Rightarrow 3P(A \cap B) = 0.4 + P(B) \quad - \text{I}$$

- AND ALSO WE HAVE

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$\Rightarrow 0.6 = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = 0.6\,P(B) \quad - \text{II}$$

- COMBINING EQUATIONS (I) & (II) WE OBTAIN

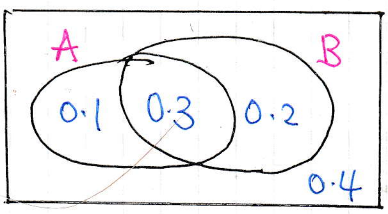
$$\Rightarrow 3[0.6\,P(B)] = 0.4 + P(B)$$

$$\Rightarrow 1.8\,P(B) = 0.4 + P(B)$$

$$\Rightarrow 0.8\,P(B) = 0.4$$

$$\Rightarrow P(B) = 0.5 \quad \& \quad P(A \cap B) = 0.3$$

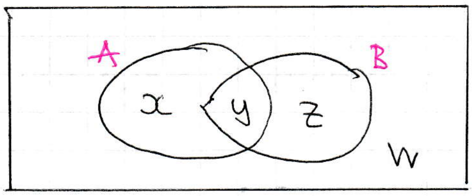
- FINALLY COMPLETING A VENN DIAGRAM



$$P(B|A') = \frac{P(B \cap A')}{P(A')} = \frac{0.2}{0.6} = \frac{1}{3}$$

IYGB - MMS PAPER U - QUESTION 1

ALTERNATIVE / VARIATION



$P(A) = 0.4 \implies x + y = 0.4$

$P(A|B) = 0.6 \implies \dfrac{y}{y+z} = 0.6$

$P(A \cup B) = 2P(A \cap B) \implies x + y + z = 2y$

$x + y + z + w = 1$

TIDY EQUATIONS

$$\left.\begin{array}{ll} ① & x+y = 0.4 \\ ② & y = 0.6(y+z) \\ ③ & x+y+z = 2y \\ ④ & x+y+z+w = 1 \end{array}\right\} \implies \begin{array}{ll} ① & x+y = 0.4 \\ ② & 0.4y = 0.6z \\ ③ & x-y+z = 0 \\ ④ & x+y+z+w = 1 \end{array}$$

$$\longrightarrow \boxed{y = \tfrac{3}{2}z}$$

SUB $y = \frac{3}{2}z$ INTO THE OTHER 3 EQUATIONS

$$\left.\begin{array}{ll} ① & x + \frac{3}{2}z = 0.4 \\ ③ & x - \frac{3}{2}z + z = 0 \\ ④ & x + \frac{3}{2}z + z + w = 1 \end{array}\right\} \implies \begin{array}{ll} ① & x + \frac{3}{2}z = 0.4 \\ ② & x - \frac{1}{2}z = 0 \\ ③ & x + \frac{5}{2}z + w = 1 \end{array}$$

SUBTRACT ① and ②: $2z = 0.4$, $z = 0.2$

HENCE WE OBTAIN x, y & w

$x = \frac{1}{2}z = \frac{1}{2} \times 0.2 = 0.1$

$y = \frac{3}{2}z = \frac{3}{2} \times 0.2 = 0.3$

$w = 1 - x - \frac{5}{2}z = 1 - 0.1 - \frac{5}{2} \times 0.2 = 0.4$

FINALLY

$P(B|A') = \dfrac{P(B \cap A')}{P(A')} = \ldots$ looking at Venn diagram

$= \dfrac{z}{z+w} = \dfrac{0.2}{0.6} = \dfrac{1}{3}$

IYGB – MMS PAPER U – QUESTION 2

$$P(X=r+1) = \begin{cases} \frac{2}{3}P(X=r) & r = 1,2,3,4,5,\ldots \\ 0 & \end{cases}$$

USE A CONDITIONAL PROBABILITY TABLE

r	1	2	3	4	5	6	...
$P(X=r)$	k	$\frac{2}{3}k$	$\frac{4}{9}k$	$\frac{8}{27}k$	$\frac{16}{81}k$	$\frac{32}{243}k$	..

$$\sum P(X=r) = 1$$

$$\Rightarrow k\left(1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \ldots\right) = 1$$

THIS IS A CONVERGENT G.P. WITH $a=1$ & $r=\frac{2}{3}$ & $S_\infty = \frac{a}{1-r}$

$$\Rightarrow k\left(\frac{1}{1-\frac{2}{3}}\right) = 1$$

$$\Rightarrow k \times 3 = 1$$

$$\Rightarrow k = \frac{1}{3}$$

$$\therefore P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{2}{3}k + \frac{4}{9}k + \frac{8}{27}k$$

$$= \frac{2}{9} + \frac{4}{27} + \frac{8}{81}$$

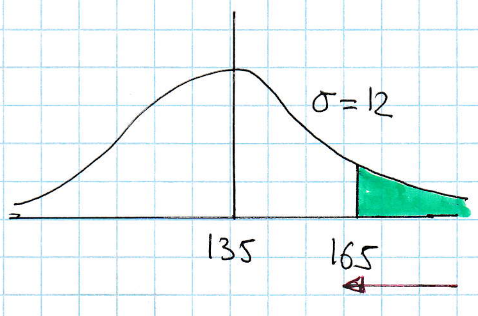
$$= \frac{38}{81}$$

IYGB – MMS PAPER U – QUESTION 3

a)

