IYGB - FS1 PAPER Q - QUESTION 1

a)

x	1	2	4	5
$F(x)$	$\frac{3}{20}$	$\frac{2k+3}{20}$	$\frac{k+5}{10}$	$\frac{k+2}{4}$
$P(X=x)$	$\frac{3}{20}$	$\left(\frac{1}{5}\right)$	$\left(\frac{7}{20}\right)$	$\left(\frac{3}{10}\right)$

$$F(5) = P(X \leq 5) = 1$$

$$\frac{k+2}{4} = 1$$

$$k+2 = 4$$

$$k = 2$$

b) FILL IN THE ABOVE TABLE (IN TERMS OF k, OR WITH NUMBERS)

- $\frac{2k+3}{20} - \frac{3}{20} = \frac{7}{20} - \frac{3}{20} = \frac{4}{20} = \frac{1}{5}$
- $\frac{k+5}{10} - \frac{2k+3}{20} = \frac{7}{10} - \frac{7}{20} = \frac{7}{20}$
- $\frac{k+2}{4} - \frac{k+5}{10} = 1 - \frac{7}{10} = \frac{3}{10}$

I) $E(X) = \sum x P(X=x)$

$$E(X) = \left(1 \times \frac{3}{20}\right) + \left(2 \times \frac{1}{5}\right) + \left(4 \times \frac{7}{20}\right) + \left(5 \times \frac{3}{10}\right)$$

$$E(X) = \frac{3}{20} + \frac{2}{5} + \frac{7}{5} + \frac{3}{2}$$

$$E(X) = 3.45$$

IYGB – FS1 PAPER Q – QUESTION 1

b) $E(X^2) = \sum x^2 P(X=x)$

$$E(X^2) = \left(1^2 \times \tfrac{3}{20}\right) + \left(2^2 \times \tfrac{1}{5}\right) + \left(4^2 \times \tfrac{7}{20}\right) + \left(5^2 \times \tfrac{3}{10}\right)$$

$$= \tfrac{3}{20} + \tfrac{4}{5} + \tfrac{28}{5} + \tfrac{15}{2}$$

$$= 14.05$$

c) FIND THE VARIANCE FIRST

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\text{Var}(X) = 14.05 - 3.45^2$$

$$\text{Var}(X) = 2.1475$$

HENCE USING $\text{Var}(aX+b) = a^2\,\text{Var}(X)$

$$\text{Var}(20X-2) = 20^2\,\text{Var}(X)$$

$$= 400 \times 2.1475$$

$$= 859$$

IYGB - FS1 PAPER Q - QUESTION 2

$X \sim Po(\lambda)$

$\Longrightarrow P(X=8) = P(X=9)$

$\Longrightarrow \dfrac{e^{-\lambda} \times \lambda^8}{8!} = \dfrac{e^{-\lambda} \times \lambda^9}{9!}$ DIVIDE THROUGH BY $e^{-\lambda} \neq 0$

$\Longrightarrow \dfrac{\lambda^8}{8!} = \dfrac{\lambda^9}{9!}$

$\Longrightarrow \dfrac{9!}{8!} = \dfrac{\lambda^9}{\lambda^8}$

$\Longrightarrow 9 = \lambda$

NOW WE CAN CALCULATE $P(4 < X \leq 10)$

$$
\begin{aligned}
P(4 < X \leq 10) &= P(5 \leq X \leq 10) \\
&= P(X \leq 10) - P(X \leq 4) \\
&= 0.7060\ldots - 0.05496\ldots \\
&= 0.6510
\end{aligned}
$$

IYGB - FS1 PAPER Q - QUESTION 3

a) START WITH THE PROBABILITY MASS FUNCTION OF $B(n,p)$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

