IYGB - FM2 PAPER P - QUESTION 1

START BY FINDING THE VOLUME OF REVOLUTION FIRST

$$V = \pi \int_{x_1}^{x_2} (y(x))^2 \, dx = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx$$

$$= \pi \left[\tfrac{1}{2}x^2 \right]_0^4 = \pi [8 - 0) = 8\pi$$

NOW SETTING UP A "MOMENTS" EQUATION, ρ = DENSITY

$$\Rightarrow M\bar{x} = \rho\pi \int_{x_1}^{x_2} (y(x))^2 x \, dx$$

$$\Rightarrow V\rho\bar{x} = \rho\pi \int_0^4 (\sqrt{x})^2 x \, dx$$

$$\Rightarrow 8\pi\rho\bar{x} = \rho\pi \int_0^4 x^2 \, dx$$

$$\Rightarrow 8\bar{x} = \left[\tfrac{1}{3}x^3 \right]_0^4$$

$$\Rightarrow 8\bar{x} = \tfrac{64}{3} - 0$$

$$\therefore \bar{x} = \tfrac{8}{3}$$

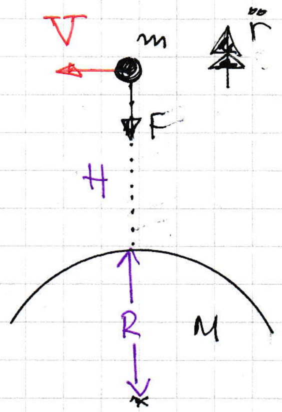
FORMULA IS OF COURSE QUOTABLE

$$\bar{x} = \frac{\int_{x_1}^{x_2} y^2 x \, dx}{\int_{x_1}^{x_2} y^2 \, dx} = \frac{\int_0^4 x^2 \, dx}{\int_0^4 x \, dx} = \frac{\left[\tfrac{1}{3}x^3\right]_0^4}{\left[\tfrac{1}{2}x^2\right]_0^4}$$

$$= \frac{64/3}{8} = \frac{8}{3}$$

IYGB - FM2 PAPER P - QUESTION 2

STARTING WITH A DIAGRAM



$$\Rightarrow \quad F = G\frac{mM}{r^2}$$

WHEN $r=R$, $F=mg$

$$\Rightarrow \quad \cancel{m}g = G\frac{\cancel{m}M}{R^2}$$

$$\Rightarrow \quad \boxed{G = \frac{gR^2}{M}}$$

NOW CIRCULAR MOTION

$$m\ddot{r} = -F \quad \Rightarrow \quad m\left(-\frac{V^2}{R+H}\right) = -\frac{GmM}{(R+H)^2}$$

$$\Rightarrow \quad \frac{mV^2}{R+H} = \frac{GmM}{(R+H)^2}$$

$$\Rightarrow \quad V^2 = \frac{GM}{R+H}$$

BUT $G = \dfrac{gR^2}{M}$

$$\Rightarrow \quad V^2 = \frac{gR^2}{\cancel{M}} \times \frac{\cancel{M}}{R+H}$$

$$\Rightarrow \quad V^2 = \frac{gR^2}{R+H}$$

$$\Rightarrow \quad R+H = \frac{gR^2}{V^2}$$

$$\Rightarrow \quad H = \frac{gR^2}{V^2} - R$$

AS REQUIRED

IYGB - FM2 PAPER P - QUESTION 3

$$\boxed{\ddot{x} = -\frac{3}{10}v^{\frac{1}{3}}\ , \quad t=0\ , \quad v=8\ , \quad x=0}$$

