IYGB - FP1 PAPER U - QUESTION 1

START BY FORMING A DOT PRODUCT

$$\underline{a} = (\sin\theta, 2\cos 2\theta, \sin\theta)$$
$$\underline{b} = (3, -1, -1)$$

$$\Longrightarrow \underline{a} \cdot \underline{b} = 0$$
$$\Longrightarrow (\sin\theta, 2\cos 2\theta, \sin\theta) \cdot (3, -1, -1) = 0$$
$$\Longrightarrow 3\sin\theta - 2\cos 2\theta - \sin\theta = 0$$
$$\Longrightarrow 2\sin\theta - 2\cos 2\theta = 0$$
$$\Longrightarrow \sin\theta - \cos 2\theta = 0$$

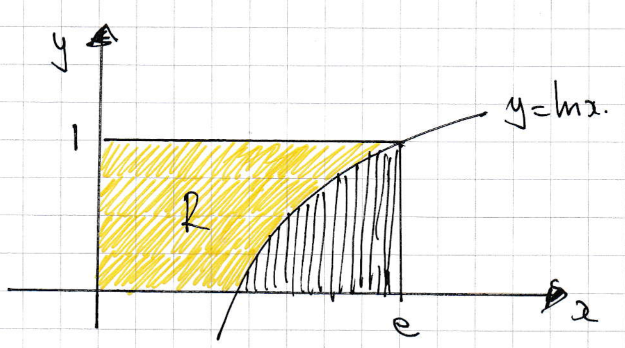
BY TRIGONOMETRIC IDENTITIES

$$\Longrightarrow \sin\theta - (1 - 2\sin^2\theta) = 0$$
$$\Longrightarrow \sin\theta - 1 + 2\sin^2\theta = 0$$
$$\Longrightarrow 2\sin^2\theta + \sin\theta - 1 = 0$$
$$\Longrightarrow (2\sin\theta - 1)(\sin\theta + 1) = 0$$
$$\Longrightarrow \sin\theta = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$
$$\Longrightarrow \theta = \begin{cases} \begin{cases} \frac{\pi}{6} \pm 2n\pi \\ \frac{5\pi}{6} \pm 2n\pi \end{cases} \\ \begin{cases} -\frac{\pi}{2} \pm 2n\pi \\ \frac{3\pi}{2} \pm 2n\pi \end{cases} \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

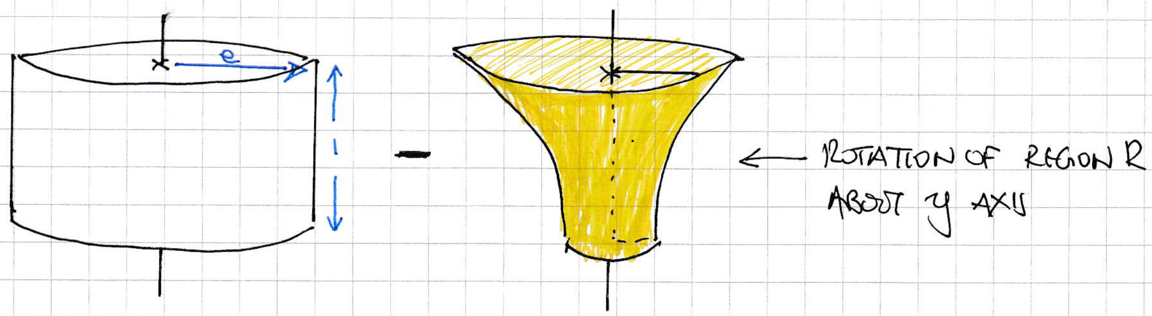
IYGB - FP1 PAPER U - QUESTION 2

LOOKING AT THE DIAGRAM



$y = \ln x$
$e^y = x$
$x^2 = e^{2y}$

REQUIRED VOLUME IS GIVEN BY



VOLUME = "$\pi r^2 h$" $= \pi \times e^2 \times 1$
$= \pi e^2$

$$V = \pi \int_{y_1}^{y_2} [x(y)]^2 \, dy$$

$$V = \pi \int_0^1 e^{2y} \, dy$$

$$V = \pi \left[\tfrac{1}{2} e^{2y} \right]_0^1$$

$$V = \pi \left[\tfrac{1}{2} e^2 - \tfrac{1}{2} \right]$$

HENCE WE FINALLY HAVE

$$V = \pi e^2 - \pi \left[\tfrac{1}{2} e^2 - \tfrac{1}{2} \right] = \pi \left[e^2 - \tfrac{1}{2} e^2 + \tfrac{1}{2} \right]$$