NOW NOTE THAT $(A+B)^n = \sum_{r=0}^{n} \left[\binom{n}{r} A^r B^{n-r}\right]$

BY THE DEFINITION OF P.G.F

$$G_X(t) = \sum_{x=0}^{n} \left[P(X=x)\, t^x\right] = \sum_{x=0}^{n} \left[\binom{n}{x} p^x (1-p)^{n-x} t^x\right]$$

$$= \sum_{x=0}^{n} \left[\binom{n}{x} (pt)^x (1-p)^{n-x}\right]$$

COMPARE WITH

$$\sum_{r=0}^{n} \left[\binom{n}{r} A^r B^{n-r}\right] = (A+B)^n$$

$$= \left[pt + (1-p)\right]^n$$

$$= (1-p+pt)^n$$

b) NOW $E(X) = G_X'(1)$

$$\Rightarrow \frac{d}{dt}(1-p+pt)^n = n(1-p+pt)^{n-1} \times p = np(1-p+pt)^{n-1}$$

LET $t=1$ $\quad E(X) = np(1-p+p)^{n-1} = np \times 1^{n-1} = np$

DIFFERENTIATE WITH RESPECT TO t, ONCE MORE

$$\Rightarrow \frac{d}{dt}\left[np(1-p+pt)^{n-1}\right] = np(n-1)(1-p+pt)^{n-2} \times p$$

$$= n(n-1)p^2(1-p+pt)^{n-2}$$

LET $t=1$

$$n(n-1)p^2(1-p+p)^{n-2} = n(n-1)p^2$$

IYGB - FS1 PAPER P - QUESTION 3

THUS SO FAR WE HAVE $G_X'(1) = np$ & $G_X''(1) = n(n-1)p^2$

$$\Rightarrow \text{Var}(X) = G_X''(1) + G_X'(1) - [G_X'(1)]^2$$
$$\Rightarrow \text{Var}(X) = n(n-1)p^2 + np - (np)^2$$
$$\Rightarrow \text{Var}(X) = (n^2-n)p^2 + np - n^2p^2$$
$$\Rightarrow \text{Var}(X) = \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$
$$\Rightarrow \text{Var}(X) = np - np^2$$
$$\Rightarrow \text{Var}(X) = np(1-p)$$

IYGB - QUESTION 4 - FS1 PAPER Q

SETTING HYPOTHESES

H_0 : THERE IS NO ASSOCIATION BETWEEN GENDER & FILM TYPE PREFERENCE, IE THE EVENTS ARE INDEPENDENT

H_1 : THERE IS ASSOCIATION BETWEEN GENDER & FILM PREFERENCE, IE THE EVENTS ARE NOT INDEPENDENT

SETTING A MULTIPURPOSE TABLE

	ACTION FILM	COMEDY FILM	ROMANCE FILM	TOTAL
MALE	239 222.216 1.267	185 188.94 0.082	140 152.844 1.079	564
FEMALE	155 171.784 1.640	150 146.06 0.106	131 118.156 1.396	436
TOTAL	394	335	271	1000

= ACTUAL DATA - OBSERVED FREQUENCY O_i

= EXPECTED FREQUENCY E_i (IF INDEPENDENT)

= CONTRIBUTIONS $\frac{(O_i - E_i)^2}{E_i}$

SUMMARIZING THE REST OF THE AUXILIARIES

- DEGREES OF FREEDOM $\nu = (r-1)(c-1) = (2-1)(3-1) = 2$
- $\chi_2^2(5\%) = 5.991$
- $\sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = 5.570$

AS $5.570 < 5.991$ THERE IS EVIDENCE OF INDEPENDENCE, IE IT APPEARS THAT THERE IS NO ASSOCIATION BETWEEN GENDER & TYPE OF FILM - THERE IS NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER Q - QUESTION 5

$$\boxed{\begin{array}{l} X = \text{NO OF CALLS PER MINUTE} \\ X \sim Po(7) \end{array}}$$

a) P(more than 5 but at most 10) $= P(5 < X \leqslant 10)$

$$= P(6 \leqslant X \leqslant 10)$$

$$= P(X \leqslant 10) - P(X \leqslant 5)$$

$$= 0.9015 - 0.3007$$

$$= 0.6008$$

b) READJUST THE RATE TO $\frac{1}{2}$ MINUTE

$$\boxed{\begin{array}{l} Y = \text{NO OF CALLS PER } \tfrac{1}{2} \text{ MINUTE} \\ Y \sim Po(3.5) \end{array}}$$

