IYGB - MMS PAPER T - QUESTION 1

a)

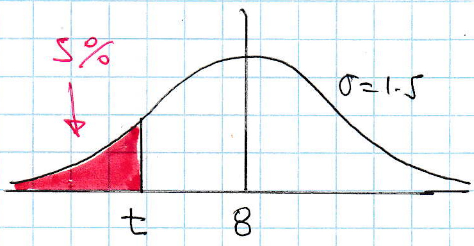
T = TIME IN WHICH THE DRUG ACTS (MINUTES)

$T \sim N(8, 1.5^2)$

$\Rightarrow P(T < t) = 5\%$

$\Rightarrow P(T > t) = 95\%$

$\Rightarrow P\left(Z > \frac{t-8}{1.5}\right) = 0.95$



INVERSION

$\Rightarrow \frac{t-8}{1.5} = -\Phi^{-1}(0.95)$

$\Rightarrow \frac{t-8}{1.5} = -1.6449$

$\Rightarrow t-8 = -2.46738$

$\Rightarrow t = 5.53265 \simeq 5.53$

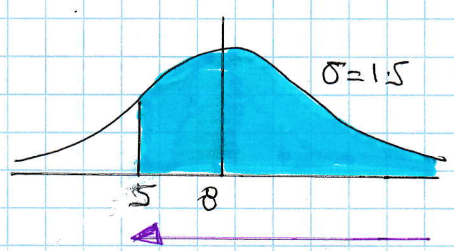
b)

START BY CALCULATING $P(T>5)$

$P(T>5) = P\left(Z > \frac{5-8}{1.5}\right)$

$= \Phi(-2)$

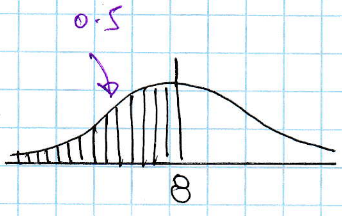
$= 0.9772$

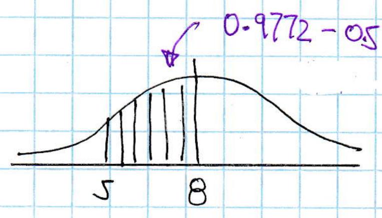


NEXT WE LOOK FOR THE CONDITIONAL PROBABILITY, USING TWO SEPARATE DIAGRAMS FOR CLARITY

$P(T>5 \mid T<8) = \ldots$

IYGB – MMS PAPER T – QUESTION 1





"GIVEN LESS THAN 8"

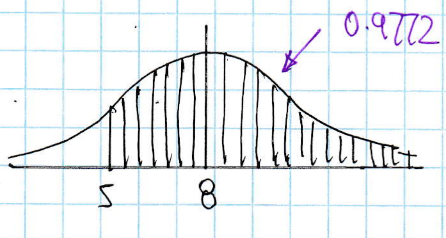
0.5

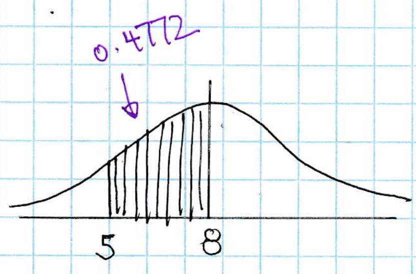
"THEN MORE THAN 5"

0.4772

$\therefore$ REQUIRED PROBABILITY $= \dfrac{0.4772}{0.5} = 0.9544$

c) REDRAWING TWO GRAPHS





"GIVEN MORE THAN 5"

0.9772 (PART b)

"THEN UNDER 8 MINUTES"

0.4772 (SAME DIAGRAM)

$\therefore$ REQUIRED PROBABILITY $= \dfrac{0.4772}{0.9772} = 0.4883$

IYGB - MMS PAPER T - QUESTION 2

a) WRITE ALL THE OUTCOMES FROM $X \sim B(8, \frac{1}{4})$ SUCH THAT $P(X_1 + X_2 \le 3)$

- $(0,3)\ (3,0) = \binom{8}{0}(\frac{1}{4})^0(\frac{3}{4})^8 \times \binom{8}{3}(\frac{1}{4})^3(\frac{3}{4})^5 \times 2 \text{ WAYS} = 0.041525\ldots$
- $(2,1)\ (1,2) = \binom{8}{2}(\frac{1}{4})^2(\frac{3}{4})^6 \times \binom{8}{1}(\frac{1}{4})^1(\frac{3}{4})^7 \times 2 \text{ WAYS} = 0.16638\ldots$
- $(2,0)\ (0,2) = \binom{8}{2}(\frac{1}{4})^2(\frac{3}{4})^6 \times \binom{8}{0}(\frac{1}{4})^0(\frac{3}{4})^8 \times 2 \text{ WAYS} = 0.0623628\ldots$
- $(1,1) = \binom{8}{1}(\frac{1}{4})^1(\frac{3}{4})^7 \times \binom{8}{1}(\frac{1}{4})^1(\frac{3}{4})^7 = 0.071272\ldots$
- $(1,0)\ (0,1) = \binom{8}{1}(\frac{1}{4})^1(\frac{3}{4})^7 \times \binom{8}{0}(\frac{1}{4})^0(\frac{3}{4})^8 \times 2 \text{ WAYS} = 0.053453\ldots$
- $(0,0) = \binom{8}{0}(\frac{1}{4})^0(\frac{3}{4})^8 \times \binom{8}{0}(\frac{1}{4})^0(\frac{3}{4})^8 = 0.010022\ldots$

ADDING GIVES 0.405

b) Y IS AN OBSERVATION OF 2 FROM 10 "GOES"

$Y \sim B(10, 0.31146)$

$P(X=2) = \binom{8}{2}(\frac{1}{4})^2(\frac{3}{4})^6 = \frac{5103}{16384} \simeq 0.31146$