STARTING BY OBTAINING AN EXPRESSION FOR $V = f(t)$

$$\Rightarrow \frac{dv}{dt} = -\frac{3}{10}v^{\frac{1}{3}}$$

$$\Rightarrow 1\,dv = -\frac{3}{10}v^{\frac{1}{3}}\,dt$$

$$\Rightarrow \int_{v=8}^{v} v^{-\frac{1}{3}}\,dv = \int_{t=0}^{t} -\frac{3}{10}\,dt$$

$$\Rightarrow \left[\frac{3}{2}v^{\frac{2}{3}}\right]_8^v = \left[-\frac{3}{10}t\right]_0^t$$

$$\Rightarrow \frac{3}{2}v^{\frac{2}{3}} - \frac{3}{2}\times 4 = -\frac{3}{10}t - 0$$

$$\Rightarrow \frac{3}{2}v^{\frac{2}{3}} - 6 = -\frac{3}{10}t$$

$$\Rightarrow v^{\frac{2}{3}} - 4 = -\frac{2}{10}t$$

$$\Rightarrow v^{\frac{2}{3}} = 4 - \frac{1}{5}t$$

$$\Rightarrow \left(v^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$$

$$\Rightarrow v = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$$

FINALLY BY WRITING $V = \frac{dx}{dt}$ OBTAIN AN EXPRESSION FOR $x = g(t)$

$$\Rightarrow \frac{dx}{dt} = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$$

$$\Rightarrow 1\,dx = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}dt$$

$$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t} \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}dt$$

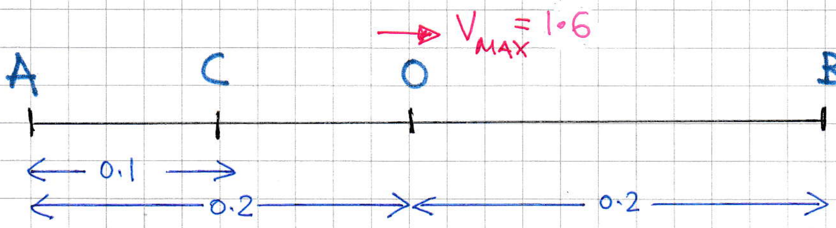
$$\Rightarrow \left[x\right]_0^x = \left[-2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}\right]_0^t$$

$$\Rightarrow x - 0 = \left[2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}\right]_t^0$$

$$\Rightarrow x = 64 - 2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}$$

IYGB - FP2 PAPER P - QUESTION 4

a) PUT THE INFORMATION GIVEN IN A DIAGRAM



USING $|V_{MAX}| = a\omega$

$$\Rightarrow 1.6 = 0.2\omega$$
$$\Rightarrow \omega = 8$$

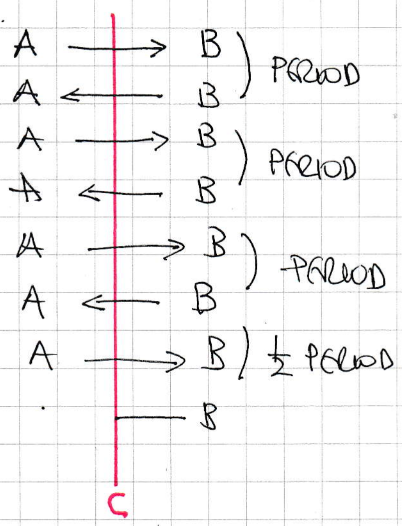
NOW USING $v^2 = \omega^2(a^2 - x^2)$

$$\Rightarrow v^2 = 8^2(0.2^2 - 0.1^2)$$
$$\Rightarrow v^2 = 1.92$$
$$\Rightarrow |v| \simeq 1.39 \text{ ms}^{-1} \quad // \; 3 \text{ s.f}$$

b) "PASSING THROUGH C FOR THE EIGHTH TIME"

SETTING $t=0$, AT B WITH AMPLITUDE $+0.2$ AT B

$$x = a\cos\omega t$$
$$x = 0.2\cos 8t$$
$$-0.1 = 0.2\cos 8t$$
$$\cos 8t = -\frac{1}{2}$$
$$8t = \frac{2\pi}{3}$$
$$t = \frac{\pi}{12}$$