$$P(Y=0) = \frac{e^{-3.5} \times 3.5^0}{0!} = 0.0302$$

$$\boxed{\begin{array}{l} W = \text{NO OF } \tfrac{1}{2} \text{ INTERVALS WITHOUT A CALL} \\ W \sim B(10, 0.0302) \end{array}}$$

$$P(W \geqslant 1) = 1 - P(W \leqslant 0) = 1 - P(W=0) = 1 - \binom{10}{0}(0.0302)^0(0.9698)^{10}$$

$$= 1 - 0.7359\ldots \simeq 0.2641$$

c) SETTING SUITABLE HYPOTHESES

$H_0: \lambda = 7$

$H_1: \lambda > 7$

WHERE λ REPRESENTS THE AVERAGE RATE OF CALLS PER MINUTE, IN GENERAL

IYGB - FS1 PAPER Q - QUESTION 5

TESTING AT 5% SIGNIFICANCE ON THE BASIS THAT $x = 13$

$$P(X \geq 13) = 1 - P(X \leq 12)$$
$$= 1 - 0.9730$$
$$= 0.0270$$
$$= 2.7\% < 5\%$$

THERE IS SIGNIFICANT EVIDENCE, AT THE 5% LEVEL, TO SUPPORT THE TELEPHONE OPERATOR'S CLAIM

SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER Q - QUESTION 6

X = NUMBER OF FAULTY TILES

$X \sim B(10, 0.1)$

a) $H_0: p = 0.1$
$H_1: p > 0.1$

- C.R. $= \{5, 6, 7, \dots, 10\}$
- SIZE OF TEST = P(TYPE I ERROR)

= "REJECT H_0 WHEN H_0 IS TRUE"

= ACTUAL SIGNIFICANCE

= ... tables

$= P(X \geq 5)$

$= 1 - P(X \leq 4)$

$= 1 - 0.9984$

$= 0.0016$

b) POWER OF A TEST = 1 − P(TYPE II ERROR)

= 1 − P(REJECTING H_1 WHEN H_1 IS TRUE)

$k = 1 - P(X \leq 4)$ ← FROM $X \sim B(10, 0.25)$

$k = 1 - 0.9219$

$k = 0.0781$

c) SECOND TEST, $X \sim B(25, 0.1)$, C.R $= \{10, 11, 12, \dots, 25\}$

P(TYPE I ERROR) = SIZE OF THE TEST = "REJECT H_0 WHEN H_0 IS TRUE"