$P(Y=5) = \binom{10}{5}(0.31146)^5(1-0.31146)^5$

$= 0.1143$

IYGB - MMS PAPER T - QUESTION 3

START BY RECORDING ALL THE OUTCOMES IN AN ORGANIZED WAY, INCLUDING PROBABILITIES

$W\,W$ $\qquad \frac{2}{5}\times\frac{1}{4} = \frac{1}{10}$

$\cancel{W}\,W\,W$
$W\,\cancel{W}\,W$ $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\right)\times 2 = \frac{2}{10}$ OR $\frac{1}{10}$ EACH "BRANCH"

$\cancel{W}\,\cancel{W}\,W\,W$
$\cancel{W}\,W\,\cancel{W}\,W$
$W\,\cancel{W}\,\cancel{W}\,W$ $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\right)\times 3 = \frac{3}{10}$ OR $\frac{1}{10}$ EACH "BRANCH"

$W\,\cancel{W}\,\cancel{W}\,\cancel{W}\,W$
$\cancel{W}\,W\,\cancel{W}\,\cancel{W}\,W$
$\cancel{W}\,\cancel{W}\,W\,\cancel{W}\,W$
$\cancel{W}\,\cancel{W}\,\cancel{W}\,W\,W$ $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\times 1\right)\times 4 = \frac{4}{10}$ OR $\frac{1}{10}$ EACH "BRANCH".

NOW WE PICK OUT OF ALL THE BRANCHES THAT HAVE W IN THE SECOND SLOT / NOTE ALL BRANCHES TURNED OUT TO HAVE PROBABILITY OF $\frac{1}{10}$

$$\Longrightarrow \text{REQUIRED PROBABILITY} = \frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}} = \frac{1}{4}$$

IYGB - MMS PAPER T - QUESTION 4

X = NUMBER OF CORRECT QUESTIONS (OUT OF n)

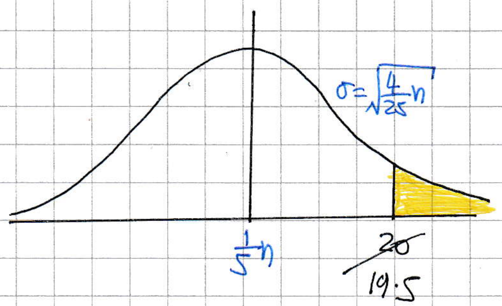
$X \sim B(n, 0.2)$

WE REQUIRE THAT THE PROBABILITY OF A PASS IS LESS THAN 2.5%

$$\Longrightarrow P(X \geqslant 20) < 0.025$$

APPROXIMATE BY A NORMAL DISTRIBUTION

$$\left.\begin{array}{l} E(X) = np = n \times 0.2 = \frac{1}{5}n \\ Var(X) = np(1-p) = \frac{1}{5}n \times \frac{4}{5} = \frac{4}{25}n \end{array}\right\} \Longrightarrow Y \sim N\left(\tfrac{1}{5}n, \tfrac{4}{25}n\right)$$



HENCE WE NOW HAVE

$$\Longrightarrow P(X \geqslant 20) < 0.025$$

$$\Longrightarrow P(Y > 19.5) < 0.025$$

$$\Longrightarrow 1 - P(Y > 19.5) < 0.025$$

$$\Longrightarrow -P(Y < 19.5) < -0.975$$

$$\Longrightarrow P(Y < 19.5) > 0.975$$

$$\Longrightarrow P\left(z < \frac{19.5 - \frac{1}{5}n}{\sqrt{\frac{4}{25}n}}\right) > 0.975$$

$\Downarrow$ INVERTING

$$\frac{19.5 - \frac{1}{5}n}{\frac{2}{5}\sqrt{n}} > +\Phi^{-1}(0.975)$$

IYGB - MMS PAPER T - QUESTION 4

$$\Rightarrow \quad \frac{19.5 - \frac{1}{5}n}{\frac{2}{5}\sqrt{n}} > 1.96$$

$$\Rightarrow \quad 19.5 - \frac{1}{5}n > 0.784\sqrt{n}$$

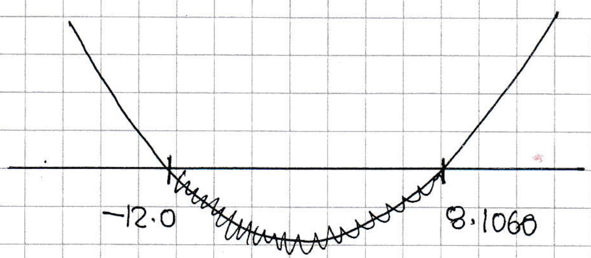
$$\Rightarrow \quad 0 > \frac{1}{5}n + 0.784\sqrt{n} - 19.5$$

$$\Rightarrow \quad n + 3.92\sqrt{n} - 97.5 < 0$$

BY QUADRATIC FORMULA OBTAIN CRITICAL VALUES

$$\sqrt{n} = \frac{-3.92 \pm \sqrt{3.92^2 - 4\times1\times(-97.5)}}{2\times1}$$

$$\sqrt{n} = \begin{cases} 8.10685\ldots \\ -12.0268\ldots \end{cases}$$



$\sqrt{n} < 8.1068\ldots$ (IGNORING NEGATIVE)

$n < 65.72\ldots$

$n = 65$

IYGB - MMS PAPER T - QUESTION 5

$$P(X=x) = \begin{cases} k & x=1 \\ \frac{1}{2}P(X=x-1) & x=2,3,4 \\ 0 & \text{OTHERWISE} \end{cases}$$

FORMING A TABLE OF PROBABILITIES

x	1	2	3	4
$P(X=x)$	k	$\frac{1}{2}k$	$\frac{1}{4}k$	$\frac{1}{8}k$

$\Rightarrow k+\frac{1}{2}k+\frac{1}{4}k+\frac{1}{8}k=1$

$\Rightarrow k\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)=1$

$\Rightarrow \frac{15}{8}k=1$

$\Rightarrow k=\frac{8}{15}$

HENCE WE NOW HAVE

$P(X=1)=\frac{8}{15}$, $P(X=2)=\frac{4}{15}$, $P(X=3)=\frac{2}{15}$, $P(X=4)=\frac{1}{15}$

