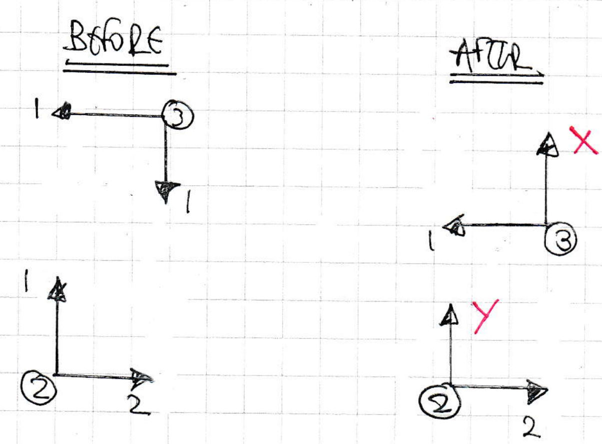
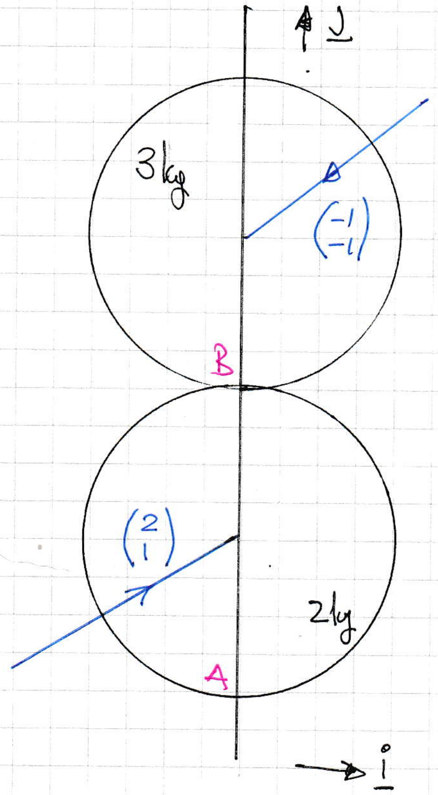
IYGB - FM1 PAPER Q - QUESTION 1

DRAWING A DIAGRAM





BY CONSERVATION OF MOMENTUM ALONG $\underline{j}$

$$(2 \times 1) - (3 \times 1) = 3X + 2Y$$

$$3X + 2Y = -1$$

BY RESTITUTION ALONG $\underline{j}$

$$e = \frac{\text{SEP}}{\text{APP}} \Rightarrow \frac{1}{2} = \frac{X - Y}{1 + 1}$$

$$\Rightarrow X - Y = 1$$

SOLVING THE EQUATION

$$\left.\begin{array}{r} 3X + 2Y = -1 \\ X - Y = 1 \end{array}\right\} \Rightarrow X = Y + 1$$

$$\Rightarrow 3(Y+1) + 2Y = -1$$

$$\Rightarrow 5Y + 3 = -1$$

$$\Rightarrow 5Y = -4$$

$$\Rightarrow Y = -\frac{4}{5} = -0.8$$ (IE IT REBOUNDS)

& $X = -0.8 + 1$

$X = 0.2$

IYGB - FM1 PAPER Q - QUESTION 1

IT SUFFICES TO WORK OUT THE KINETIC ENERGY CHANGE IN THE DIRECTION WHICH MOMENTUM IS EXCHANGED ONLY

$$\text{K.E BEFORE} = \frac{1}{2}\times 3\times 1^2 + \frac{1}{2}\times 2\times 1^2 = \frac{3}{2} + 1 = 2.5$$

$$\text{KE AFTER} = \frac{1}{2}\times 3\times X^2 + \frac{1}{2}\times 2\times Y^2 = \frac{3}{2}\times\left(\frac{1}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