$$= \pi \left[\tfrac{1}{2} e^2 + \tfrac{1}{2} \right]$$

$$= \tfrac{1}{2} \pi \left[e^2 + 1 \right]$$

AS REQUIRED

IYGB - FP1 PAPER U - QUESTION 3

a) WRITE THE EQUATIONS IN PARAMETRIC

$$\frac{x-8}{1} = \frac{y+1}{-1} = \frac{z-2}{1} = \lambda \Rightarrow \underline{r}_1 = (8,-1,2) + \lambda(1,-1,1)$$

$$\frac{x-3}{-1} = \frac{y-4}{1} = \frac{z-1}{1} = \mu \Rightarrow \underline{r}_2 = (3,4,1) + \mu(-1,1,1)$$

$$\underline{r}_1 = (\lambda+8, -\lambda-1, \lambda+2)$$

$$\underline{r}_2 = (-\mu+3, \mu+4, \mu+1)$$

EQUATE $\underline{j}$ & $\underline{k}$ (EQUATING $\underline{i}$ & $\underline{j}$ YIELD NOTHING)

$$\left.\begin{array}{l} \underline{j}: \ -\lambda-1 = \mu+4 \\ \underline{k}: \ \lambda+2 = \mu+1 \end{array}\right\} \text{ ADDING } \Rightarrow 1 = 2\mu+5$$

$$\Rightarrow -4 = 2\mu$$

$$\underline{\mu = -2} \quad \& \quad \underline{\lambda = -3}$$

CHECK $\underline{i}$

$$\lambda+8 = -3+8 = 5$$

$$-\mu+3 = -(-2)+3 = 5$$

INDEED THE LINES INTERSECT AT

$P(5,2,-1)$

b) DOTTING THE DIRECTION VECTORS

$$(1,-1,1)\cdot(-1,1,1) = |1,-1,1|\,|-1,1,1|\cos\theta$$

$$-1-1+1 = \sqrt{1+1+1}\sqrt{1+1+1}\cos\theta$$

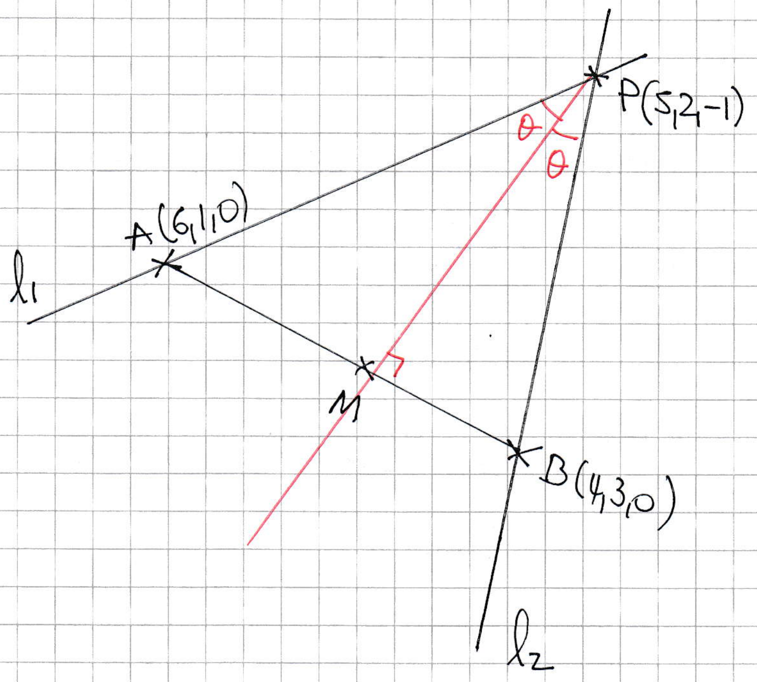
$$-1 = \sqrt{3}\sqrt{3}\cos\theta$$

$$\cos\theta = -\frac{1}{3}$$

$\therefore$ ACUTE ANGLE WILL HAVE $\cos\theta = \frac{1}{3}$

IYGB - FP1 PAPER U - QUESTION 3

c) LOOKING AT THE DIAGRAM BELOW



$$|\overrightarrow{AP}| = |\underline{p}-\underline{a}| = |(5,2,-1)-(6,1,0)| = |-1,1,-1| = \sqrt{3}$$

$$|\overrightarrow{BP}| = |\underline{p}-\underline{b}| = |(5,2,-1)-(4,3,0)| = |1,-1,-1| = \sqrt{3}$$