NOW ODD & EVEN

odd	Even
$\frac{8}{15}+\frac{2}{15}$	$\frac{4}{15}+\frac{1}{15}$
$\frac{2}{3}$	$\frac{1}{3}$

EVEN SUM $\Rightarrow$ EEE: $\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$

$\left.\begin{matrix} OOE \\ OEO \\ EOO \end{matrix}\right\}$ $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times 3=\frac{12}{27}$

$\therefore P(Y \text{ IS EVEN})=\frac{13}{27}$

IYGB — MMS PAPER T — QUESTION 5

NEXT THE PROBABILITY THAT $Y>9$ OR $Y \geqslant 10$ FROM

EEE OR EOO, OEO, OOE

$$4+4+4=12 \geqslant 10 \implies \frac{1}{15}\times\frac{1}{15}\times\frac{1}{15}$$

$$4+3+3=10 \geqslant 10 \quad (3 \text{ WAYS}) \implies \frac{1}{15}\times\frac{2}{15}\times\frac{2}{15}\times 3$$

$$4+4+2=10 \geqslant 10 \quad (3 \text{ WAYS}) \implies \frac{1}{15}\times\frac{1}{15}\times\frac{4}{15}\times 3$$

ADD $\frac{1}{135}$

$\therefore$ THE REQUIRED PROBABILITY IS GIVEN BY

$$P(Y>9 \mid Y \text{ is EVEN}) = \frac{\frac{1}{135}}{\frac{13}{27}} = \frac{1}{65}$$

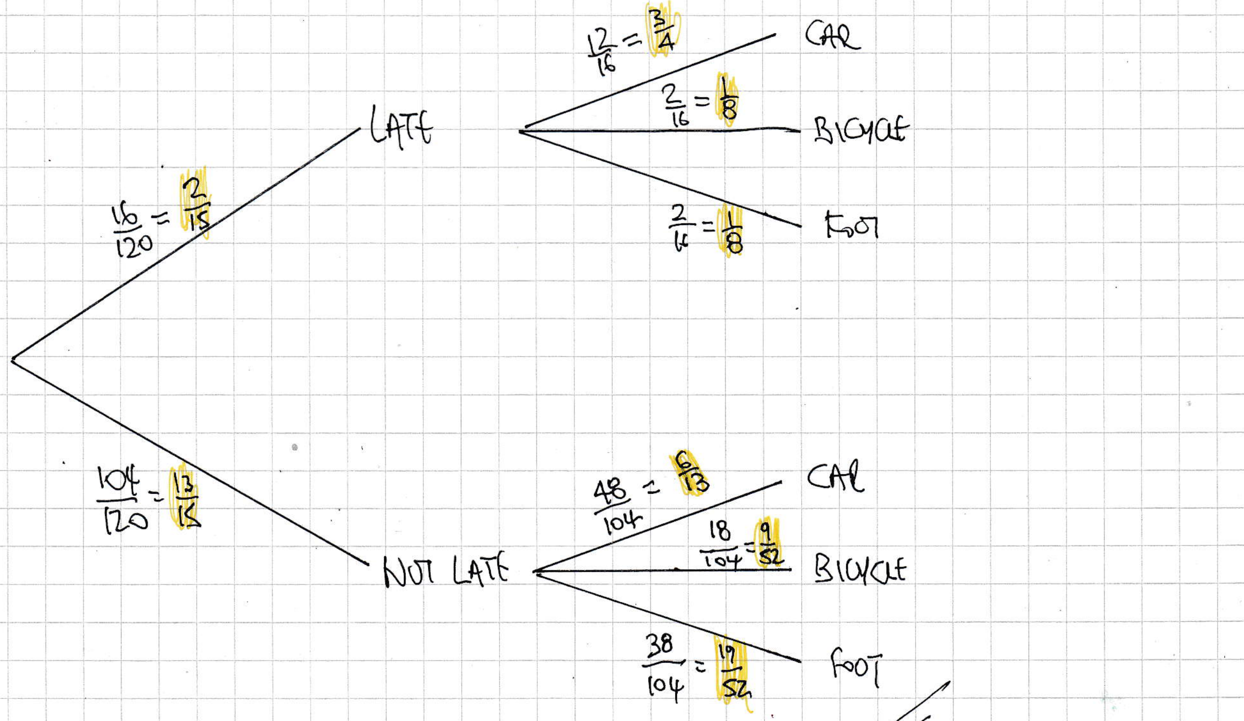
IYGB - MMS PAPER T - QUESTION 6

HERE IT IS BEST TO REVERSE BY USING A TWO WAY TABLE BASED ON A HYPOTHETICAL TOTAL (120) ← L.C.M OF DENOMINATORS

	CAR	BICYCLE	FOOT	TOTAL
LATE	$60 \times \frac{1}{5} = \underline{12}$	$20 \times \frac{1}{10} = \underline{2}$	$40 \times \frac{1}{20} = \underline{2}$	16
NOT LATE	48	18	38	104
TOTAL	$\frac{1}{2} \times 120 = \underline{60}$	$\frac{1}{6} \times 120 = \underline{20}$	$\frac{1}{3} \times 120 = \underline{40}$	120

FILL THE "ROW TOTALS" AT THE BOTTOM 60, 20, 40
THEN FILL THE LATE TOTALS 12, 2, 2
FINALLY FILL THE REST (IN BLUE)