IYGB – FP2 PAPER P – QUESTION 4

NEXT WE FIND THE PERIOD OF THE MOTION

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8} = \frac{\pi}{4}$$

THE REQUIRED TIME IS

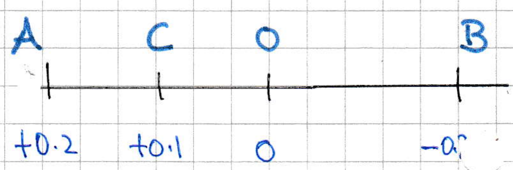
$$\underbrace{3 \times \frac{\pi}{4}}_{\text{3 PERIODS}} + \underbrace{\frac{1}{2} \times \frac{\pi}{4}}_{\text{HALF PERIOD}} + \frac{\pi}{12} = \frac{23}{24}\pi$$

FROM B TO C AT THE VERY END

ALTERNATIVE FOR PART (b) BY A DIRECT TRIG EQUATION

- SET $t=0$ AT A, SO + AMPLITUDE IS NOW AT A
- WE REQUIRE THE 8^{TH} POSITIVE SOLUTION OF THE EQUATION

$$+0.1 = 0.2\cos 8t$$



- SOLVING THE EQUATION

$$\Rightarrow \cos 8t = \frac{1}{2}$$

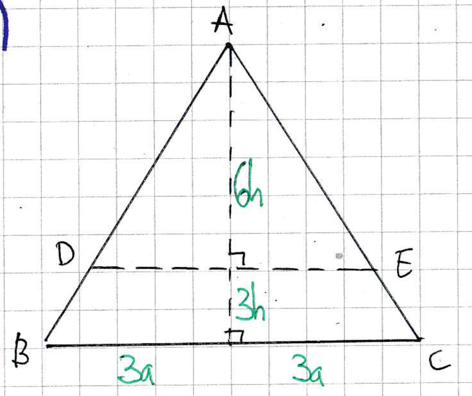
$$\Rightarrow \left(\begin{array}{l} 8t = \frac{\pi}{3} + 2n\pi \\ 8t = \frac{5\pi}{3} + 2n\pi \end{array}\right. \quad n = 0,1,2,3,\ldots$$

$$\Rightarrow \left(\begin{array}{l} t = \frac{\pi}{24}[1+6n] \\ t = \frac{\pi}{24}[5+6n] \end{array}\right.$$

$$\Rightarrow t = \underset{\text{1st}}{\frac{\pi}{24}}, \underset{\text{2nd}}{\frac{5\pi}{24}}, \underset{\text{3rd}}{\frac{7\pi}{24}}, \underset{\text{4th}}{\frac{11\pi}{24}}, \underset{\text{5th}}{\frac{13\pi}{24}}, \underset{\text{6th}}{\frac{17\pi}{24}}, \underset{\text{7th}}{\frac{19\pi}{24}}, \underset{\text{8th}}{\frac{23\pi}{24}}$$

IYGB - FM2 PAPER P - QUESTION 5

a)



BY SIMILARITY

$|DE| = \frac{2}{3} \times 6a = 4a$

- AREA OF ABC $= \frac{1}{2} \times 6a \times 9h = 27ah$
- AREA OF ADE $= \frac{1}{2} \times 4a \times 6h = 12ah$
- AREA OF TRAPEZIUM $= 27ah - 12ah = 15ah$

FORMING A STANDARD TABLE

	△	⏢	△
MASS RATIO	~~12ah~~ 4	~~15ah~~ 5	~~27ah~~ 9
DISTANCE OF CENTRE OF MASS FROM BC	$3h + \frac{1}{3}(6h)$	$\bar{y}$	$\frac{1}{3}(9h)$