HENCE $\overset{\triangle}{APB}$ IS ISOSCELES SO THE PERPENDICULAR BISECTOR OF $\hat{APB}$ MUST GO THROUGH THE MIDPOINT OF AB

$$M\left(\frac{6+4}{2}, \frac{3+1}{2}, \frac{0+0}{2}\right) \Longrightarrow M(5,2,0)$$

$$\overrightarrow{MP} = \underline{p}-\underline{m} = (5,2,-1)-(5,2,0) = (0,0,-1)$$

IE MULTIPLE OF $\underline{k}$

$\therefore$ PARALLEL TO $\underline{k}$

IYGB - FP1 PAPER U - QUESTION 4

USING THE LINEARITY OF THE SIGMA OPERATOR

$$\sum_{r=k}^{n}\left[\lambda f(r) + \mu g(r)\right] \equiv \lambda \sum_{r=k}^{n} f(r) + \mu \sum_{r=k}^{n} g(r)$$

MANIPULATING AS FOLLOWS

$$\sum_{r=1}^{20}(r-10) = 200$$

$$\sum_{r=1}^{20} r - 10\sum_{r=1}^{20} 1 = 200$$

$$\sum_{r=1}^{20} r - 10 \times 20 = 200$$

$$\sum_{r=1}^{20} r = 400$$

FINALLY USING THE SECOND FACT

$$\sum_{r=1}^{20}(r-10)^2 = 2800$$

$$\sum_{r=1}^{20}\left(r^2 - 20r + 100\right) = 2800$$

$$\sum_{r=1}^{20} r^2 - 20\sum_{r=1}^{20} r + 100\sum_{r=1}^{20} 1 = 2800$$

$$\sum_{r=1}^{20} r^2 - 20 \times 400 + 100 \times 20 = 2800$$

$$\sum_{r=1}^{20} r^2 - 8000 + 2000 = 2800$$

$$\sum_{r=1}^{20} r^2 = 8800$$

IYGB - FP1 PAPER U - QUESTION 5

SUPPOSE THAT THE RESULT HOLDS FOR $n=1$

$$\text{L.H.S} = \sum_{r=1}^{1} \frac{r \times 2^r}{(r+2)!} = \frac{1 \times 2^1}{3!} = \frac{2}{6} = \frac{1}{3}$$

$$\text{R.H.S} = 1 - \frac{2^2}{3!} = 1 - \frac{4}{6} = 1 - \frac{2}{3} = \frac{1}{3}$$

I.E RESULT HOLDS FOR $n=1$

NEXT SUPPOSE THAT THE RESULT HOLD FOR $n=k$, $k \in \mathbb{N}$

$$\sum_{r=1}^{k} \left(\frac{r \times 2^r}{(r+2)!} \right) = 1 - \frac{2^{k+1}}{(k+2)!}$$

$$\sum_{r=1}^{k} \left(\frac{r \times 2^r}{(r+2)!} \right) + \frac{(k+1) \times 2^{k+1}}{(k+3)!} = 1 - \frac{2^{k+1}}{(k+2)!} + \frac{(k+1) \times 2^{k+1}}{(k+3)!}$$

$$\sum_{r=1}^{k+1} \left[\frac{r \times 2^r}{(r+2)!} \right] = 1 - \frac{(k+3) \times 2^{k+1}}{(k+3)(k+2)!} + \frac{(k+1) \times 2^{k+1}}{(k+3)!}$$

$$\sum_{r=1}^{k+1} \left[\frac{r \times 2^r}{(r+2)!} \right] = 1 - \frac{(k+3) \times 2^{k+1}}{(k+3)!} + \frac{(k+1) \times 2^{k+1}}{(k+3)!}$$

$$\sum_{r=1}^{k+1} \left[\frac{r \times 2^r}{(r+2)!} \right] = 1 + \frac{(k+1) \times 2^{k+1} - (k+3) \times 2^{k+1}}{(k+3)!}$$

$$\sum_{r=1}^{k+1} \left[\frac{r \times 2^r}{(r+2)!} \right] = 1 + \frac{-2 \times 2^{k+1}}{k+3}$$