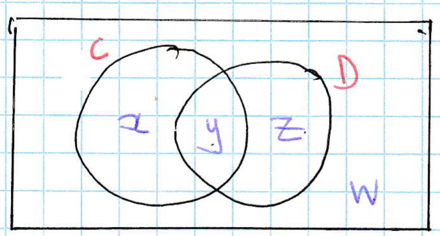
HENCE USING PROBABILITIES OUT OF THE TABLE THE TREE DIAGRAM IS REVERSED



IYGB - MMS PAPER T - QUESTION 7

$P(C) = \frac{1}{3}$ • $P(D) = \frac{7}{36}$ • $P[(C \cap D') \cup (C' \cap D)] = \frac{13}{36}$

a) FILL IN A VENN DIAGRAM



$P(C) = \frac{1}{3} \Rightarrow x + y = \frac{1}{3}$

$P(D) = \frac{7}{36} \Rightarrow y + z = \frac{7}{36}$

$P[(C \cap D') \cup (C' \cap B)] = \frac{13}{36} \Rightarrow x + z = \frac{13}{36}$

& $x + y + z + w = 1$

SOLVING THE 4 EQUATIONS

$x + y = \frac{1}{3}$

$y + z = \frac{7}{36}$ ADD THESE $2x + 2y + 2z = \frac{8}{9}$

$x + z = \frac{13}{36}$

$x + y + z + w = 1 \Leftarrow$ DOUBLE THIS $\underbrace{2x + 2y + 2z}_{\frac{8}{9}} + 2w = 2$

$\frac{8}{9} + 2w = 2$

$2w = \frac{10}{9}$

$w = \frac{5}{9}$

USING

$x + y + z + w = 1$

$x + y + z + \frac{5}{9} = 1$

$x + y + z = \frac{4}{9}$

BUT $x + z = \frac{13}{36}$

$\frac{13}{36} + y = \frac{4}{9}$

$y = \frac{1}{12}$ //

$\therefore P(C \cap D) = \frac{1}{12}$

IYGB - MMS PAPER T - QUESTION 7

b) PROCEED AS FOLLOWS

$$\boxed{P(C) = \frac{k}{k+2} \qquad P(D) = \frac{2}{k}}$$

IF MUTUALLY EXCLUSIVE

$$\Rightarrow P(C \cup D) = P(C) + P(D) - \cancel{P(C \cap D)}$$

$$\Rightarrow P(C \cup D) = P(C) + P(D)$$

$$\Rightarrow P(C \cup D) = \frac{k}{k+2} + \frac{2}{k}$$

$$\Rightarrow P(C \cup D) = \frac{k^2+2k+2}{k(k+2)}$$

$$\Rightarrow P(C \cup D) = \frac{k^2+2k+4}{k^2+2k}$$

$$\Rightarrow P(C \cup D) = 1 + \frac{4}{k^2+2k}$$

$$\Rightarrow P(C \cup D) > 1$$

$\therefore$ C & D CANNOT BE MUTUALLY EXCLUSIVE //

IYGB - MMS PAPER T - QUESTION 8

USING $\underline{v} = \underline{u} + \underline{a}t$ FOR EACH PARTICLE

- $\underline{v}_A = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}\frac{1}{4}\\-\frac{1}{2}\end{pmatrix}t = \begin{pmatrix}\frac{1}{4}t+1\\2-\frac{1}{2}t\end{pmatrix}$
- $\underline{v}_B = \begin{pmatrix}2\\-2\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\-\frac{1}{4}\end{pmatrix}t = \begin{pmatrix}2+\frac{1}{2}t\\-2-\frac{1}{4}t\end{pmatrix}$

IF THE VELOCITIES ARE PARALLEL THEY MUST BE IN PROPORTION

$$\frac{\frac{1}{4}t+1}{2-\frac{1}{2}t} = \frac{2+\frac{1}{2}t}{-2-\frac{1}{4}t} \Longrightarrow \frac{t+4}{8-2t} = \frac{8+2t}{-8-t}$$

$$\Longrightarrow \frac{t+4}{2t-8} = \frac{2t+8}{t+8}$$

$$\Longrightarrow (2t-8)(2t+8) = (t+8)(t+4)$$

$$\Longrightarrow 4t^2-64 = t^2+12t+32$$

$$\Longrightarrow 3t^2-12t-96=0$$

$$\Longrightarrow t^2-4t-32=0$$

$$\Longrightarrow (t-8)(t+4)=0$$

$$\Longrightarrow t = \begin{cases}8\\ \cancel{-4}\end{cases}$$

NOW FORMING EQUATIONS FOR THE POSITION VECTORS USING $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r}_A = \begin{pmatrix}7\\2\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix}\times 8 + \frac{1}{2}\begin{pmatrix}\frac{1}{4}\\-\frac{1}{2}\end{pmatrix}8^2 = \begin{pmatrix}23\\2\end{pmatrix} \quad \text{OR} \quad A(23,2)$$

$$\underline{r}_B = \begin{pmatrix}1\\-1\end{pmatrix} + \begin{pmatrix}2\\-2\end{pmatrix}\times 8 + \frac{1}{2}\begin{pmatrix}\frac{1}{2}\\-\frac{1}{4}\end{pmatrix}8^2 = \begin{pmatrix}33\\-25\end{pmatrix} \quad \text{OR} \quad B(33,-25)$$

$$|AB| = \sqrt{(33-23)^2+(-25-2)^2} = \sqrt{100+729} = \sqrt{829} \simeq 28.79$$

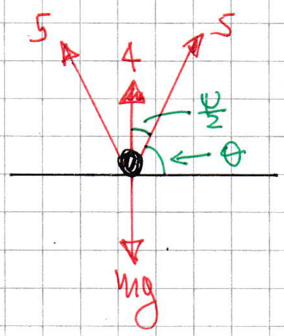
IYGB - MMS PAPER T - QUESTION 9

USING THE COSINE RULE

$$\Rightarrow 8^2 = 5^2 + 5^2 - 2\times5\times5\times\cos\phi$$
$$\Rightarrow 64 = 25 + 25 - 50\cos\phi$$
$$\Rightarrow 50\cos\phi = -14$$
$$\Rightarrow \cos\phi = -\frac{14}{50}$$
$$\Rightarrow \cos\phi = -\frac{7}{25} \text{ (OBTUSE)}$$

$$\therefore \cos\psi = +\frac{7}{25} \text{ (ACUTE)}$$

NOW DRAWING A DIAGRAM IN EQUILIBRIUM



NOTE THAT

- EQUILIBRIUM ON A SMOOTH PLANE CAN ONLY BE ACHIEVED IF THE 5N FORCES ARE SYMMETRICAL ABOUT THE VERTICAL
- THE 5N FORCES CANNOT ACT INTO THE PLANE AS THIS WILL PRODUCE A NEGATIVE m