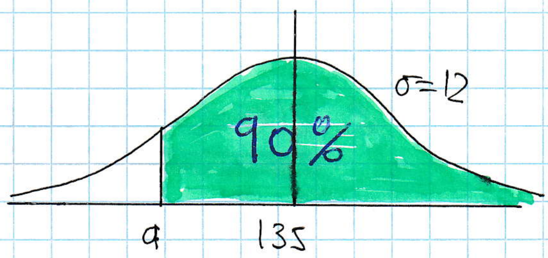
X = NUMBER OF MILES ON A FULL TANK

$X \sim N(135, 12^2)$



$$P(X>165) = 1 - P(X<165)$$
$$= 1 - P\left(Z < \frac{165-135}{12}\right)$$
$$= 1 - \Phi(2.5)$$
$$= 1 - 0.9938$$
$$= \underline{0.0062}$$

b) PUT INFORMATION INTO A DIAGRAM



$$\Rightarrow P(X>a) = 90\%$$
$$\Rightarrow P\left(Z > \frac{a-135}{12}\right) = 0.9$$

INVERTING

$$\Rightarrow \frac{a-135}{12} = -\Phi^{-1}(0.9)$$
$$\Rightarrow \frac{a-135}{12} = -1.2816$$
$$\Rightarrow a - 135 = -15.3792$$
$$\Rightarrow a = 119.62$$

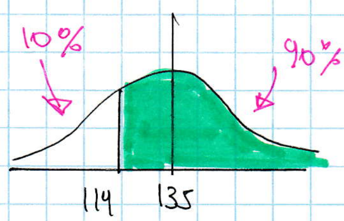
$\therefore$ 119 MILES !!

IYGB - MMS PAPER U - QUESTION 3

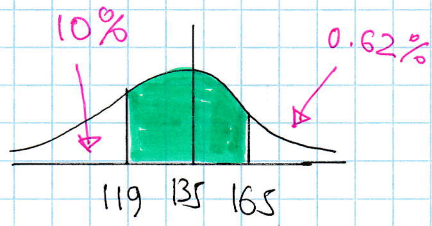
c) PUTTING INFORMATION FOR CONDITIONAL PROBABILITY INTO TWO SEPARATE DIAGRAMS

$P(X \quad 165 \mid X > 119)$

90%



90% (GIVEN)



89.38%

$\therefore$ REQUIRED PROBABILITY $= \dfrac{0.8938}{0.9} = 0.9931$

IYGB – MMS PAPER U – QUESTION 4

a) $\sum_{r=1}^{20}(x_r-10) = 220 \qquad \sum_{r=1}^{20}(x_r-10)^2 = 2720 \qquad n=20$

$$\sum_{r=1}^{20}(x_r-10)^2 = \sum_{r=1}^{20}\left[x_r^2 - 20x_r + 100\right]$$

$$2720 = \sum_{r=1}^{20} x_r^2 - 20\sum_{r=1}^{20} x_r + 100\sum_{r=1}^{20} 1$$

$$2720 = \sum_{r=1}^{20} x_r^2 - 20\sum_{r=1}^{20} x_r + 100\times 20$$

BY INSPECTION $\sum_{r=1}^{20} x = 220 + 20\times 10 = 420$ OR BY USING A DETAILED METHOD

$$\sum_{r=1}^{20}(x_r-10) = 220$$

$$\sum_{r=1}^{20} x_r - 10\sum_{r=1}^{20} 1 = 220$$

$$\sum_{r=1}^{20} x_r - 10\times 20 = 220$$

$$\sum_{r=1}^{20} x_r = 420$$

RETURNING TO THE MAIN LINE

$$\Rightarrow 2720 = \sum_{r=1}^{20} x_r^2 - 20\times 420 + 2000$$

$$\Rightarrow \sum_{r=1}^{20} x_r^2 = 9120$$

IYGB - MMS PAPER U - QUESTION 4

b) • $\bar{x} = \frac{\sum x}{n} = \frac{420}{20} = 21$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{9120}{20} - 21^2} = \sqrt{15}$

≈ 3.87

ALTERNATIVE USING THE CODED VALUES

$y = x - 10$ so $\sum y = 220$ & $\sum y^2 = 2720$

$\bar{y} = \frac{\sum y}{n} = \frac{220}{20} = 11$

$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = \sqrt{\frac{2720}{20} - 11^2} = \sqrt{15}$

UNCODING

$\bar{x} = \bar{y} + 10 = 21$

$\sigma_x = \sigma_y = \sqrt{15}$ (UNAFFECTED BY SUBTRACTION)

IYGB – MMS PAPER U – QUESTION 5

$P(A) = 0.4$ $\quad P(A \cup B) = 0.58$ $\quad P(C) = 0.4$ $\quad P(B' \cap C') = 0.4$

A & B ARE INDEPENDENT $\qquad$ A & C ARE MUTUALLY EXCLUSIVE

STARTING FROM "A & B ARE INDEPENDENT"

$\Longrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

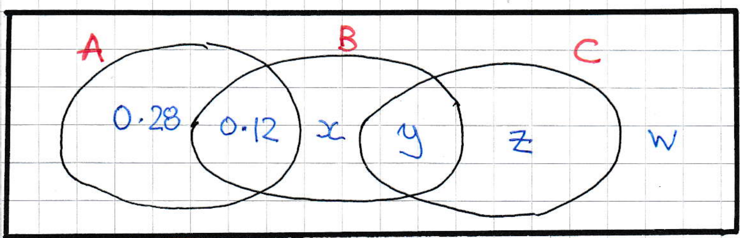
$\Longrightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B) \leftarrow$ INDEPENDENCE