$\Rightarrow 4 \times 5h + 5\bar{y} = 27h$

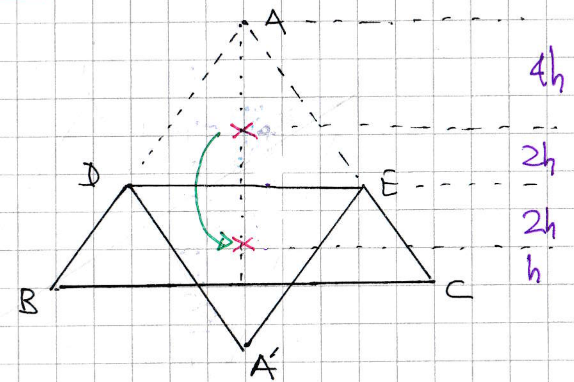
$\Rightarrow 20h + 5\bar{y} = 27h$

$\Rightarrow 5\bar{y} = 7h$

$\Rightarrow \bar{y} = \frac{7}{5}h$

AS REQUIRED

b) LOOKING AT A DIAGRAM OF THE FOLDED LAMINA



NEW TABLE NOW

	⏢	▽	⏢▽
MASS RATIO	5	4	9
DISTANCE OF CENTRE OF MASS FROM BC	$\frac{7}{5}h$	h	$\bar{x}$

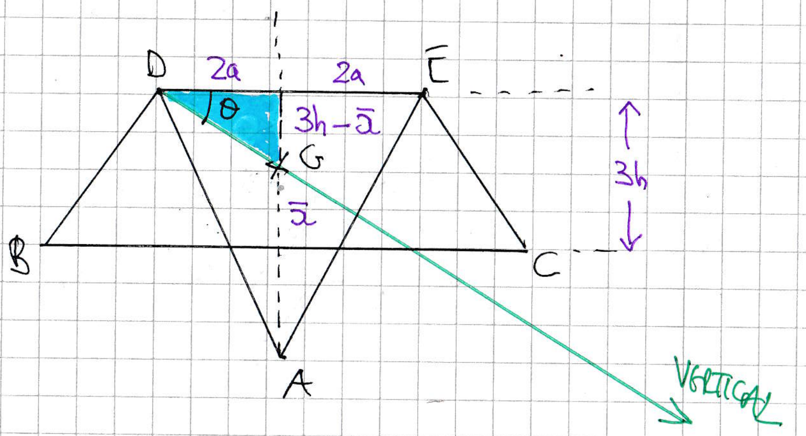
$\Rightarrow 5 \times \frac{7}{5}h + 4h = 9\bar{x}$

$\Rightarrow 11h = 9\bar{x}$

$\Rightarrow \bar{x} = \frac{11}{9}h$

IYGB - FM1 PAPER P - QUESTION 5

c) FINALLY LOOKING AT THE DIAGRAM BELOW



$$\tan\theta = \frac{3h-\bar{x}}{2a}$$

$$\frac{2}{9} = \frac{3h-\bar{x}}{2a}$$

$$4a = 27h - 9\bar{x}$$

$$4a = 27h - 9\left(\frac{11}{9}h\right)$$

$$4a = 27h - 11h$$

$$4a = 16h$$

$$a = 4h$$

IYGB - FM2 PAPER P - QUESTION 6

- BY ENERGIES TAKING THE LEVEL OF "O" AS THE ZERO POTENTIAL

$$\Rightarrow KE_{\alpha} + P.E_{\alpha} = KE_{\theta} + P.E_{\theta}$$

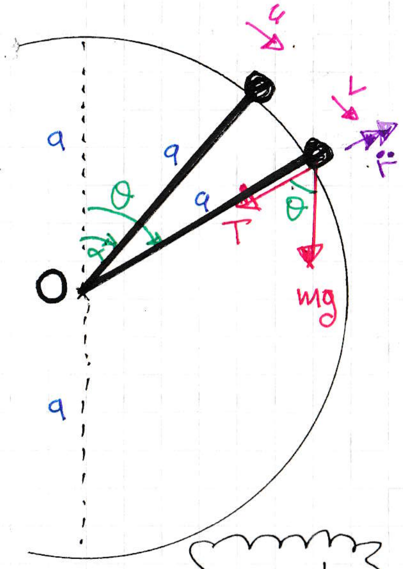
$$\Rightarrow \tfrac{1}{2}\cancel{m}u^2 + \cancel{m}ga\cos\alpha = \tfrac{1}{2}\cancel{m}v^2 + \cancel{m}ga\cos\theta$$