NEED THE EXACT VALUE OF $\cos\frac{\psi}{2}$

$$\cos 2A = 2\cos^2 A - 1$$
$$\cos\psi = 2\cos^2\tfrac{\psi}{2} - 1$$
$$\frac{7}{25} = 2\cos^2\frac{\psi}{2} - 1$$
$$\frac{7}{25} + \frac{25}{25} = 2\cos^2\frac{\psi}{2}$$
$$\frac{32}{25} = 2\cos^2\frac{\psi}{2}$$
$$\cos^2\frac{\psi}{2} = \frac{16}{25}$$

$$\therefore \cos\frac{\psi}{2} = +\frac{4}{5} \quad (\psi/2 \text{ ACUTE})$$

IYGB - MMS PAPER T - QUESTION 9

RESOLVING VERTICALLY WE HAVE

$$\Rightarrow 2 \times 5\cos\frac{\psi}{2} + 4 = mg$$

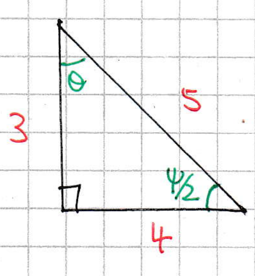
$$\Rightarrow 10 \times \frac{4}{5} + 4 = mg$$

$$\Rightarrow 8 + 4 = mg$$

$$\Rightarrow m = \frac{12}{9.8}$$

$$\Rightarrow m = \frac{120}{98} = \frac{60}{49}$$

FINALLY TO FIND θ, EITHER USE COMPLEMENTARY RELATIONSHIPS OR COMPOUND ANGLE IDENTITIES OR A STANDARD TRIANGLE



$$\theta + \frac{\psi}{2} = 90°$$

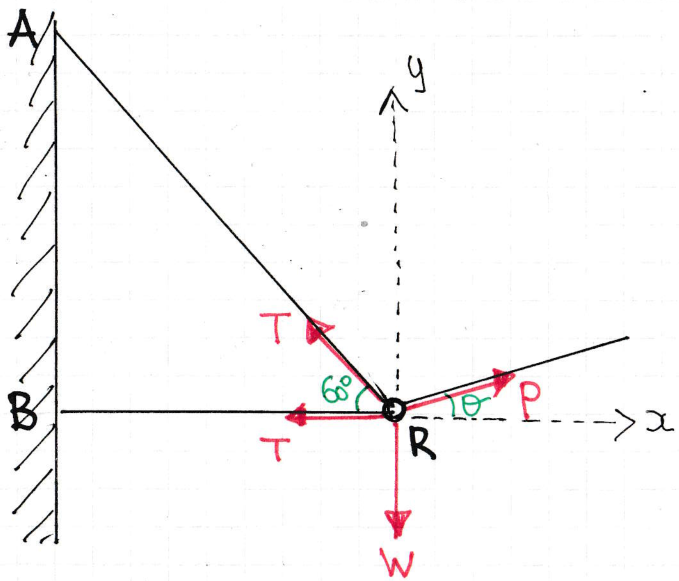
$$\cos\frac{\psi}{2} = \frac{4}{5}$$

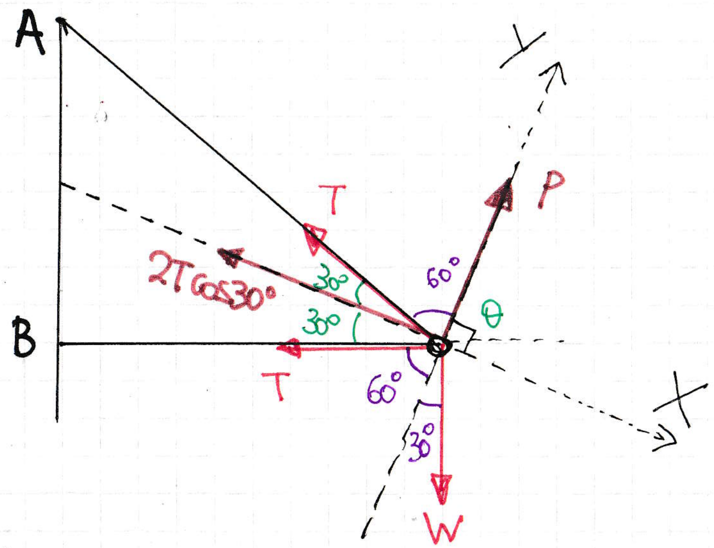
$\therefore$ IT IS A 3:4:5

$$\therefore \cos\theta = \frac{3}{5}$$

IYGB - MMS PAPER T - QUESTION 10

DRAWING A DIAGRAM - NOTE THAT THE TENSION IS THE SAME ON BOTH SIDES (THREADED)





- LOOKING IN THE SECOND DIAGRAM, THE RESULTANT OF THE TWO TENSIONS ACTS ALONG THE ANGLE BISECTOR OF $A\hat{R}B$ AND HAS MAGNITUDE $2T\cos 30$
- NOW WE OBSERVE:

  THIS TENSION (RESULTANT) HAS TO BALANCE THE WEIGHT, AND P - HOWEVER $2T\cos 30$ WILL ONLY HAVE TO BALANCE THE WEIGHT IF P ACTS ALONG THE "Y AXIS"