$\Longrightarrow 0.58 = 0.4 + P(B) - 0.4P(B)$

$\Longrightarrow 0.18 = 0.6P(B)$

$\Longrightarrow P(B) = 0.3$

PUTTING THE KNOWN INFORMATION INTO A VENN DIAGRAM, NOTING THAT A & C ARE MUTUALLY EXCLUSIVE & $P(A \cap B) = P(A)P(B) = 0.12$



- $P(C) = 0.5 \Rightarrow y + z = 0.4$ — I
- $P(B' \cap C') = 0.4 \Rightarrow 0.28 + w = 0.4$ — II
- $P(A \cup B) = 0.58 \Rightarrow z + w = 0.42$ — III
  $\Rightarrow x + y + 0.28 + 0.12 = 0.58$ — IV

FROM II : $w = 0.12$
FROM III : $z = 0.3$
FROM I : $y = 0.1$
FROM IV : $x = 0.08$

$\therefore P\big((B \cap C') \cup (B' \cap C' \cap A')\big)$

$= (x + 0.12) + (w)$

$= 0.32$

IYGB – MMS PAPER U – QUESTION 6

- START BE DEFINING VARIABLES & DISTRIBUTIONS

X = NO OF CUSTOMERS WHO BUY INSURANCE

$X \sim B(160, 0.35)$

- APPROXIMATE BY NORMAL

  - $E(X) = np = 160 \times 0.35 = 56$
  - $Var(X) = np(1-p) = 56 \times 0.65 = 36.4$

$\boxed{Y \sim N(56, 36.4)}$

- WE NOW HAVE

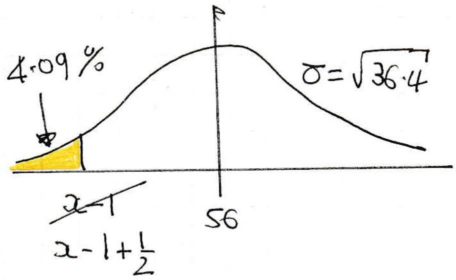
$\Rightarrow P(X < x) = 4.09\%$

$\Rightarrow P(X \leq x-1) = 0.0409$

$\Rightarrow P(Y < x - \tfrac{1}{2}) = 0.0409$

$\Rightarrow P(Y > x - \tfrac{1}{2}) = 0.9591$

$\Rightarrow P\left(Z > \dfrac{x - \frac{1}{2} - 56}{\sqrt{36.4}}\right) = 0.9591$



↓ INVERTING

$\Rightarrow \dfrac{x - 56.5}{\sqrt{36.4}} = -\Phi^{-1}(0.9591)$

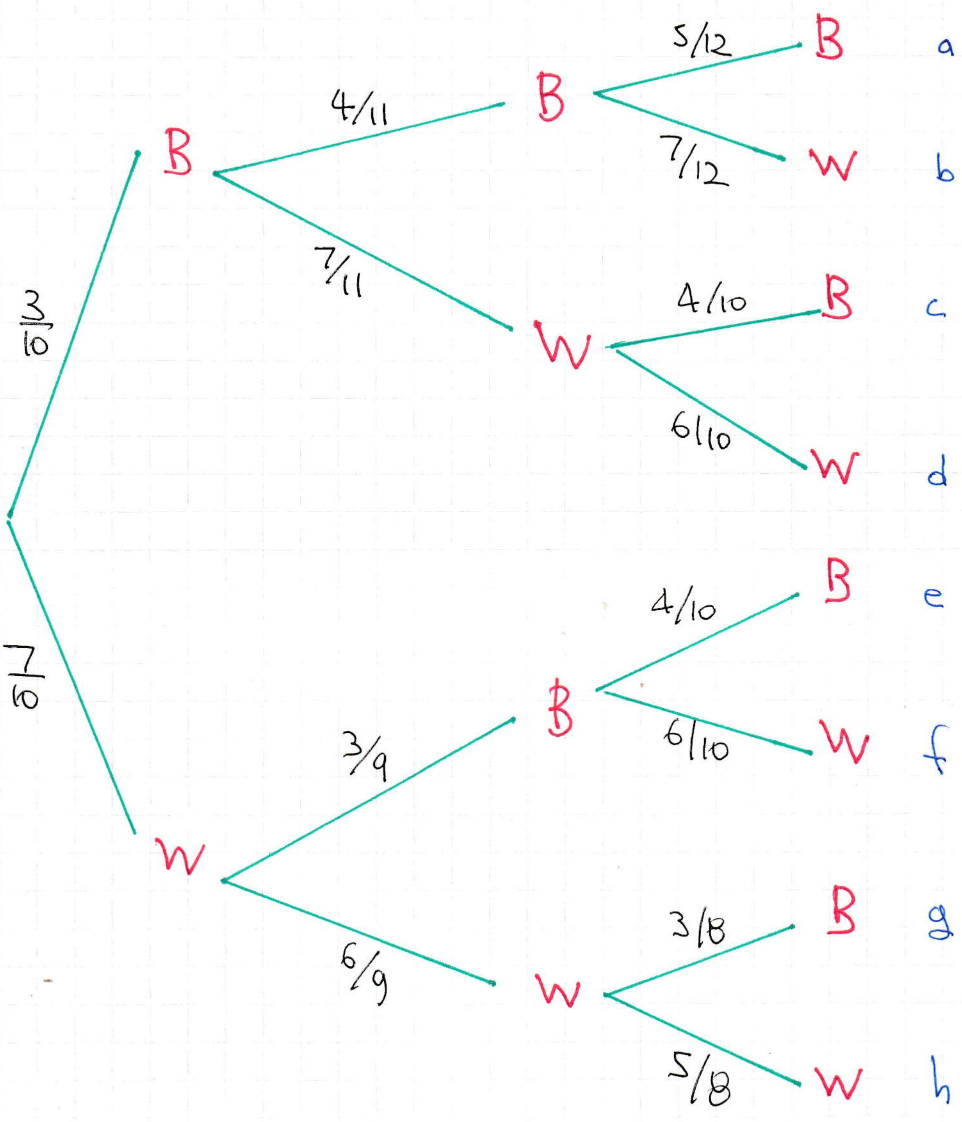
$\Rightarrow \dfrac{x - 56.5}{\sqrt{36.4}} = -1.74$