= ACTUAL SIGNIFICANCE

$= P(X \geq 10)$

$= 1 - P(X \leq 9)$

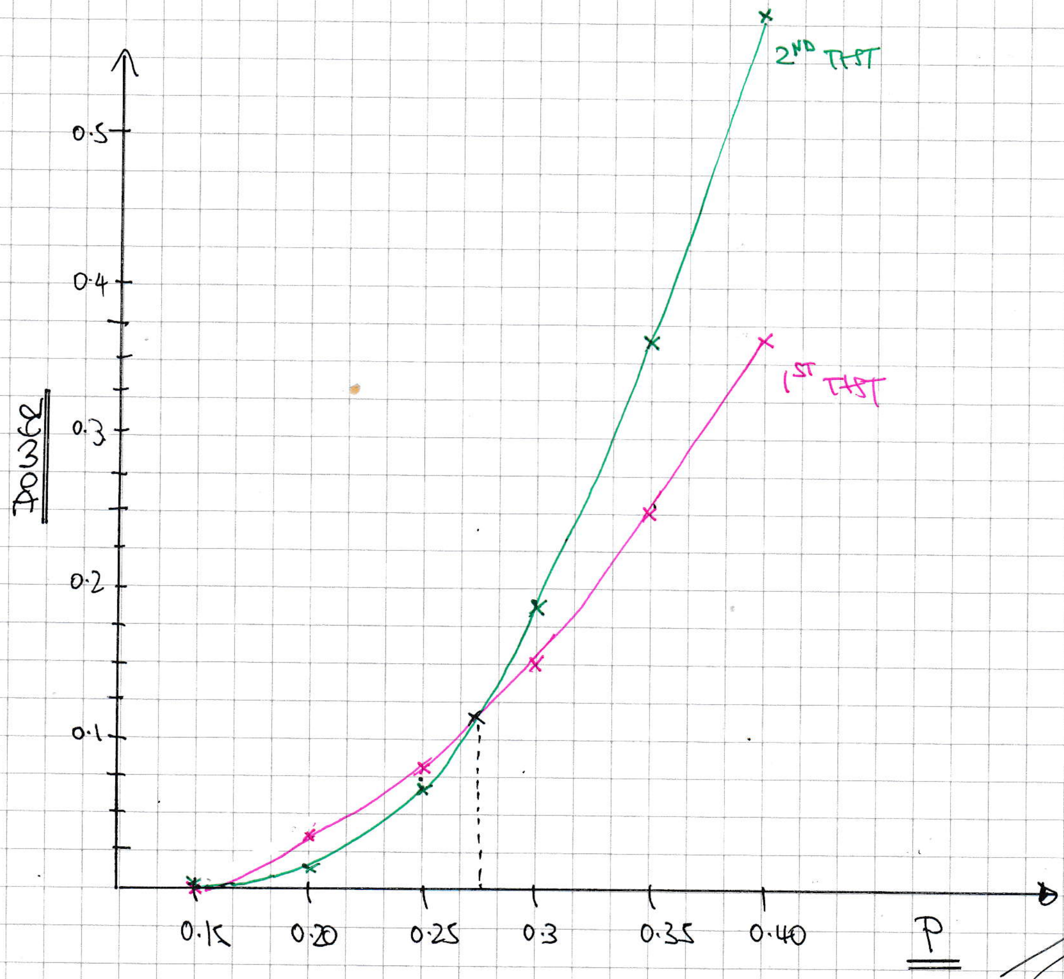
= tables

$= 1 - 0.9999$

$= 0.0001$

IYGB – FS1 PAPER Q – QUESTION 6

d)



e) p WHEN GRAPHS INTERSECT IS APPROX 0.27

IF p IS SMALLER THAN 0.27 1ST TEST IS BETTER, WHILE IF p IS GREATER THAN 0.27, THE SECOND TEST IS BETTER (SMALLER ERRORS)

IYGB - FS1 PAPER Q - QUESTION 7

a) MODEL AS THE X = NUMBER OF SUCCESSES, WITH GEOMETRIC

$$X \sim Geo(0.4) \implies P(X \leqslant 7) = 1 - P(X \geqslant 8)$$

"7 FAILURES"

$$= 1 - 0.6^7$$

$$= \underline{0.9720}$$

b I) JUST A SIMPLE BINOMIAL, X = NUMBER OF SUCCESSES

$$X \sim B(7, 0.4) \implies P(X=3) = \binom{7}{3}(0.4)^3(0.6)^4 = \underline{0.2903}$$

II) SET UP A NEGATIVE BINOMIAL, X = NUMBER OF SUCCESSES AGAIN

$$X \sim NB(3, 0.4) \implies P(X=7) = \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4$$

$$= \underline{0.1244}$$

III) FROM PART b II, $X \sim NB(3, 0.4)$

$$P(X \leqslant 7) = P(X = 3, 4, 5, 6, 7)$$

$$= (0.4)^3 + \binom{3}{2}(0.4)^2 \times 0.6 \times 0.4 + \binom{4}{2}(0.4)^2(0.6)^2 \times 0.4$$

$$+ \binom{5}{2}(0.4)^2(0.6)^3 \times 0.4 + \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4$$

$$= (0.4)^3\left[1 + 3 \times 0.6 + 6 \times 0.6^2 + 10 \times 0.6^3 + 15 \times 0.6^4\right]$$

$$= 0.064 \times 9.064$$

$$= \underline{0.5801}$$