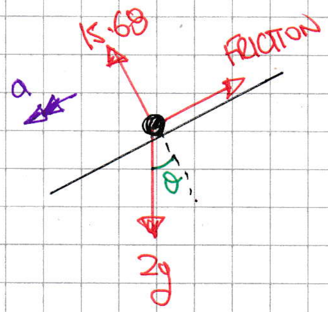
- THIS OCCURS WHEN $\theta = 60°$
- AND RESOLVING ALONG THE X & Y, $P_{MIN} = W\cos 30°$

$$P_{MIN} = \frac{\sqrt{3}}{2}W$$

IYGB – MMS PAPER T – QUESTION 11

IN THIS QUESTION IT TAKES A WHILE TO SEE WHAT CALCULATIONS ARE RELEVANT SO IN A FIRST ATTEMPT IT IS TYPICAL TO OBTAIN "ITEMS" NOT REQUIRED (NOT SHOWN HERE)

LOOKING AT THE JOURNEY A TO B, I.E. THE ACCELERATING SECTION



$15.68 = 2g\cos\theta$

$15.68 = 19.6\cos\theta$

$\cos\theta = 0.8$

& CONSEQUENTLY $\sin\theta = 0.6$

LOOKING AT THE DECELERATING SECTION FROM B TO C

$a = \frac{\Delta v}{\Delta t} = \frac{5-9}{16-6} = \frac{-4}{10} = -0.4$

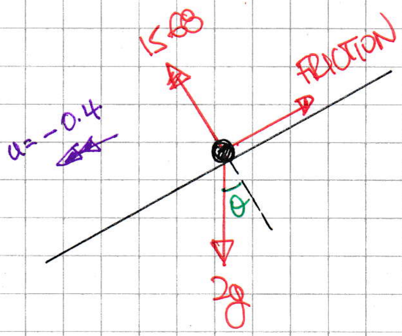
$\Rightarrow$ "$F = ma$"

$\Rightarrow 2g\sin\theta - \text{FRICTION} = 2(-0.4)$

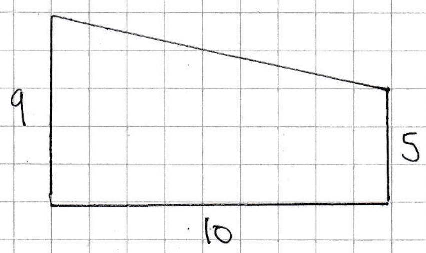
$\Rightarrow 2(9.8)(0.6) - \text{FRICTION} = -0.8$

$\Rightarrow \text{FRICTION} = 12.56$

FRICTION IN THE BC SECTION



NEXT LOOKING AT THE SPEED TIME GRAPH



$\text{AREA} = \frac{9+5}{2} \times 10$

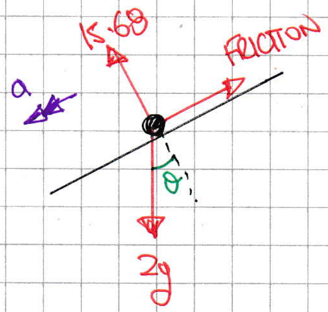
$\text{AREA} = 7 \times 10$

$\therefore$ DISTANCE B TO C IS 70

IYGB – MMS PAPER T – QUESTION 11

IN THIS QUESTION IT TAKES A WHILE TO SEE WHAT CALCULATIONS ARE RELEVANT SO IN A FIRST ATTEMPT IT IS TYPICAL TO OBTAIN "ITEMS" NOT REQUIRED (NOT SHOWN HERE)

LOOKING AT THE JOURNEY A TO B, I.E. THE ACCELERATING SECTION



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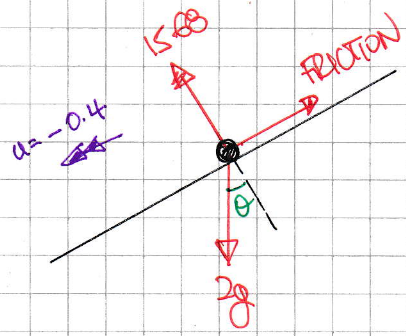
$a = \frac{\Delta v}{\Delta t} = \frac{5-9}{16-6} = \frac{-4}{10} = -0.4$



$\Rightarrow$ "$F = ma$"

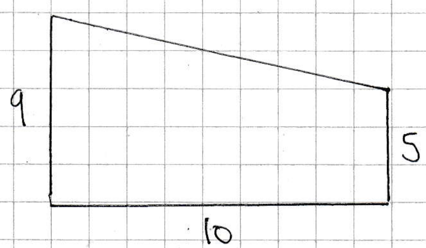
$\Rightarrow 2g\sin\theta - \text{FRICTION} = 2(-0.4)$

$\Rightarrow 2(9.8)(0.6) - \text{FRICTION} = -0.8$

$\Rightarrow \text{FRICTION} = 12.56$

FRICTION IN THE BC SECTION

NEXT LOOKING AT THE SPEED TIME GRAPH



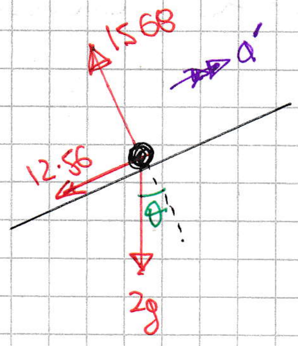
$\text{AREA} = \frac{9+5}{2} \times 10$

$\text{AREA} = 7 \times 10$

$\therefore$ DISTANCE B TO C IS 70

IYGB - MMS PAPER T - QUESTION 11

NOW THE JOURNEY BACK UP FROM C TO B



$\Longrightarrow$ "$F = ma$"

$\Longrightarrow -12.56 - 2g\sin\theta = 2a'$

$\Longrightarrow -12.56 - 2g(0.6) = 2a'$

$\Longrightarrow -24.32 = 2a'$

$\Longrightarrow a' = -12.16 \text{ ms}^{-2}$

FINALLY KINEMATICS FOR THE JOURNEY B TO C

$u = ?$
$a = -12.16$
$s = 70$
$t =$
$v = 0$

"$v^2 = u^2 + 2as$"

