IYGB – FP2 PAPER U – QUESTION 1

a) $\boxed{f'(x) - 5f(x) = 3g(x) \quad \& \quad g'(x) + 4g(x) = -6f(x)}$

DIFFERENTIATE THE FIRST O.D.E. WITH RESPECT TO x

$\Rightarrow f''(x) - 5f'(x) = 3g'(x)$

$g'(x) = -4g(x) - 6f(x)$

$\Rightarrow f''(x) - 5f'(x) = 3[-4g(x) - 6f(x)]$

$\Rightarrow f''(x) - 5f'(x) = -12g(x) - 18f(x)$

$\Rightarrow f''(x) - 5f'(x) = -4[3g(x)] - 18f(x)$

$f'(x) - 5f(x) = 3g(x)$

$\Rightarrow f''(x) - 5f'(x) = -4[f'(x) - 5f(x)] - 18f(x)$

$\Rightarrow f''(x) - 5f'(x) = -4f'(x) + 20f(x) - 18f(x)$

$\Rightarrow f''(x) - f'(x) - 2f(x) = 0$

b) SOLVING THE 2ND ORDER O.D.E (AUXILIARY EQUATION ETC)

$\lambda^2 - \lambda - 2 = 0$

$(\lambda + 1)(\lambda - 2) = 0$

$\lambda = \begin{cases} -1 \\ 2 \end{cases}$

$\therefore$ GENERAL SOLUTION

$$f(x) = Ae^{2x} + Be^{-x}$$

DIFFERENTIATING TO OBTAIN $g(x)$

$$f'(x) = 2Ae^{2x} - Be^{-x}$$

IYGB - FP2 PAPER U - QUESTION 1

$$\Rightarrow 3g(x) = f'(x) - 5f(x)$$
$$\Rightarrow 3g(x) = 2Ae^{2x} - Be^{-x} - 5\left[Ae^{2x} + Be^{-x}\right]$$
$$\Rightarrow 3g(x) = 2Ae^{2x} - Be^{-x} - 5Ae^{2x} - 5Be^{-x}$$
$$\Rightarrow 3g(x) = -3Ae^{2x} - 6Be^{-x}$$
$$\Rightarrow g(x) = -Ae^{2x} - 2Be^{-x}$$

APPLY CONDITIONS $f(0) = 1$ $\quad g(0) = 3$

$f(x) = Ae^{2x} + Be^{-x}$

$1 = A + B$

$g(x) = -Ae^{2x} - 2Be^{-x}$

$3 = -A - 2B$

$A + B = -3 - B$

$-3 - B = 1$

$\underline{-4 = B}$

&

$A + B = 1$

$A - 4 = 1$

$\underline{A = 5}$

FINALLY WE HAVE

$$f(x) = 5e^{2x} - 4e^{-x} \quad \& \quad g(x) = 8e^{-x} - 5e^{2x}$$

IYGB - FP2 PAPER U - QUESTION 2

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1-\sqrt{\arcsin x}}{\sqrt{1-x^2}\,\arcsin x}\,dx = \ldots \quad \text{BY SUBSTITUTION}$$

$u = \sqrt{\arcsin x}$

$u^2 = \arcsin x$

$2u \dfrac{du}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$dx = 2u\sqrt{1-x^2}\,du$

$x = \frac{1}{2} \longmapsto u = \sqrt{\frac{\pi}{6}}$

$x = \frac{\sqrt{3}}{2} \longmapsto u = \sqrt{\frac{\pi}{3}}$

BY TRANSFORMING THE INTEGRAL

$$= \int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{3}}} \frac{1-u}{\cancel{\sqrt{1-x^2}}\,\cancel{u^2}}\left(\cancel{2u}\,\cancel{\sqrt{1-x^2}}\,du\right)$$

$$= \int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{3}}} \frac{2-2u}{u}\,du = \int_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{3}}} \frac{2}{u} - 2\,du$$

$$= \Big[2\ln|u| - 2u\Big]_{\sqrt{\frac{\pi}{6}}}^{\sqrt{\frac{\pi}{3}}} = \left(2\ln\sqrt{\tfrac{\pi}{3}} - 2\sqrt{\tfrac{\pi}{3}}\right) - \left(2\ln\sqrt{\tfrac{\pi}{6}} - 2\sqrt{\tfrac{\pi}{6}}\right)$$