$\Rightarrow x = 46.0021\ldots$

$\therefore \underline{x = 46}$

IYGB – MMS PAPER U – QUESTION 7

- STARTING WITH A TREE DIAGRAM (VERY UNUSUAL)



- P(2 BLACKS | BOTH COLOURS WERE DRAWN) CAN BE FOUND AS

$$\frac{b+c+e}{b+c+d+e+f+g} =$$

$$= \frac{\left(\frac{3}{10}\times\frac{4}{11}\times\frac{7}{12}\right)+\left(\frac{3}{10}\times\frac{7}{11}\times\frac{4}{10}\right)+\left(\frac{7}{10}\times\frac{3}{9}\times\frac{4}{10}\right)}{\left(\frac{3}{10}\times\frac{4}{11}\times\frac{7}{12}\right)+\underbrace{\left(\frac{3}{10}\times\frac{7}{11}\times\frac{4}{10}\right)+\left(\frac{3}{10}\times\frac{7}{11}\times\frac{6}{10}\right)}+\underbrace{\left(\frac{7}{10}\times\frac{3}{9}\times\frac{4}{10}\right)+\left(\frac{7}{10}\times\frac{3}{9}\times\frac{6}{10}\right)}+\left(\frac{7}{10}\times\frac{6}{9}\times\frac{3}{8}\right)}$$

$$= \frac{\frac{7}{30}}{\frac{175}{264}} = \frac{44}{125} = 0.352$$

IYGB - MMS PAPER U - QUESTION 8

NO OF SCOOPS	1	2	3
PROBABILITY	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

PROCEED AS FOLLOWS

$\Rightarrow$ P(MORE THAN n SCOOPS OF ICE CREAM WERE ORDERED BY n CUSTOMERS) > 0.99

$\Rightarrow$ P(AT LEAST ONE CUSTOMER BOUGHT MORE THAN 1 SCOOP) > 0.99

$\Rightarrow$ 1 − P(ALL n CUSTOMERS BOUGHT 1 SCOOP) > 0.99

$$\Rightarrow 1-\left(\frac{1}{6}\right)^n > 0.9999$$

$$\Rightarrow -\left(\frac{1}{6}\right)^n > -0.0001$$

$$\Rightarrow \left(\frac{1}{6}\right)^n < 0.0001$$

$$\Rightarrow 6^n > 10000$$

$$\Rightarrow \log_{10}(6^n) > \log_{10}(10000)$$

$$\Rightarrow n\log_{10}6 > 4$$

$$\Rightarrow n > \frac{4}{\log_{10}6}$$

$$\Rightarrow n > 5.140388\ldots$$

$$\therefore\ n = 6$$

IYGB - MMS PAPER U - QUESTION 9

a) INTEGRATING BACKWARDS TO GET A VELOCITY EXPRESSION

$\Rightarrow \quad a = 4t - 9$

$\Rightarrow \quad v = \int 4t - 9 \, dt$

$\Rightarrow \quad v = 2t^2 - 9t + C$

APPLY $t = 1 \quad v = -3$

$\Rightarrow \quad -3 = 2 - 9 + C$

$\Rightarrow \quad 4 = C$

$\Rightarrow \quad v = 2t^2 - 9t + 4$

NOW v IS MINIMUM WHEN THE ACCELERATION IS ZERO, IE $\frac{dv}{dt} = 0$

$4t - 9 = 0$

$4t = 9$

$t = \frac{9}{4}$

(WE MAY ALSO COMPLETE THE SQUARE HERE)

$\therefore \; v\left(\frac{9}{4}\right) = 2\left(\frac{9}{4}\right)^2 - 9\left(\frac{9}{4}\right) + 4 = -\frac{49}{8}$