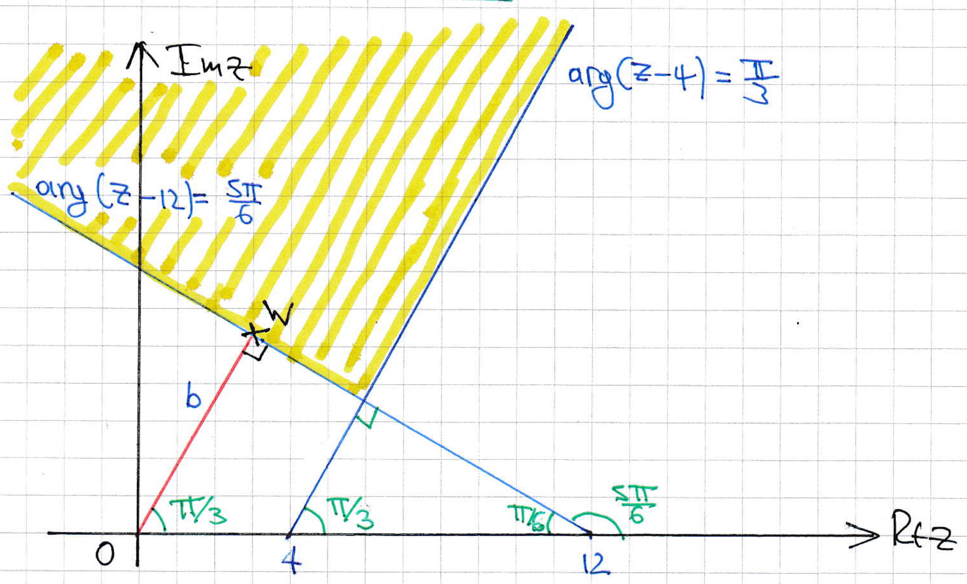
$$\sum_{r=1}^{k+1} \left[\frac{r \times 2^r}{(r+2)!} \right] = 1 - \frac{2^{k+2}}{(k+3)!} = 1 - \frac{2^{(k+1)+1}}{[(k+1)+2]!}$$