$$= \ln\frac{\pi}{3} - 2\sqrt{\frac{\pi}{3}} - \ln\frac{\pi}{6} + 2\sqrt{\frac{\pi}{6}}$$

$$= \left(\ln\frac{\pi}{3} + \ln\frac{6}{\pi}\right) - 2\left[\sqrt{\frac{\pi}{3}} + \sqrt{\frac{\pi}{6}}\right]$$

$$= \ln 2 - 2\left(\sqrt{\frac{\pi}{3}} - \sqrt{\frac{\pi}{6}}\right)$$

IYGB - FP2 PAPER V - QUESTION 3

WRITE FIRSTLY IN SIGMA NOTATION

$$\underbrace{\frac{1}{1\times3}-\frac{2}{3\times5}+\frac{3}{5\times7}-\frac{4}{7\times9}+\frac{5}{9\times11}-\dots}_{n \text{ terms}} = \sum_{r=1}^{n}\frac{(-1)^{r+1}\times r}{(2r-1)(2r+1)}$$

IGNORING THE $(-1)^{r+1}$ TERM IN THE SUMMAND, OBTAIN THE PARTIAL FRACTIONS

$$\frac{r}{(2r-1)(2r+1)} = \frac{\frac{1/2}{2}}{2r-1}+\frac{\frac{-1/2}{-2}}{2r+1} = \frac{\frac{1}{4}}{2r-1}+\frac{\frac{1}{4}}{2r+1}$$

$$\frac{4r}{(2r-1)(2r+1)} \equiv \frac{1}{2r-1}+\frac{1}{2r+1}$$

NOW WE HAVE FOR DIFFERENT VALUES OF r

$r=1$	$\frac{4}{1\times3}=\frac{1}{1}+\frac{1}{3}$	$\frac{4}{1\times3}=\frac{1}{1}+\cancel{\frac{1}{3}}$
$r=2$	$\frac{8}{3\times5}=\frac{1}{3}+\frac{1}{5}$	$-\frac{8}{3\times5}=-\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}$
$r=3$	$\frac{12}{5\times7}=\frac{1}{5}+\frac{1}{7}$	$\frac{12}{5\times7}=\cancel{\frac{1}{5}}+\cancel{\frac{1}{7}}$
$r=4$	$\frac{16}{7\times9}=\frac{1}{7}+\frac{1}{9}$	$-\frac{16}{7\times9}=-\cancel{\frac{1}{7}}-\cancel{\frac{1}{9}}$
$\vdots$	$\vdots$	
$r=n$	$\frac{4n}{(2n-1)(2n+1)}=\frac{1}{2n-1}+\frac{1}{2n+1}$	$(-1)^{n+1}\frac{4n}{(2n-1)(2n+1)}=\left(\cancel{\frac{1}{2n-1}}+\frac{1}{2n+1}\right)(-1)^{n+1}$

ADDING BOTH SIDES

$$\Rightarrow \sum_{r=1}^{n}\left[\frac{(-1)^{r+1}(4r)}{(2r-1)(2r+1)}\right] = 1+\frac{(-1)^{n+1}}{2n+1}$$

$$\Rightarrow 4\sum_{r=1}^{n}\frac{r(-1)^{r+1}}{(2r-1)(2r+1)} = 1-\frac{(-1)^{n}}{2n+1}$$

$$\Rightarrow \sum_{r=1}^{n}\frac{r(-1)^{r+1}}{(2r-1)(2r+1)} = \frac{1}{4}-\frac{(-1)^{n}}{4(2n+1)}$$

+ −

IYGB − FP2 PAPER U − QUESTION 4

a) START WITH $w = e^{\frac{2\pi}{5}i}$

$$w^5 = \left(e^{\frac{2\pi}{5}i}\right)^5 = e^{2\pi i} = \cos 2\pi + \cancel{i\sin 2\pi} = 1$$