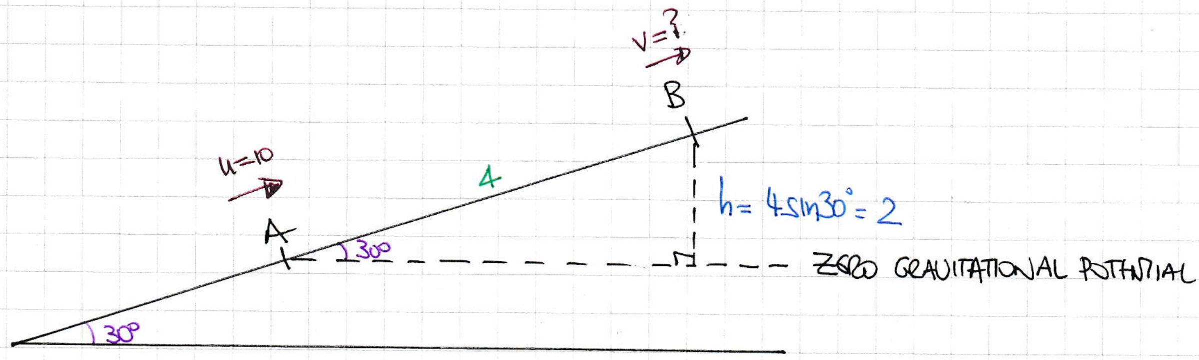
$$= 0.06 + 0.64$$

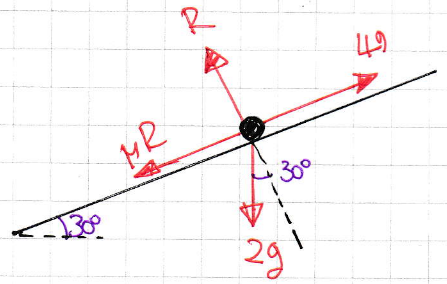
$$= 0.7$$

$\therefore$ A LOSS OF $2.5 - 0.7 = 1.8\ \text{J}$

IYGB - FM1 PAPER Q - QUESTION 2

STARTING WITH A DIAGRAM





$\mu = \frac{1}{2}$

FORMING AN ENERGY EQUATION

$\Longrightarrow K.E_A + \cancel{P.E_A} + W_{IN} - W_{OUT} = K.E_B + P.E_B$

$\Longrightarrow \frac{1}{2}mu^2 + 0 + (49 \times 4) - \mu R \times 4 = \frac{1}{2}mv^2 + mgh$

$\Longrightarrow \frac{1}{2} \times 2 \times 10^2 + 196 - \frac{1}{2}(2g\cos 30) \times 4 = \frac{1}{2} \times 2 \times v^2 + 2 \times 9.8 \times 2$

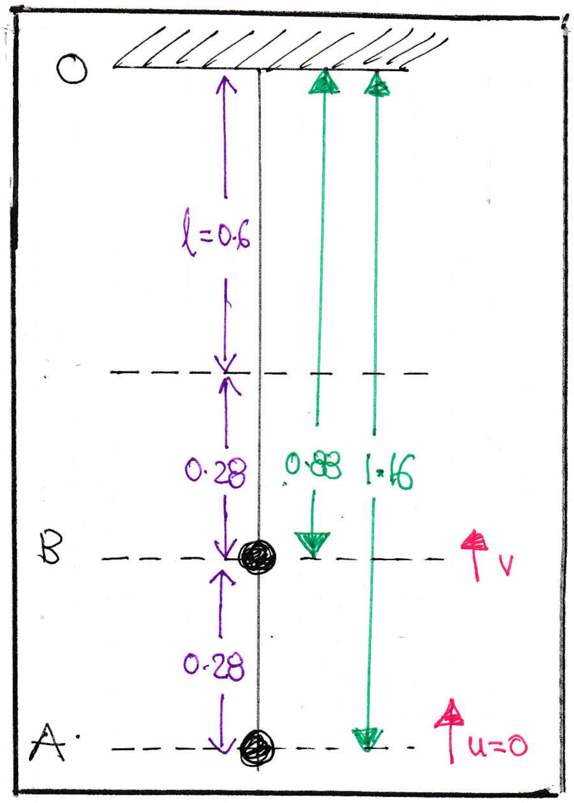
$\Longrightarrow 100 + 196 - \frac{98}{5}\sqrt{3} = v^2 + 39.2$