$$\Rightarrow u^2 + 2ag\cos\alpha = v^2 + 2ag\cos\theta$$

$$\Rightarrow u^2 + \tfrac{2}{3}ag = v^2 + 2ag\cos\theta$$

$$\Rightarrow v^2 = u^2 + \tfrac{2}{3}ag - 2ag\cos\theta$$

$$\Rightarrow v^2 = u^2 + \tfrac{2}{3}ag(1 - 3\cos\theta)$$



$\cos\alpha = \frac{1}{3}$

- USING THIS EXPRESSION WE FIND THE SPEED AT THE HIGHEST AND LOWEST POINTS OF THE PATH

- $V^2_{TOP} = u^2 - \frac{4}{3}ag \quad (\theta = 0°)$

- $V^2_{BOTTOM} = u^2 + \frac{8}{3}ag \quad (\theta = 180°)$

- NEXT THE EQUATION OF MOTION (RADIALLY), IN THE GENERAL POSITION OF THE PATH

$$m\ddot{r} = -T - mg\cos\theta$$

$$T = -m\ddot{r} - mg\cos\theta$$

$$T = -m\left(-\frac{v^2}{a}\right) - mg\cos\theta$$

$$T = \frac{m}{a}v^2 - mg\cos\theta$$

- WE CAN NOW SUB THE TWO VELOCITIES, FOUND EARLIER

- $T_{TOP} = \frac{m}{a}\left[u^2 - \frac{4}{3}ag\right] - mg \times 1$ $\quad \theta = 0°$

- $T_{BOTTOM} = \frac{m}{a}\left[u^2 + \frac{8}{3}ag\right] - mg(-1)$ $\quad \theta = 180°$

IYGB - FM2 PAPER P - QUESTION 6

$$\Rightarrow \begin{cases} T_{TOP} = \frac{mu^2}{a} - \frac{4}{3}mg - mg = \frac{mu^2}{a} - \frac{7}{3}mg \\ T_{BOTTOM} = \frac{mu^2}{a} + \frac{8}{3}mg + mg = \frac{mu^2}{a} + \frac{11}{3}mg \end{cases}$$

- FINALLY WE ARE GIVEN $T_{MAX} = 10T_{MIN}$

$$\Rightarrow \frac{mu^2}{a} + \frac{11}{3}mg = 10\left[\frac{mu^2}{a} - \frac{7}{3}mg\right]$$

$$\Rightarrow \frac{\cancel{m}u^2}{a} + \frac{11}{3}\cancel{m}g = \frac{10\cancel{m}u^2}{a} - \frac{70}{3}\cancel{m}g$$

$$\Rightarrow 27g = \frac{9u^2}{a}$$

$$\Rightarrow 27ag = 9u^2$$

$$\Rightarrow u^2 = 3ag$$

IYGB - FM2 PAPER P - QUESTION 7

a) WORKING WITH ENERGIES

$$\Rightarrow \text{P.E. LOST} = \text{E.E GAINED}$$

$$\Rightarrow mgh = \frac{\lambda}{2\ell}x^2$$

$$\Rightarrow 0.75g \times 4 = \frac{78}{2\ell}x^2$$

$$\Rightarrow \frac{x^2}{\ell} = \frac{49}{65}$$

BUT $x + \ell = 4$

$$\Rightarrow \frac{x^2}{4-x} = \frac{49}{65}$$

$$\Rightarrow 65x^2 = 196 - 49x$$

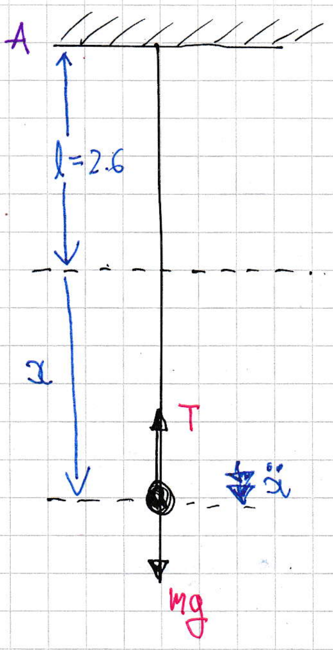
$$\Rightarrow 65x^2 + 49x - 196 = 0$$

$$\Rightarrow x = \frac{-49 \pm \sqrt{53361}}{2 \times 65} = \begin{cases} 1.4 \\ \cancel{-2.1538\ldots} \end{cases}$$