$0 = u^2 + 2(-12.16) \times 70$

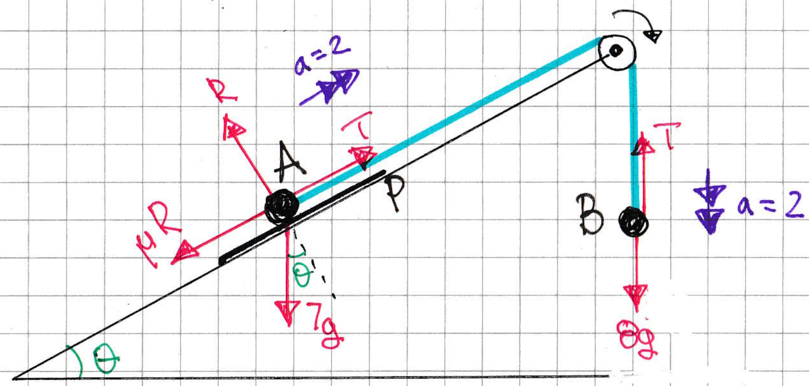
$u^2 = 1702.4$

$u = 41.26 \text{ ms}^{-1}$

I.E $U = 41.26$

IYGB - MMS PAPER T - QUESTION 12

START WITH A DIAGRAM WHICH IGNORES THE PLATE, IE WE CONSIDER THE REST OF THE SYSTEM AS THE PLATE IS IN EQUILIBRIUM



$$\left\{ \tan\theta = \frac{3}{4},\ \sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5} \right\}$$

LET μ BE THE COEFFICIENT OF FRICTION BETWEEN A & P

(B): $8g - T = 8a$

$8g - 8a = T$

$T = 62.4\text{ N}$

(A): $T - \mu R - 7g\sin\theta = ma$

$\Rightarrow 62.4 - \mu(7g\cos\theta) - 7g\sin\theta = 7 \times 2$

$\Rightarrow 62.4 - 54.88\mu - 41.16 = 14$

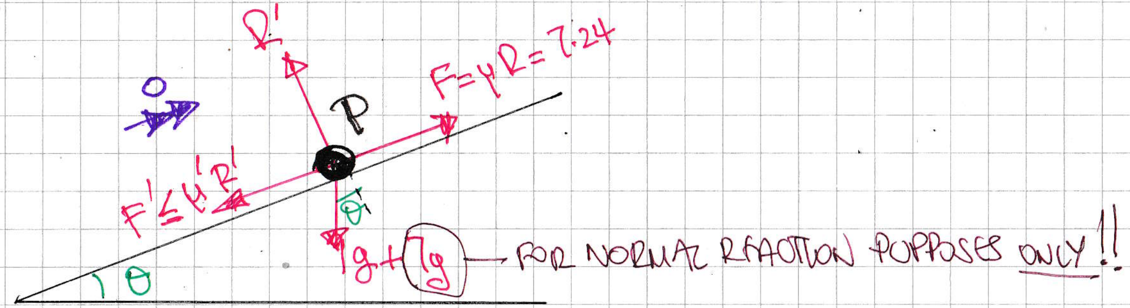
$\Rightarrow 54.88\mu = 7.24$

$\Rightarrow \mu = \frac{181}{1372} \simeq 0.132$

NOW LOOKING AT THE PLATE IN EQUILIBRIUM AND LET

- μ' = BE THE COEFFICIENT OF FRICTION BETWEEN THE PLATE & THE PLANE
- $F = \mu R = \frac{181}{1372} \times 7g\cos\theta = 7.24$ (FROM A)
- R' = NORMAL REACTION BETWEEN THE PLATE & THE PLANE

IYGB - MMS PAPER T - QUESTION 12



(I): $R' = 8g\cos\theta$

$R' = 62.72\text{ N}$

(II): $F = F' - 1g\sin\theta$ (EQUILIBRIUM)

$7.24 = F' - 5.88$

$F' = 1.36\text{ N}$

FINALLY WE OBTAIN

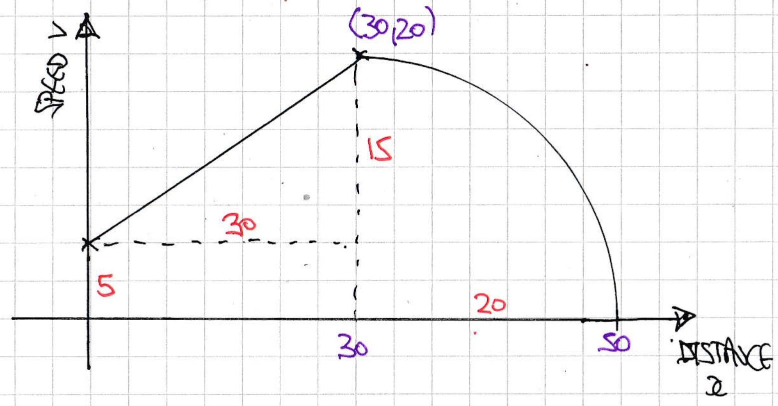
$F' \leq \mu' R'$

$1.36 \leq \mu' \times 62.72$

$\mu \geq \frac{17}{784}$ ($\mu \geq 0.0217$)

IYGB - MMS PAPER I - QUESTION 13

STARTING WITH THE "DISTANCE - SPEED" GRAPH



- GRADIENT OF LINE $= \frac{\Delta v}{\Delta x} = \frac{15}{30} = \frac{1}{2}$
- EQUATION OF LINE $v = \frac{1}{2}x + 5$
- EQUATION OF THE CIRCLE IS GIVEN BY

$$(x-30)^2 + v^2 = 20^2$$
$$v^2 = 400 - (x-30)^2$$
$$v = +\sqrt{400-(x-30)^2}$$

THE EQUATION OF THE GRAPH IS GIVEN BY

$$v = \begin{cases} \frac{1}{2}x + 5 & 0 \leq x \leq 30 \\ \sqrt{400-(x-30)^2} & 30 < x \leq 50 \end{cases}$$