NOW $1+w+w^2+w^3+w^4$ IS A GEOMETRIC SERIES, $a=1$, $r=w$

$$\Rightarrow S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_5 = \frac{1(1-w^5)}{1-w}$$

$$\Rightarrow 1+w+w^2+w^3+w^4 = \frac{1(1-1)}{1-e^{\frac{2\pi}{5}i}} = 0$$

ALTERNATIVE

IF $w^5=1$

$w^5-1=0$

$(w-1)(w^4+w^3+w^2+w+1)=0$

BUT $w \neq 1$

$\therefore w^4+w^3+w^2+w+1=0$

b) $\left[z-(w+w^4)\right]\left[z-(w^2+w^3)\right]=0$

$$\Rightarrow z^2 - (w+w^2+w^3+w^4)z + (w+w^4)(w^2+w^3) = 0$$

$$\Rightarrow z^2 - (-1)z + (w^3+w^4+w^6+w^7) = 0$$

AS $1+w+w^2+w^3+w^4=0$

$w+w^2+w^3+w^4=-1$

$w^6 = e^{\frac{12\pi i}{5}} = e^{\frac{2\pi i}{5}} = w$

$w^7 = e^{\frac{14\pi i}{5}} = e^{\frac{4\pi i}{5}} = w^2$

$$\Rightarrow z^2+z+(w+w^2+w^3+w^4)=0$$

$$\Rightarrow z^2+z-1=0$$

IYGB - FP2 PAPER U - QUESTION 4

SOLVING THE QUADRATIC IN Z WHOSE SOLUTIONS ARE $w+w^4$, w^2+w^3

$$z = \frac{-1 \pm \sqrt{5}}{2}$$

NOW WE HAVE

$$w+w^4 = e^{\frac{2\pi}{5}i} + e^{\frac{8\pi}{5}i} = e^{\frac{2\pi}{5}i} + e^{-\frac{2\pi}{5}i} = 2\cosh\left(\tfrac{2\pi i}{5}\right) = 2\cos\tfrac{2\pi}{5}$$

$$w^2+w^3 = e^{\frac{4\pi}{5}i} + e^{\frac{6\pi}{5}i} = e^{\frac{4\pi}{5}i} + e^{-\frac{4\pi}{5}i} = 2\cosh\left(\tfrac{4\pi}{5}i\right) = 2\cos\tfrac{4\pi}{5}$$

FINALLY TO MATCH THEM CORRECTLY

$\cos\frac{2\pi}{5}$ IS ACUTE SO POSITIVE

$2\cos\frac{4\pi}{5}$ IS OBTUSE SO NEGATIVE

$$\therefore \quad 2\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{2} \qquad\qquad 2\cos\frac{4\pi}{5} = \frac{-1-\sqrt{5}}{2}$$

$$\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4} \qquad\qquad \cos\frac{4\pi}{5} = \frac{-1-\sqrt{5}}{4}$$

IYGB - FP2 PAPER U - QUESTION 5

- START BY DIRECT DIFFERENTIATION — NOTE THE FUNCTION IS EVEN SO WE NEED DERIVATIVES UP TO x^6

$$\Rightarrow y = \ln(1+\cosh x)$$

$$\Rightarrow \frac{dy}{dy} = \frac{\sinh x}{1+\cosh x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(1+\cosh x)\cosh x - \sinh x(\sinh x)}{(1+\cosh x)^2} = \frac{\cosh x + \cosh^2 x - \sinh^2 x}{(1+\cosh x)^2}$$

$$= \frac{\cosh x + 1}{(1+\cosh x)^2} = \frac{1}{1+\cosh x}$$

- OBTAINING 4 MORE DERIVATIVES DIRECTLY IS DIFFICULT TO WE MAY PROCEED AS FOLLOWS

$$y = \ln(1+\cosh x) = -\ln\left(\frac{1}{1+\cosh x}\right) = -\ln\left(\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow -y = \ln\left(\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow e^{-y} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = e^{-y}}$$

- CONTINUE THE DIFFERENTIATIONS W.R.T x

$$\Rightarrow \frac{d^3y}{dx^3} = -e^{-y}\frac{dy}{dx}$$

$$\Rightarrow \frac{d^4y}{dx^4} = e^{-y}\left(\frac{dy}{dx}\right)^2 - e^{-y}\frac{d^2y}{dx^2} = e^{-y}\left(\frac{dy}{dx}\right)^2 - e^{-2y}$$

$$\Rightarrow \frac{d^5y}{dx^5} = -e^{-y}\left(\frac{dy}{dx}\right)^3 + 2e^{-y}\frac{dy}{dx}\frac{d^2y}{dx^2} + 2e^{-2y}\frac{dy}{dx}$$