$\therefore$ MINIMUM VELOCITY IS -6.125 ms^{-1}

b) SOLVING $v = 0$ BY FACTORIZING

$2t^2 - 9t + 4 = 0$

$(2t - 1)(t - 4) = 0$

$t = \begin{cases} \frac{1}{2} \\ 4 \end{cases}$

IYGB - MMS PAPER U - QUESTION 9

c) OBTAIN AN EXPRESSION FOR THE DISPLACEMENT SUBJECT TO $t=0$ $x=0$ (ARBITRARY)

$$V = 2t^2 - 9t + 4$$

$$x = \int 2t^2 - 9t + 4 \, dt$$

$$x = \tfrac{2}{3}t^3 - \tfrac{9}{2}t^2 + 4t + D$$

$$t=0 \quad x=0 \implies D=0$$

$$x = \tfrac{2}{3}t^3 - \tfrac{9}{2}t^2 + 4t$$

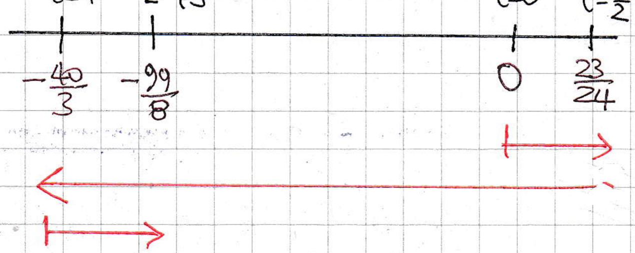
$$\boxed{x = \tfrac{1}{6}t\left[4t^2 - 27t + 24\right]}$$

$$x(\tfrac{1}{2}) = \frac{23}{24} = 0.958333\ldots$$

$$x(4) = -\frac{40}{3} = -13.333\ldots$$

$$x(4.5) = -\frac{99}{8} = -12.375$$

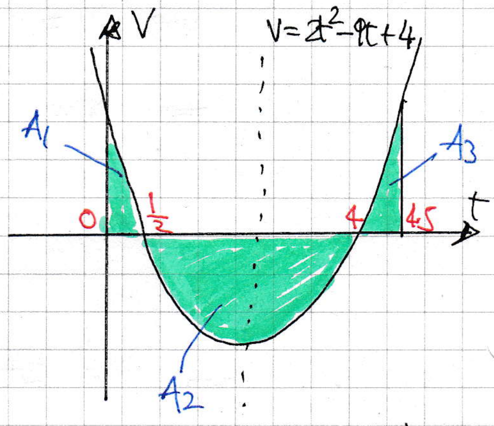
DRAWING A DIAGRAM



$$\therefore\ d = \frac{23}{24} + \left(\frac{23}{24} + \frac{40}{3}\right) + \left(\frac{40}{3} - \frac{99}{8}\right)$$

$$= \frac{389}{24} \approx 16.21 \text{ m}$$

ALTERNATIVE BY SPEED TIME GRAPH CONSIDERATIONS



• BY SYMMETRY $A_1 = A_3 = \int_0^{\frac{1}{2}} v \, dt$

$$A_1 + A_3 = 2\int_0^{\frac{1}{2}} 2t^2 - 9t + 4 \, dt$$

$$= 2\left[\tfrac{2}{3}t^3 - \tfrac{9}{2}t^2 + 4t\right]_0^{\frac{1}{2}}$$

$$= 2\left[\left(\tfrac{1}{12} - \tfrac{9}{8} + 2\right) - 0\right]$$

$$= \frac{23}{12}$$

• $A_2 = \left|\int_{\frac{1}{2}}^{4} 2t^2 - 9t + 4 \, dt\right|$

$$= \left|\left[\tfrac{2}{3}t^3 - \tfrac{9}{2}t^2 + 4t\right]_{\frac{1}{2}}^{4}\right|$$

