IYGB GCE

Mathematics FP4

Advanced Level

Practice Paper R

Difficulty Rating: 3.4533/1.5707

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A sequence of integers is defined recursively by the relation

$$a_{n+1} = a_n - 4 \,,\ a_1 = 3 \,,\ n = 1, 2, 3, \ldots$$

Prove by induction that its n^{th} term is given by

$$a_n = 7 - 4n \,,\ n = 1, 2, 3, \ldots$$ (5)

Question 2

There are 18 people available for selection.

It is required that these people should be grouped into 4 teams, one team of 6 people, one team of 5 people, one team of 4 people and one team of 3 people.

Calculate the number of ways that these teams could be selected. (5)

Question 3

The integral I_n is defined for $n \geq 0$ as

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx \,,\ n \in \mathbb{N} \,.$$

Show clearly that …

a) … $I_n = \dfrac{1}{n-1} - I_{n-2} \,,\ n \geq 1$. (5)

b) … $I_4 = \dfrac{1}{12}(3\pi - 8)$. (5)

Question 4

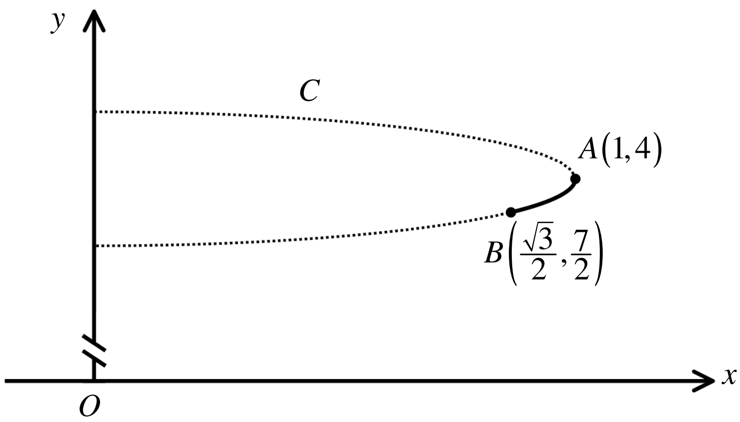
The binary operation $\otimes$ is defines over the real numbers by

$$p \otimes q \equiv p + q + pq$$

a) State the identity element of the above binary operation and hence find the inverse of 4. **(4)**

b) Use algebra to determine whether the above binary operation is associative. **(5)**

Question 5



The curve C, shown above, is given parametrically by the equations

$$x = \operatorname{sech} t, \quad y = 4 - \tanh t, \quad t \in \mathbb{R}$$

a) Show that the length of the arc of C from $A(1,4)$ to $B\left(\frac{\sqrt{3}}{2}, \frac{7}{2}\right)$ is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \ \ dx .$$ **(5)**

b) Use the substitution $u = e^t$, to find the exact value of s. **(6)**

Question 6

a) Find a Cartesian equation for each of the following loci defined in the complex plane by the following relationships.

i. $\left|\dfrac{z-2}{z+1-\mathrm{i}}\right| = 1$. **(4)**

ii. $\mathrm{Re}\left(\dfrac{1}{\bar{z}}\right) = \dfrac{1}{2}$. **(5)**

b) Shade in a standard Argand diagram the region defined by

$$\left|\frac{z-2}{z+1-\mathrm{i}}\right| \geq 1 \ \cap \ \mathrm{Re}\left(\frac{1}{\bar{z}}\right) \leq \frac{1}{2}.$$ **(3)**

Question 7

Let $a \in \mathbb{N}$ with $\frac{1}{5}a \notin \mathbb{N}$.

a) Show that the remainder of the division of a^2 by 5 is either 1 or 4. **(5)**

b) Given further that $b \in \mathbb{N}$ with $\frac{1}{5}b \notin \mathbb{N}$, deduce that $\frac{1}{5}\left(a^4 - b^4\right) \in \mathbb{N}$. **(6)**

Question 8

The numbers z and w satisfy the relationship

$$w = \frac{z+9\mathrm{i}}{1+\mathrm{i}z}, \quad z \neq \mathrm{i}.$$

a) Given that $w \in \mathbb{R}$, find the possible values of z. **(4)**

b) Given instead that $z \in \mathbb{R}$, find a Cartesian equation of the locus of the point represented by w, in an Argand diagram. **(8)**