$\Longrightarrow v^2 = 222.8518042\ldots$

$\Longrightarrow v \approx 14.93 \text{ ms}^{-1}$

IYGB - FM1 PAPER Q - QUESTION 3

BY ENERGIES TAKING THE LEVEL OF "A" AS THE POINT OF ZERO GRAVITATIONAL POTENTIAL LEVEL



$$\Rightarrow \cancel{KE_A} + \cancel{PE_A} + EE_A = KE_B + PE_B + EE_B$$

$$\Rightarrow \frac{\lambda}{2l}x_1^2 = \frac{1}{2}mv^2 + mgh + \frac{\lambda}{2l}x_2^2$$

$$\Rightarrow \frac{\lambda x_1^2}{l} = mv^2 + 2mgh + \frac{\lambda}{l}x_2^2$$

$$\Rightarrow \frac{\lambda}{ml}x_1^2 = v^2 + 2gh + \frac{\lambda}{ml}x_2^2$$

$$\Rightarrow v^2 = \frac{\lambda}{ml}\left(x_1^2 - x_2^2\right) - 2gh$$

$$\Rightarrow v^2 = \frac{47}{0.5 \times 0.6}\left[0.56^2 - 0.28^2\right] - 2(9.8)(0.28)$$

$$\Rightarrow v^2 = \frac{47}{0.3} \times \frac{147}{625} - \frac{686}{125}$$

$$\Rightarrow v^2 = \frac{4606}{125} - \frac{686}{125}$$

$$\Rightarrow v^2 = \frac{784}{25}$$

$$\Rightarrow |v| = \frac{28}{5} = 5.6 \text{ ms}^{-1}$$

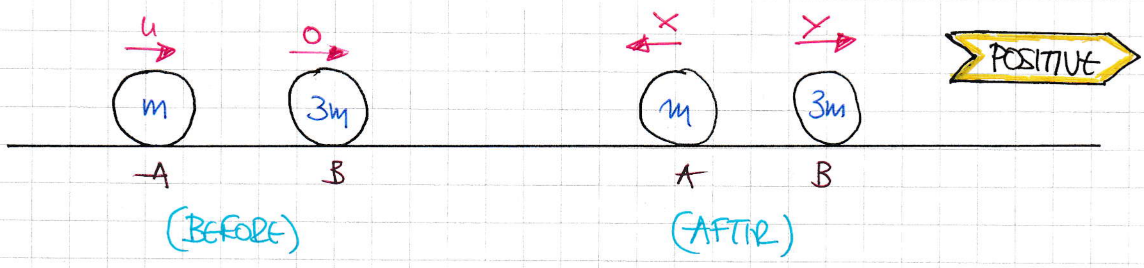
$\lambda = 47$ N

$l = 0.6$ m

$m = 0.5$

IYGB - FM1 PAPER Q - QUESTION 4

a) STARTING WITH A BEFORE AND AFTER DIAGRAM



BY CONSERVATION OF MOMENTUM

$$mu + 0 = -mX + 3mY$$

$$\boxed{u = -X + 3Y}$$

BY RESTITUTION CONSIDERATIONS

$$e = \frac{SEP}{APP}$$

$$e = \frac{X+Y}{u}$$

$$\boxed{X + Y = eu}$$

ADDING THE EQUATIONS

$$\Rightarrow 4Y = u + eu$$

$$\Rightarrow 4Y = u(1+e)$$

$$\Rightarrow Y = \tfrac{1}{4}u(1+e) \quad \leftarrow \text{SPEED OF B}$$

AND FINALLY USING $X = 3Y - u$

$$X = \tfrac{3}{4}u(1+e) - u = \tfrac{1}{4}u\left[3(1+e) - 4\right] = \tfrac{1}{4}u(3+3e-4)$$

$$X = \tfrac{1}{4}u(3e-1) \quad \leftarrow \text{SPEED OF A}$$

b) AS X IS MARKED "CORRECTLY" IN THE DIAGRAM $X > 0$

$$\Rightarrow \tfrac{1}{4}u(3e-1) > 0$$

$$\Rightarrow 3e - 1 > 0$$

$$\Rightarrow 3e > 1$$

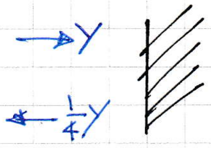
$$\Rightarrow e > \tfrac{1}{3}$$

IYGB − FM1 PAPER Q − QUESTION 4

THUS WE NOW HAVE

$$\frac{1}{3} < e < 1$$
$$\frac{4}{3} < e+1 < 2$$
$$\frac{4}{3}u < u(e+1) < 2u$$
$$\frac{1}{3}u < \frac{1}{4}u(e+1) < \frac{1}{2}u$$
$$\frac{1}{3}u < Y < \frac{1}{2}u$$