INTEGRATING THE FIRST SECTION

$$v = \frac{dx}{dt} = \frac{1}{2}x + 5$$

$$\Longrightarrow \frac{1}{\frac{1}{2}x+5}\,dx = 1\,dt$$

$$\Longrightarrow \int_{x=0}^{30} \frac{1}{\frac{1}{2}x+5}\,dx = \int_{t=0}^{t} 1\,dt$$

$$\Longrightarrow \int_{0}^{30} \frac{2}{x+10}\,dx = \int_{0}^{t} 1\,dt$$

$$\Longrightarrow \Big[2\ln(x+10)\Big]_{0}^{30} = \Big[t\Big]_{0}^{t}$$

$$\Longrightarrow 2\ln 40 - 2\ln 10 = t - 0$$

$$\Longrightarrow t = 2\left[\ln 40 - \ln 10\right]$$

$$\Longrightarrow t = 2\ln 4$$

$$\Longrightarrow t = 4\ln 2$$

IYGB - MMS PAPER T - QUESTION 13

MOVING INTO THE SECOND SECTION OF THE GRAPH

$$v = \frac{dx}{dt} = \sqrt{400 - (x-30)^2}$$

$$\Rightarrow \frac{1}{\sqrt{400-(x-30)^2}}\,dx = 1\,dt$$

$$\Rightarrow \int_{30}^{50} \frac{1}{\sqrt{20^2-(x-30)^2}}\,dx = \int_{4\ln 2}^{t} 1\,dt$$

USING THE RESULT GIVEN

$$\Rightarrow \left[\arcsin\left(\frac{x-30}{20}\right)\right]_{30}^{50} = \left[t\right]_{4\ln 2}^{t}$$

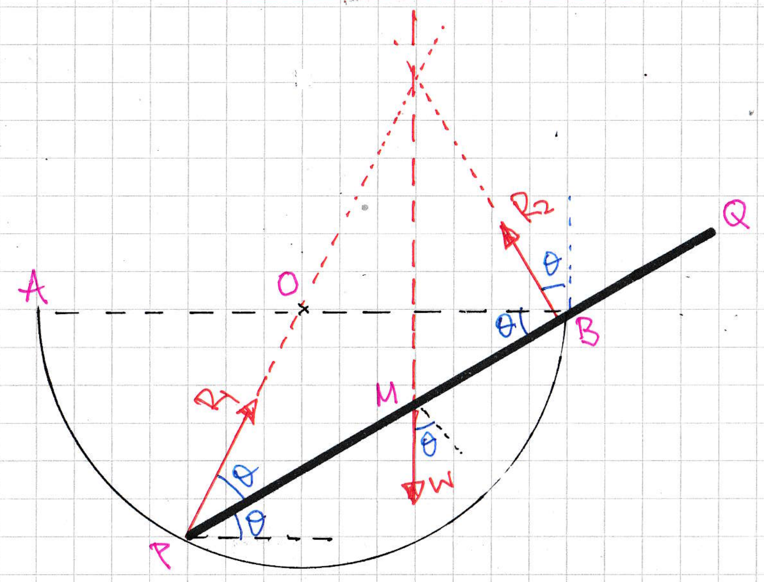
$$\Rightarrow \arcsin 1 - \cancel{\arcsin 0} = t - 4\ln 2$$

$$\Rightarrow t = \frac{\pi}{2} + 4\ln 2$$

$$\Rightarrow t = \frac{1}{8}\left[4\pi + \ln 2\right]$$

IYGB - MMS PAPER T - QUESTION 14

START WITH A DETAILED DIAGRAM



- AS CONTACTS ARE SMOOTH, R_1 IS $\perp$ AT P, SO IT MUST PASS THROUGH O
- FOR THE SAME REASON (SMOOTHNESS), R_2 IS $\perp$ TO THE ROD AT B
- THE THREE FORCES, W, R_1, R_2 MUST BE CONCURRENT
- TRIANGLE $\hat{POB}$ IS ISOSCELES, SO LOADS OF ANGLES CAN BE MARKED
- $|PM| = \frac{L}{2}$

BY TRIGONOMETRY (SPLIT ISOSCELES TRIANGLE INTO 2)

$$|PB| = 2|OB|\cos\theta$$
$$|PB| = 2a\cos\theta$$

$$\therefore |MB| = |PB| - |PM| = 2a\cos\theta - \frac{L}{2}$$

RESOLVING ALONG THE ROD

$$R_1\cos\theta = W\sin\theta$$

MOMENTS ABOUT B

$$W\cos\theta \times |MB| = R_1\sin\theta \times |PB|$$

DIVIDING EQUATIONS

$$\Rightarrow \frac{\cancel{R_1}\sin\theta|PB|}{\cancel{R_1}\cos\theta} = \frac{\cancel{W}\cos\theta|MB|}{\cancel{W}\sin\theta}$$

$$\Rightarrow |MB| = \frac{\sin^2\theta}{\cos^2\theta}|PB|$$

$$\Rightarrow 2a\cos\theta - \frac{L}{2} = \frac{\sin^2\theta}{\cos^2\theta}(2a\cos\theta)$$

$$\Rightarrow 2a\cos\theta - \frac{L}{2} = \frac{2a\sin^2\theta}{\cos\theta}$$

IYGB - MMS PAPER T - QUESTION 14

$$\Longrightarrow 2a\cos\theta - \frac{2a\sin^2\theta}{\cos\theta} = \frac{L}{2}$$

$$\Longrightarrow \frac{2a\cos^2\theta - 2a\sin^2\theta}{\cos\theta} = \frac{L}{2}$$

$$\Longrightarrow \frac{L}{2} = \frac{2a(\cos^2\theta - \sin^2\theta)}{\cos\theta}$$

$$\Longrightarrow \frac{L}{2} = \frac{2a\cos 2\theta}{\cos\theta}$$

$$\therefore L = \frac{4a\cos 2\theta}{\cos\theta}$$

AS REQUIRED