$$= \left[\tfrac{2}{3}t^3 - \tfrac{9}{2}t^2 + 4t\right]_4^{\frac{1}{2}}$$

$$= \frac{23}{24} - \left(-\frac{40}{3}\right)$$

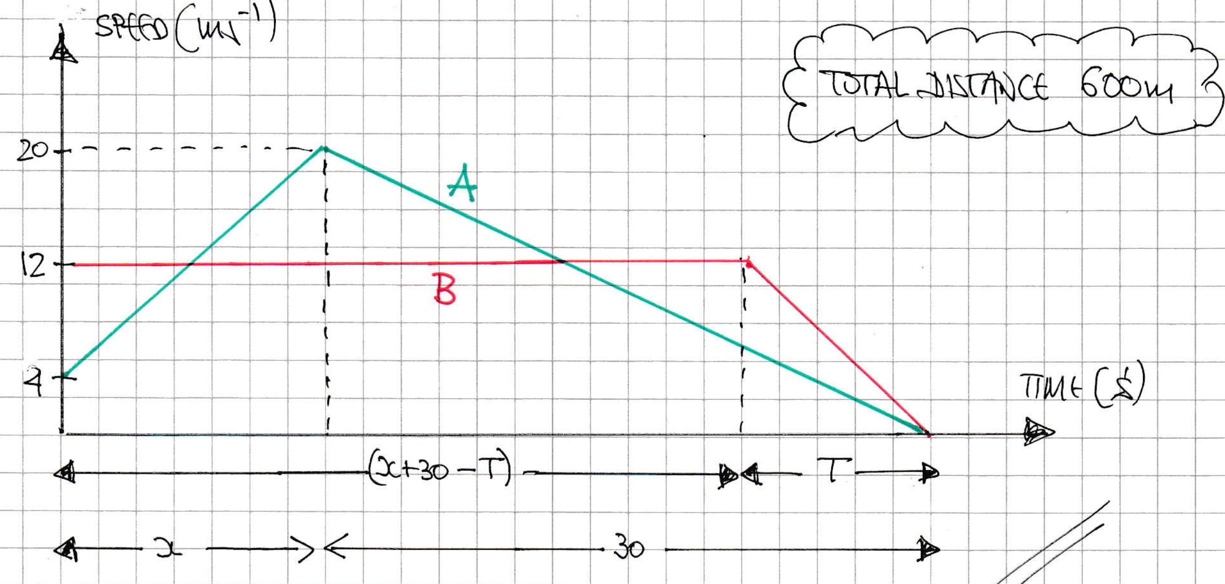
$$= \frac{343}{24}$$

$$\therefore\ d = \frac{23}{12} + \frac{343}{24} = \frac{389}{24}$$

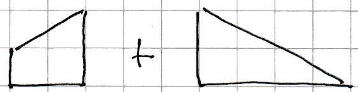
$$\approx 16.21 \text{ m}$$

IYGB - MMS PAPER U - QUESTION 10

a)



b) LOOKING AT THE MOTION OF A

 $= 600$

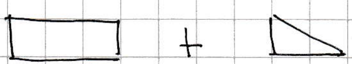
$$\left(\frac{4+20}{2}\right)x + \frac{1}{2}\times 20\times 30 = 600$$

$$12x + 300 = 600$$

$$12x = 300$$

$$x = 25$$

LOOKING AT THE MOTION OF B

 $= 600$

$$12(x+30-T) + \frac{1}{2}\times 12\times T = 600$$

$$12(55-T) + 6T = 600$$

$$660 - 12T + 6T = 600$$

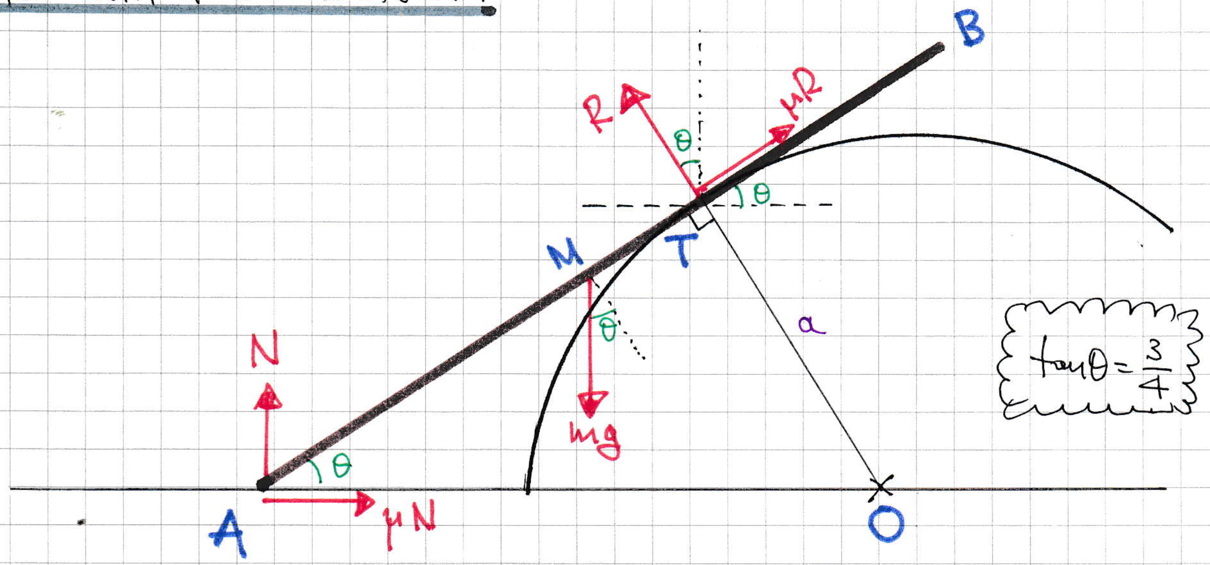
$$60 = 6T$$

$$T = 10$$

$$\therefore \text{DECELERATION OF B} = \frac{\Delta v}{\Delta t} = \frac{12}{10} = 1.2\ ms^{-2}$$

IYGB - MMS PAPER U - QUESTION 11

START WITH A GOOD DIAGRAM



FIRSTLY BY SIMPLE TRIGONOMETRY

$$\Rightarrow \frac{|OT|}{|AT|} = \tan\theta$$

$$\Rightarrow \frac{a}{|AT|} = \frac{3}{4}$$

$$\Rightarrow |AT| = \frac{4}{3}a$$

RESOLVING & TAKING MOMENTS YIELDS THE FOLLOWING EQUATIONS

$(\uparrow)$: $N + R\cos\theta + \mu R\sin\theta = mg$

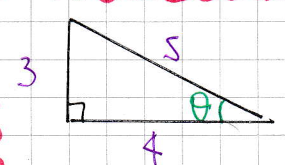
$(\rightarrow)$: $\mu N + \mu R\cos\theta = R\sin\theta$