IF THE RESULT HOLDS FOR $n=k \in \mathbb{N}$, THEN IT MUST ALSO HOLD FOR $n=k+1$

SINCE THE RESULT HOLDS FOR $n=1$, THEN THE RESULT HOLDS FOR ALL n

IYGB – FP1 PAPER U – QUESTION 6

a) SKETCHING AS APPROPRIATE



b) MARKING THE POSITION OF w IN THE DIAGRAM, SO THAT |w| IS MIN

BY SIMPLE TRIGONOMETRY

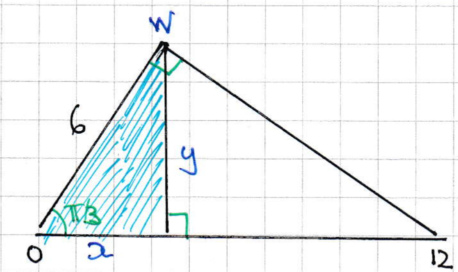
$$\frac{b}{12} = \cos\frac{\pi}{3}$$

$$\frac{b}{12} = \frac{1}{2}$$

$$b = 6$$

$$\therefore \ |w| = 6$$

BY TRIGONOMETRY AGAIN



$$x = 6\cos\frac{\pi}{3} = 3$$

$$y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$$

$$\therefore \ w = 3 + 3\sqrt{3}\,i$$

IYGB – FP1 PAPER U – QUESTION 7

a) NEED THE PLANE NORMAL FIRST

$(2,2,1) \Rightarrow \quad 2c+8-12=k$

$k = 2c-4$

$(6,-7,-1) \Rightarrow \quad 6c-28+12=k$

$k=6c-16$

$2c-4=6c-16$

$12=4c$

$c=3$

&

$(k=2)$

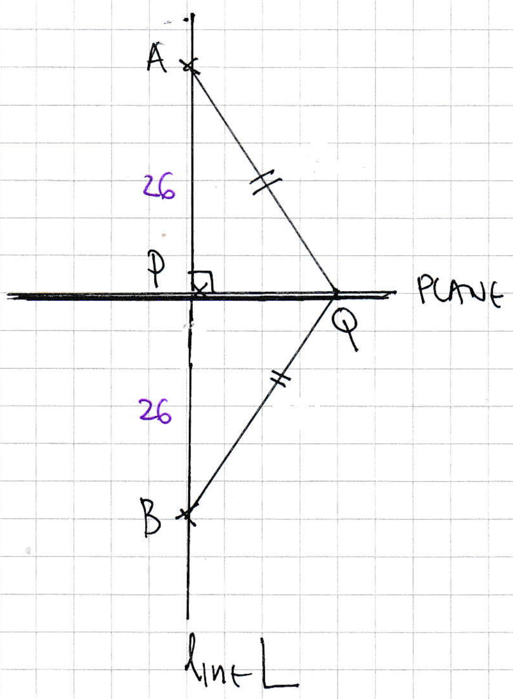
$\therefore \; 3x+4y-12z=2$

$\therefore \; \underline{n} = (3,4,-12)$

$\therefore \; \underline{r} = (2,2,1) + \lambda(3,4,-12)$

$\therefore \; \underline{r} = (2+3\lambda,\, 2+4\lambda,\, 1-12\lambda)$

b) LOOKING AT A DIAGRAM



- $|PQ| = |\underline{q}-\underline{p}|$

$= |(6,-7,-1)-(2,2,1)|$

$= |4,-9,-2|$

$= \sqrt{16+81+4}$

$= \sqrt{101}$

$\simeq 10.0$

- REQUIRED AREA

$\frac{1}{2}|AB||PQ|$

$= \frac{1}{2} \times 2 \times 26\sqrt{101} \simeq 261$

IYGB - FP1 PAPER U - QUESTION 8

BY LONG DIVISION AS $(z-2-i)$ MUST BE A FACTOR

$$
\begin{array}{r|l}
 & z^2+(-2z-iz)+(1+i) \\
\hline
z-2-i & z^3-(4+2i)z^2+(4+5i)z-(1+3i) \\
\hline
 & \cancel{z^3}-4z^2-2iz^2+4z+5iz-1-3i \\
 & -\cancel{z^3}+2z^2+iz^2 \\
\hline
 & -\cancel{2z^2}-\cancel{iz^2}+4z+5iz-1-3i \\
 & +\cancel{2z^2}+\cancel{iz^2}-4z-2iz \\
 & \qquad +z-2iz \\
\hline
 & \cancel{z}+\cancel{iz}\;-\cancel{1}-\cancel{3i} \\
 & -\cancel{z}-\cancel{iz}\;+\cancel{2}+\cancel{2i} \\
 & \qquad -\cancel{1}+\cancel{i} \\
\hline
 & \qquad 0
\end{array}
$$

HENCE WE NOW HAVE

$$(z-2-i)\left[z^2-(2+i)z+(1+i)\right]=0$$

BY THE QUADRATIC FORMULA

$$z=\frac{(2+i)\pm\sqrt{(2+i)^2-4\times1\times(1+i)}}{2\times1}$$

$$z=\frac{2+i\pm\sqrt{4+4i-1-4-4i}}{2}$$

$$z=\frac{2+i\pm\sqrt{-1}}{2}$$

$$z=\begin{cases}\dfrac{2+i+i}{2}=\dfrac{2+2i}{2}=1+i\\[2mm]\dfrac{2+i-i}{2}=1\end{cases}$$