IYGB - FP2 PAPER U - QUESTION 5

- TIDY BEFORE THE FINAL DIFFERENTIATION

$$\Rightarrow \frac{d^5y}{dx^5} = -e^{-y}\left(\frac{dy}{dx}\right)^3 + 2e^{-2y}\left(\frac{dy}{dx}\right) + 2e^{-2y}\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{d^5y}{dx^5} = 4e^{-2y}\frac{dy}{dx} - e^{-y}\left(\frac{dy}{dx}\right)^3$$

$$\Rightarrow \frac{d^6y}{dx^6} = -8e^{-2y}\left(\frac{dy}{dx}\right)^2 + 4e^{-2y}\frac{d^2y}{dx^2} + e^{-y}\left(\frac{dy}{dx}\right)^4 - 3e^{-y}\left(\frac{dy}{dx}\right)^2\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^6y}{dx^6} = -8e^{-2y}\left(\frac{dy}{dx}\right)^2 + 4e^{-3y} + e^{-y}\left(\frac{dy}{dx}\right)^4 - 3e^{-2y}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{d^6y}{dx^6} = 4e^{-3y} + e^{-y}\left(\frac{dy}{dx}\right)^4 - 11e^{-2y}\left(\frac{dy}{dx}\right)^2$$

- EVALUATING THESE DERIVATIVES AT $x=0$

$$y_0 = \ln 2, \quad \left.\frac{dy}{dx}\right|_{x=0} = 0, \quad \left.\frac{d^2y}{dx^2}\right|_{x=0} = e^{-\ln 2} = \frac{1}{2}$$

$$, \quad \left.\frac{d^3y}{dx^3}\right|_{x=0} = 0, \quad \left.\frac{d^4y}{dx^4}\right|_{x=0} = -e^{-2\ln 2} = -\frac{1}{4}$$

$$\left.\frac{d^5y}{dx^5}\right|_{x=0} = 0, \quad \left.\frac{d^6y}{dx^6}\right|_{x=0} = 4e^{-3\ln 2} = \frac{1}{2}$$

- HENCE WE CAN OBTAIN THE McLORIN, IGNORING ODD TERMS

$$y = y_0 + \frac{x^2}{2!}y_0'' + \frac{x^4}{4!}y_0'''' + \frac{x^6}{6!}y_0''''' + O(x^8)$$

$$y = \ln 2 + \frac{1}{4}x^2 + \left(\frac{-1}{4}\right)\left(\frac{1}{24}\right)x^4 + \frac{1}{2}\times\frac{1}{720}x^6 + O(x^8)$$

$$y = \ln 2 + \frac{1}{4}x^2 - \frac{1}{96}x^4 + \frac{1}{1440}x^6 + O(x^8)$$

IYGB - FP2 PAPER U - QUESTION 6

PROCEED AS FOLLOWS

$$\Rightarrow y = \ln\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$\Rightarrow e^y = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\Rightarrow e^y = \frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$\Rightarrow e^y = \frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}$$

$$\Rightarrow e^y = \frac{1+T}{1-T} \quad \text{WHERE } T = \tan\frac{x}{2}$$

MAKE T THE SUBJECT

$$\Rightarrow e^y - Te^y = 1 + T$$

$$\Rightarrow e^y - 1 = T + Te^y$$

$$\Rightarrow e^y - 1 = T(1 + e^y)$$

$$\Rightarrow T = \frac{e^y - 1}{e^y + 1}$$

NOW USING $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$$\Rightarrow \tan x = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}}$$

$$\Rightarrow \tan x = \frac{2T}{1 - T^2}$$

$$\Rightarrow \tan x = \frac{2\left(\frac{e^y - 1}{e^y + 1}\right)}{1 - \left(\frac{e^y - 1}{e^y + 1}\right)^2}$$