$\therefore$ NATURAL LENGTH $= 4 - 1.4 = 2.6$

AS REQUIRED

b) CONSIDER THE PARTICLE IN AN ARBITRARY POSITION WITH $0 < x < 1.4$



$$\Rightarrow m\ddot{x} = mg - T$$

$$\Rightarrow m\ddot{x} = mg - \frac{\lambda}{\ell}x$$

$$\Rightarrow \ddot{x} = g - \frac{\lambda}{m\ell}x$$

$$\Rightarrow \ddot{x} = g - \frac{78}{0.75 \times 2.6}x$$

$$\Rightarrow \ddot{x} = g - 40x$$

$$\therefore \ddot{x} = -40x + g$$

AS REQUIRED

IYGB – FM2 PAPER P – QUESTION 7

c) LET $-40x+g = -40X$

$$\Rightarrow -40\dot{x} = -40\dot{X}$$
$$\Rightarrow \ddot{X} = \ddot{x}$$

$$\therefore \quad \ddot{x} = -40x + g$$
$$\ddot{X} = (-g - 40X) + g$$
$$\ddot{X} = -40X$$

I.E S.H.M WITH $\omega^2 = 40$

d) LOOKING AT PREVIOUS PARTS

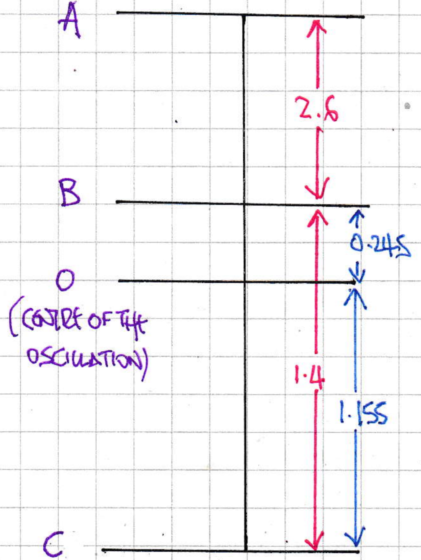
MAXIMUM x IS 1.4 $\Rightarrow$ MAXIMUM $X = 1.155$

$$-40X = -40x + g$$
$$X = x - \frac{g}{40}$$

$\therefore$ MAX SPEED $= |\omega a| = \sqrt{40} \times 1.155$

$\simeq 7.30 \text{ ms}^{-1}$

e) LOOKING AT THE DIAGRAM



• KINEMATICS (FREE FALL FROM A to B)

$u = 0$
$a = 9.8$
$s = 2.6$
$t = ?$
$v =$

$$s = \cancel{ut} + \tfrac{1}{2}at^2$$
$$2.6 = \tfrac{1}{2} \times 9.8 \times t_1^2$$
$$t_1^2 = \frac{52}{49}$$
$$t_1 \simeq 1.0612\ldots$$

IYGB - FM2 PAPER P - QUESTION 7

- NEXT THE TIME FROM B to C IS THE SAME AS THAT FROM C to B

"C to O"

$\frac{1}{4}$ PERIOD

$= \frac{1}{4} \times \frac{2\pi}{\omega}$

$= \frac{2\pi}{4\sqrt{40}}$

$\therefore t_2 \approx 0.24836$

"O to B"

USING $x = a\sin\omega t$ (STARTING AT O)

$0.245 = 1.155 \sin\sqrt{40}\,t$

$\sin(\sqrt{40}\,t) = \frac{7}{33}$

FIRST POSITIVE SOLUTION

$\sqrt{40}\,t = 0.21374\ldots$

$t_3 \simeq 0.03380$

$\therefore$ TOTAL TIME $= t_1 + t_2 + t_3$

$\simeq 1.3434\ldots$

$\simeq 1.34$