$$\therefore\ z=1,\ 1+i,\ 2+i$$

IYGB - FP1 PAPER U - QUESTION 8

ALTERNATIVE BY CONSIDERING RELATIONSHIP IN ROOTS

$$z^3 - (4+2i)z^2 + (4+5i)z - (1+3i) = 0 \quad , \quad \alpha = 2+i$$

- $\alpha+\beta+\gamma = -\frac{b}{a}$

$2+i+\beta+\gamma = 4+2i$

$\beta+\gamma = 2+i$

- $\alpha\beta\gamma = -\frac{d}{a}$

$(2+i)\beta\gamma = 1+3i$

$\beta\gamma = \frac{1+3i}{2+i}$

$\beta\gamma = \frac{(1+3i)(2-i)}{5} = \frac{2-i+6i+3}{5}$

$\beta\gamma = 1+i$

SOLVING SIMULTANEOUSLY

$\beta+\gamma = 2+i$

$\beta^2+\beta\gamma = \beta(2+i)$

$\beta^2+(1+i) = \beta(2+i)$

$\beta^2-(2+i)\beta+(1+i) = 0$

WHICH IS THE SAME QUADRATIC WE SOLVED IN Z EARLIER

$$\therefore \beta = \begin{cases} 1+i \\ 1 \end{cases}$$

& AS EQUATIONS ARE SYMMETRIC ($\beta+\gamma$ & $\beta\gamma$) $\quad \gamma = \begin{cases} 1 \\ 1+i \end{cases}$

$$\therefore z = 2+i, \ 1+i, \ 1$$

IYGB - FP1 PAPER U - QUESTION 9

- START BY OBTAINING THE STANDARD RELATIONSHIPS FOR THE QUADRATIC

$$x^2 - 3x + 4 = 0 \qquad \alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{1} = 4$$

- START FORMING THE SUM AND PRODUCT OF THE ROOTS OF THE REQUIRED QUADRATIC AS FOLLOWS

$$\boxed{\begin{aligned} A &= \alpha^3 - \beta \\ B &= \beta^3 - \alpha \end{aligned}}$$

- $A + B = (\alpha^3 - \beta) + (\beta^3 - \alpha) = \alpha^3 + \beta^3 - (\alpha + \beta)$

NOW
$$\boxed{\begin{aligned} (\alpha+\beta)^3 &= \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \\ (\alpha+\beta)^3 &= \alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta) \\ \alpha^3 + \beta^3 &= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \end{aligned}}$$

$$\Rightarrow A + B = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) - (\alpha+\beta)$$

$$\Rightarrow A + B = 3^3 - 3\times4\times3 - 3$$

$$\Rightarrow A + B = 27 - 36 - 3$$

$$\Rightarrow A + B = -12$$

- $AB = (\alpha^3 - \beta)(\beta^3 - \alpha) = \alpha^3\beta^3 - \alpha^4 - \beta^4 + \alpha\beta$

$$\Rightarrow AB = (\alpha\beta)^3 + (\alpha\beta) - (\alpha^4 + \beta^4)$$

NOW
$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha+\beta)^2 - 2\alpha\beta \\ (\alpha^2)^2 + (\beta^2)^2 &= (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 \end{aligned}$$

IYGB - FP1 PAPER U - QUESTION 9

$$\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$$
$$\alpha^4+\beta^4 = \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$$

$$\Rightarrow AB = (\alpha\beta)^3 + (\alpha\beta) - \left[\left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2\right]$$

$$\Rightarrow AB = 4^3 + 4 - \left[3^2 - 2\times4\right]^2 + 2\times4^2$$

$$\Rightarrow AB = 64 + 4 - 1 + 32$$

$$\Rightarrow AB = 99$$

- HENCE THE REQUIRED QUADRATIC IS

$$x^2 - (-12x) + (+99) = 0$$
$$x^2 + 12x + 99 = 0$$

IYGB – FP1 PAPER U – QUESTION 10

DETERMINE THE TRANSFORMATION EQUATIONS

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \Longleftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-3+4}\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}$$

HENCE WE HAVE

- $x = 3X - 2Y$
- $y = 2X - Y$

SUBSTITUTE INTO THE EQUATION $5x^2 - 16xy + 13y^2 = 25$

$$\Rightarrow 5(3X-2Y)^2 - 16(3X-2Y)(2X-Y) + 13(2X-Y)^2 = 25$$

$$\Rightarrow 5(9X^2 - 12XY + 4Y^2) - 16(6X^2 - 7XY + 2Y^2) + 13(4X^2 - 4XY + Y^2) = 25$$

$$\Rightarrow \left\{\begin{array}{l} 45X^2 - \cancel{60XY} + 20Y^2 \\ -96X^2 + \cancel{112XY} - 32Y^2 \\ 52X^2 - \cancel{52XY} + 13Y^2 \end{array}\right\} = 25$$

$$\Rightarrow X^2 + Y^2 = 25$$

OR

$$x^2 + y^2 = 25$$