MULTIPLY TOP AND BOTTOM OF THE FRACTION BY $(e^y+1)^2$ YIELDS

$$\Rightarrow \tan x = \frac{2(e^y - 1)(e^y + 1)}{(e^y + 1)^2 - (e^y - 1)^2}$$

$$\Rightarrow \tan x = \frac{2(e^{2y} - 1)}{\cancel{e^{2y}} + 2e^y \cancel{+1} - \cancel{e^{2y}} + 2e^y \cancel{-1}}$$

$$\Rightarrow \tan x = \frac{2e^{2y} - 2}{4e^y}$$

$$\Rightarrow \tan x = \frac{1}{2}e^y - \frac{1}{2}e^{-y}$$

$$\Rightarrow \tan x = \sinh y$$

FINALLY $\cosh^2 y - \sinh^2 = 1$

$$\Rightarrow \cosh y = +\sqrt{1 + \sinh^2 y}$$

$$\Rightarrow \cosh y = \sqrt{1 + \tan^2 x}$$

$$\Rightarrow \cosh y = \sec x$$

BUT $\cosh y + \sinh y = e^y$

$$\Rightarrow \sinh y + \cosh y = \tan x + \sec x$$

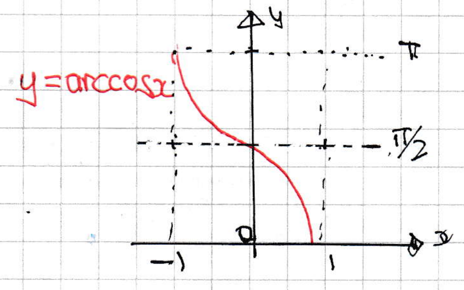
$$\Rightarrow e^y = \tan x + \sec x$$

IYGB - FP2 PAPER W - QUESTION 7

a) FOLLOWING THE SUGGESTION GIVEN

$$\Rightarrow y = \arccos x$$
$$\Rightarrow \cos y = x$$
$$\Rightarrow x = \cos y$$
$$\Rightarrow \frac{dx}{dy} = -\sin y$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{-\sin y}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{-(\pm\sqrt{1-\cos^2 y})}$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{\pm\sqrt{1-\cos^2 y}}$$
$$\Rightarrow \frac{dy}{dx} = \pm\frac{1}{\sqrt{1-x^2}}$$

BUT $y = \arccos x$ IS A STRICTLY DECREASING FUNCTION



$$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

AS REQUIRED

b) REWRITE AND USE THE CHAIN RULE & QUOTIENT RULE

$$\Rightarrow y = \arccos(1-x^2)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{1-(1-x^2)^2}} \times (-2x) = \frac{2x}{\sqrt{1-1+2x^2-x^4}} = \frac{2x}{\sqrt{2x^2-x^4}}$$

NOW AS x IS POSITIVE WE MAY TAKE IT OUT OF THE RADICAL WITHOUT THE USE OF MODULUS SIGN

$$\therefore \frac{dy}{dx} = \frac{2x}{x(2-x^2)^{1/2}}$$

$$\frac{dy}{dx} = \frac{2}{(2-x^2)^{1/2}}$$

IYGB - FP2 PAPER D - QUESTION 7

CONTINUE WITH THIS "FAKE QUOTIENT" (CAN BE REWRITTEN) AND USE THE CHAIN RULE INSTEAD

$$\frac{dy}{dx}=\frac{2}{(2-x^2)^{1/2}} \implies \frac{d^2y}{dx^2}=\frac{\cancel{(2-x^2)^{1/2}\times 0}-2\times\frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)}{\left[(2-x^2)^{1/2}\right]^2}$$

$$\implies \frac{d^2y}{dx^2}=\frac{2x(2-x^2)^{-\frac{1}{2}}}{(2-x^2)^1}$$

$$\implies \frac{d^2y}{dx^2}=\frac{2x}{(2-x^2)^{3/2}}$$

AS REQUIRED

c) DIFFERENTIATE AFTER REWRITING SINCE $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ ARE RELATED IN STRUCTURE

$$\frac{d^2y}{dx^2}=\frac{2x}{(2-x^2)^{3/2}}=\frac{2}{(2-x^2)^{1/2}}\times\frac{x}{2-x^2}=\frac{dy}{dx}\left(\frac{x}{2-x^2}\right)$$

$$\implies \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right]=\frac{d}{dx}\left[\underbrace{\frac{dy}{dx}\cdot\frac{x}{2-x^2}}\right]$$