$\overset{\curvearrowright}{A}$: $mg\cos\theta \times |AM| = R \times |AT|$

FROM THE MOMENT EQUATION WE OBTAIN

$$\Rightarrow mg\left(\frac{4}{5}\right) \times \cancel{a} = R \times \frac{4}{3}\cancel{a}$$

$$\Rightarrow R = \frac{3}{5}mg$$



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

IYGB - MMS PAPER U - QUESTION 11

USING $R = \frac{3}{5}mg$ INTO THE OTHER TWO EQUATIONS AND SIMPLIFYING

$$\Rightarrow \begin{cases} N + \left(\frac{3}{5}mg\right)\left(\frac{4}{5}\right) + \mu\left(\frac{3}{5}mg\right)\left(\frac{3}{5}\right) = mg \\ \mu N + \mu\left(\frac{3}{5}mg\right)\left(\frac{4}{5}\right) = \left(\frac{3}{5}mg\right)\left(\frac{3}{5}\right) \end{cases}$$

$$\Rightarrow \begin{cases} N + \frac{12}{25}mg + \frac{9}{25}\mu mg = mg \\ \mu N + \frac{12}{25}\mu mg = \frac{9}{25}mg \end{cases}$$

ELIMINATE N BETWEEN THE EQUATIONS

$$\Rightarrow \mu\left[\cancel{mg} - \frac{12}{25}\cancel{mg} - \frac{9}{25}\mu \cancel{mg}\right] + \frac{12}{25}\mu \cancel{mg} = \frac{9}{25}\cancel{mg}$$

$$\Rightarrow \mu\left[1 - \frac{12}{25} - \frac{9}{25}\mu\right] + \frac{12}{25}\mu = \frac{9}{25}$$

$$\Rightarrow \mu\left[25 - 12 - 9\mu\right] + 12\mu = 9$$

$$\Rightarrow 13\mu - 9\mu^2 + 12\mu = 9$$

$$\Rightarrow 9\mu^2 - 25\mu + 9 = 0$$

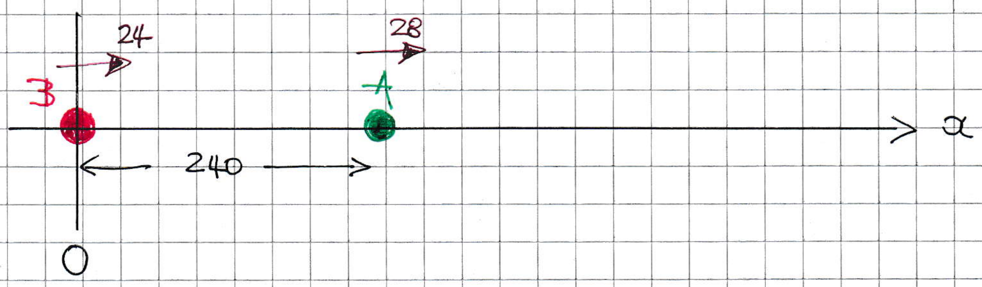
BY THE QUADRATIC FORMULA

$$\Rightarrow \mu = \frac{25 \pm \sqrt{301}}{18}$$

$$\Rightarrow \mu \approx \begin{cases} 0.425 \\ \cancel{2.35} \end{cases}$$

IYGB – MMS PAPER U – QUESTION 12

- LET THE TIME t BE MEASURED, FROM THE INSTANT WHEN B IS 240 METRES BEHIND A
- LET THE POSITION OF B, AT $t=0$ BE THE "ORIGIN"



USING $s = s_0 + ut + \frac{1}{2}at^2$, AT TIME t

$$\left.\begin{aligned} s_A &= 240 + 28t + \tfrac{1}{2}(0.1)t^2 \\ s_B &= 0 + 24t + \tfrac{1}{2}(0.2)t^2 \end{aligned}\right\} \Longrightarrow s_A = s_B$$

$$\Longrightarrow 240 + 28t + \frac{1}{20}t^2 = 24t + \frac{1}{10}t^2$$

$$\Longrightarrow 0 = \frac{1}{20}t^2 - 4t - 240$$

$$\Longrightarrow t^2 - 80t - 4800 = 0$$

$$\Longrightarrow (t+40)(t-120) = 0$$

$$\Longrightarrow t = \begin{cases} 120 \\ \cancel{-40} \end{cases}$$

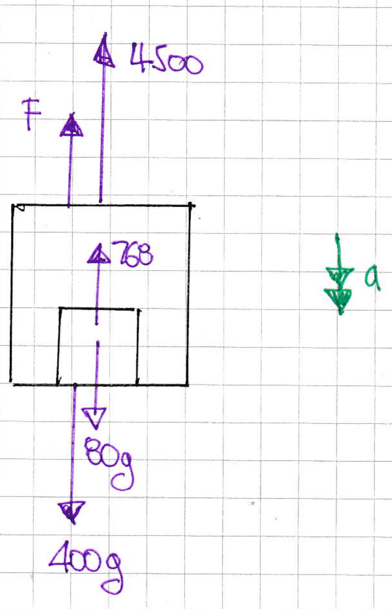
FINALLY USING $V = u + at$

$$\therefore V_B = 24 + 0.2 \times 120$$

$$V_B = 48 \text{ ms}^{-1}$$

IYGB – MMS PAPER U – QUESTION 13

STARTING WITH A DIAGRAM



LOOKING AT THE FORCES ON THE BOX WHICH IS ACCELERATING DOWN

"$F = ma$"

$80g - 768 = 80a$

$784 - 768 = 80a$

$80a = 16$

$a = 0.2 \text{ ms}^{-2}$

LOOKING AT THE FORCES ON THE LIFT TREATING IT AS AN ENTIRE SYSTEM

"$F = ma$"