c) B COLLIDES WITH THE WALL



THE CONFIGURATION NOW IS

ANOTHER COLLISION IMPLIES $\frac{1}{4}Y > X$

$$\Rightarrow \frac{1}{4}\left(\frac{1}{4}u(1+e)\right) > \frac{1}{4}u(3e-1)$$
$$\Rightarrow \frac{1}{4}(1+e) > 3e-1$$
$$\Rightarrow 1+e > 12e-4$$
$$\Rightarrow 5 > 11e$$
$$\Rightarrow e < \frac{5}{11}$$

BUT EARLIER IT WAS FOUND (PART b) THAT $\frac{1}{3} < e < 1$

$$\therefore \quad \frac{1}{3} < e < \frac{5}{11}$$

AS REQUIRED

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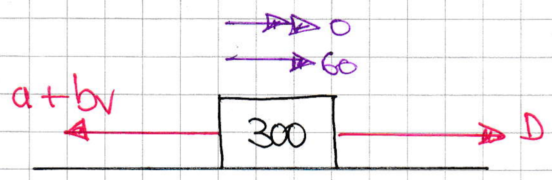
LOOKING AT THE BIKE AT MAX SPEED ON THE FLAT



- $P = Dv$

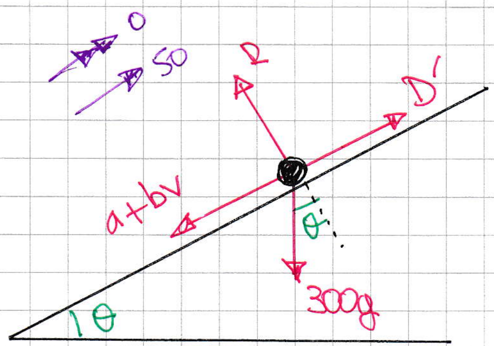
$54000 = D \times 60$

$D = 900$

- $D = a + bv$

$900 = a + 60b$

LOOKING AT THE BIKE AT MAX SPEED ON THE INCLINE



- $\sin\theta = \frac{4}{49}$

- $P = D'v$

$54000 = D' \times 50$

$D' = 1080$

- $D' = (a + bv) + 300g\sin\theta$

$1080 = a + 50v + 300g \times \frac{4}{49}$

$1080 = a + 50v + 240$

$840 = a + 50b$

SOLVING SIMULTANEOUSLY

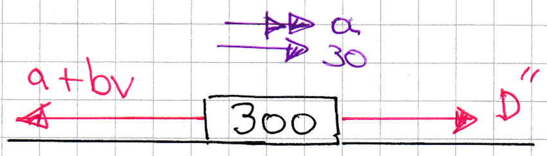
$$\left.\begin{aligned} a + 60b &= 900 \\ a + 50b &= 840 \end{aligned}\right\} \Rightarrow 10b = 60.$$

$\Rightarrow b = 6$

$\Rightarrow a = 540$

IYGB - FM1 PAPER Q - QUESTION 5

RETURNING TO THE FLAT



$$P = D''v$$
$$54000 = D'' \times 30$$
$$D'' = 1800$$

"$F = ma$"

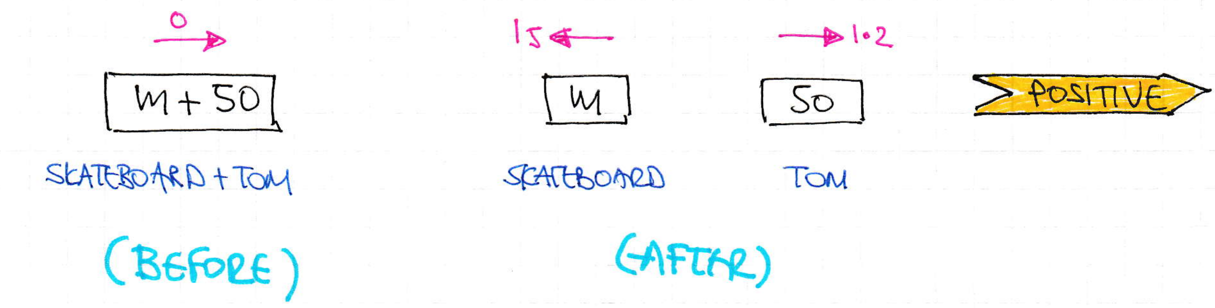
$\Longrightarrow \quad 1800 - (540 + 6 \times 30) = 300a$