PRODUCT WITH A QUOTIENT AS ONE OF ITS "FACTORS"

$$\implies \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}\times\frac{x}{2-x^2}+\frac{dy}{dx}\times\frac{(2-x^2)\times 1-x(-2x)}{(2-x^2)^2}$$

$$\implies \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}\times\frac{1}{4}x\left(\frac{4}{2-x^2}\right)+\frac{dy}{dx}\times\frac{2-x^2+2x^2}{(2-x^2)^2}$$

$$\implies \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}\times\frac{1}{4}x\left[\frac{2}{(2-x^2)^{1/2}}\right]^2+\frac{dy}{dx}\times\frac{2+x^2}{(2-x^2)^2}$$

$$\implies \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}\times\frac{1}{4}x\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}\times(2+x^2)\times\frac{1}{16}\times\frac{16}{(2-x^2)^2}$$

$$\implies \frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}\times\frac{1}{4}x\left(\frac{dy}{dx}\right)^2+\frac{1}{16}(2+x^2)\frac{dy}{dx}\left[\frac{2}{(2-x^2)^{1/2}}\right]^4$$

$$\implies \frac{d^3y}{dx^3}=\frac{1}{4}x\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^2+\frac{1}{16}(2+x^2)\frac{dy}{dx}\left(\frac{dy}{dx}\right)^4$$

$$\implies 16\frac{d^3y}{dx^3}=4x\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^2+(2+x^2)\left(\frac{dy}{dx}\right)^5$$

AS REQUIRED

IYGB - FP2 PAPER U - QUESTION 7

ALTERNATIVE APPROACH FOR PART (C)

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2x}{(2-x^2)^{3/2}}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{(2-x^2)^{3/2} \times 2 - 2x \times \frac{3}{2}(2-x^2)^{1/2}(-2x)}{\left[(2-x^2)^{3/2}\right]^2} = \frac{2(2-x^2)^{3/2} + 6x^2(2-x^2)^{1/2}}{(2-x^2)^3}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{2(2-x^2)^{1/2}\left[(2-x^2) + 3x^2\right]}{(2-x^2)^3}$$

$$\Rightarrow \frac{d^3y}{dx^3} = \frac{2(2+2x^2)}{(2-x^2)^{5/2}} = \frac{4(x^2+1)}{(2-x^2)^{5/2}}$$

$$\therefore 16\frac{d^3y}{dx^3} = 64(x^2+1) \times \frac{1}{(2-x^2)^{5/2}}$$

NOW BY VERIFICATION OF THE R.H.S NOTING THAT $16\frac{d^3y}{dx^3} = \frac{64(x^2+1)}{(2-x^2)^{5/2}}$

$$\Rightarrow 16\frac{d^3y}{dx^3} = 4x\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^2 + (2+x^2)\left(\frac{dy}{dx}\right)^5$$

$$\Rightarrow 16\frac{d^3y}{dx^3} = 4x\left[\frac{2x}{(2-x^2)^{3/2}}\right]\left[\frac{2}{(2-x^2)^{1/2}}\right]^2 + (2+x^2)\left[\frac{2}{(2-x^2)^{1/2}}\right]^5$$

$$\Rightarrow 16\frac{d^3y}{dx^3} = \frac{32x^2}{(2-x^2)^{5/2}} + (2+x^2)\frac{32}{(2-x^2)^{5/2}}$$

$$\Rightarrow 16\frac{d^3y}{dx^3} = \frac{32x^2}{(2-x^2)^{5/2}} + \frac{64}{(2-x^2)^{5/2}} + \frac{32x^2}{(2-x^2)^{5/2}}$$

$$\Rightarrow 16\frac{d^3y}{dx^3} = \frac{64x^2}{(2-x^2)^{5/2}} + \frac{64}{(2-x^2)^{5/2}}$$

$$\Rightarrow 16\frac{d^3y}{dx^3} = \frac{64(x^2+1)}{(2-x^2)^{5/2}}$$

AND THE RESULT IS VERIFIED