$400g + 80g - F - 4500 = (80 + 400)a$

$480g - F - 4500 = 480 \times 0.2$

$204 - F = 96$

$F = 108 \text{ N}$

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START WITH THE RESULTANT OF THE TWO FORCES

- $|3\underline{i} - 4\underline{j}| = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore \underline{F}_1 = 10(3\underline{i} - 4\underline{j}) = 30\underline{i} - 40\underline{j}$

- $|-7\underline{i} + 24\underline{j}| = \sqrt{49+576} = \sqrt{625} = 25$

$\therefore \underline{F}_2 = 2(-7\underline{i} + 24\underline{j}) = -14\underline{i} + 48\underline{j}$

$$\underline{F} = \underline{F}_1 + \underline{F}_2 = (30\underline{i} - 40\underline{j}) + (-14\underline{i} + 48\underline{j}) = 16\underline{i} + 8\underline{j}$$

NEXT FIND THE ACCELERATION, ONCE THE FORCES START ACTING

$$\underline{F} = m\underline{a} \implies 16\underline{i} + 8\underline{j} = 4\underline{a}$$
$$\implies \underline{a} = 4\underline{i} + 2\underline{j}$$

NOW TRACE THE JOURNEY

$t=0 \quad \underline{r} = -17\underline{i} - 50\underline{j}, \quad \underline{v} = -2\underline{i} + 2\underline{j}$

$$\underline{r} = \underline{r}_0 + \underline{v}t$$
$$\underline{r} = (-17\underline{i} - 50\underline{j}) + (-2\underline{i} + 2\underline{j}) \times 10$$
$$\underline{r} = (-17\underline{i} - 50\underline{j}) + (-20\underline{i} + 20\underline{j})$$
$$\underline{r} = -37\underline{i} - 30\underline{j}$$

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FINALLY THE LAST 10 SECONDS

$$\underline{r} = \underline{r}_0 + \underline{u}t + \tfrac{1}{2}\underline{a}t^2$$

$$\underline{r} = (-37\underline{i} - 30\underline{j}) + (-2\underline{i} + 2\underline{j}) \times 10 + \tfrac{1}{2}(4\underline{i} + 2\underline{j}) \times 10^2$$

$$\underline{r} = (-37\underline{i} - 30\underline{j}) + (-20\underline{i} + 20\underline{j}) + (200\underline{i} + 100\underline{j})$$

$$\underline{r} = 143\underline{i} + 90\underline{j}$$

HENCE THE PARTICLE TRAVELLED FROM $-17\underline{i} - 50\underline{j}$ TO $143\underline{i} + 90\underline{j}$

OR IN CO-ORDINATES FROM $(-17, -50)$ TO $(143, 90)$

USING THE DISTANCE FORMULA

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(90 - (-50))^2 + (143 - (-17))^2}$$

$$d = \sqrt{140^2 + 160^2}$$

$$d = 20\sqrt{113} \simeq 213 \text{ m}$$

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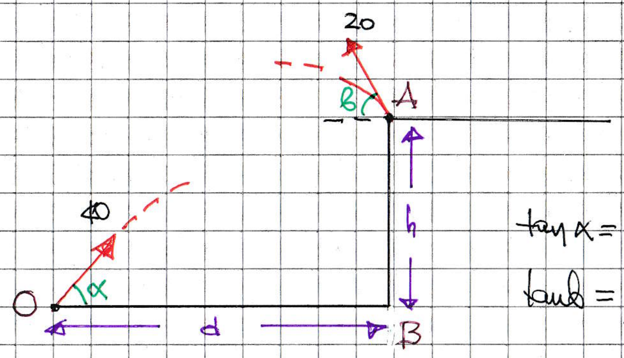
LOOKING AT THE DIAGRAM BELOW



$\tan\alpha = \frac{3}{4}$

$\tan\beta = \frac{4}{3}$

TAKING O AS THE ORIGIN

$x_1 = (40\cos\alpha)T = 40\times\frac{4}{5}T = 32T$

$x_2 = d - (20\cos\beta)T = d - 20\times\frac{3}{5}T = d - 12T$

FOR COLLISION $x_1 = x_2$

$32T = d - 12T$

$d = 44T$

SIMILARLY IN THE VERTICAL DIRECTION

$y_1 = (40\sin\alpha)T - \frac{1}{2}gT^2 = 40\times\frac{3}{5}\times T - \frac{1}{2}gT^2$

$y_2 = h + (20\sin\beta)T - \frac{1}{2}gT^2 = h + 20\times\frac{4}{5}T - \frac{1}{2}gT^2$

$$\left.\begin{aligned} y_1 &= 24T - \tfrac{1}{2}gT^2 \\ y_2 &= h + 16T - \tfrac{1}{2}gT^2 \end{aligned}\right\} \Rightarrow y_1 = y_2$$

$\Rightarrow 24T - \cancel{\frac{1}{2}gT^2} = h + 16T - \cancel{\frac{1}{2}gT^2}$

$\Rightarrow h = 8T$

DIVIDING THE EQUATIONS

$$\left.\begin{aligned} d &= 44T \\ h &= 8T \end{aligned}\right\} \Rightarrow \frac{d}{h} = \frac{44\cancel{T}}{8\cancel{T}} = \frac{11}{2}$$

$\Rightarrow d : h = 11 : 2$

// AS REQUIRED