$\Longrightarrow \quad 1080 = 300a$

$\Longrightarrow \quad a = 3.6 \ \mathrm{ms}^{-2}$

IYGB - FM1 PAPER Q - QUESTION 6

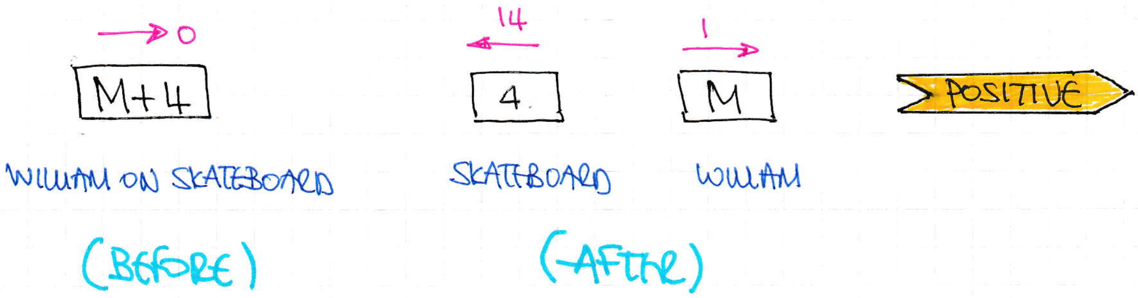
START WITH A BEFORE/AFTER DIAGRAM – LET m BE THE SKATEBOARD'S MASS



BY CONSERVATION OF MOMENTUM

$$(m+50)\times 0 = -(m\times 15) + (50\times 1.2)$$
$$0 = -15m + 60$$
$$m = 4 \text{ kg}$$

SIMILARLY WITH WILLIAM – LET M BE THE MASS OF WILLIAM



BY CONSERVATION OF MOMENTUM AGAIN

$$(M+4)\times 0 = -(14\times 4) + (M\times 1)$$
$$0 = -56 + M$$
$$M = 56$$

SO WILLIAM HAS A MASS OF 56 kg

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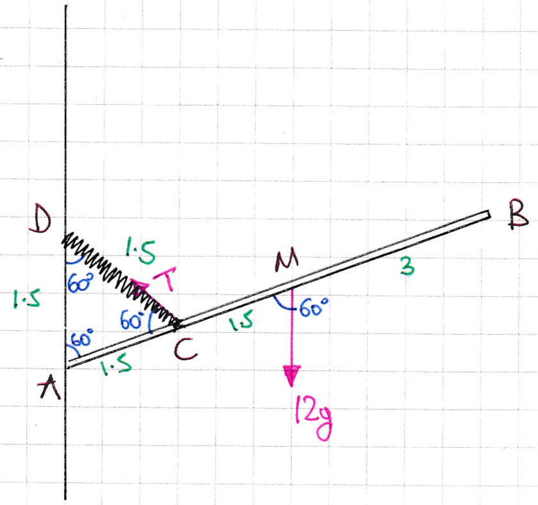
LOOKING AT A DIAGRAM $\overset{\triangle}{ACD}$ IS EQUILATERAL



IGNORING THE "REACTION FORCE(S)" AT A & TAKING MOMENTS ABOUT A

$$1.5 \times T\sin 60 = 12g \sin 60 \times 3$$

$$1.5T = 36g$$

$$T = 24g$$

BY HOOKE'S LAW

$$\Longrightarrow T = \lambda \frac{x}{a}$$

$$\Longrightarrow 24g = \lambda \left(\frac{1.5 - 1}{1}\right)$$

$$\Longrightarrow 24g = \lambda \times \frac{1}{2}$$

$$\Longrightarrow \lambda = 48g$$

$$\Longrightarrow \lambda = 470.4\text{ N}$$ i.e. 470 N

IYGB - FM1 PAPER Q - QUESTION 8

NON GEOMETRIC APPROACH

$$\underline{I} = m\underline{v} - m\underline{u}$$

$$\lambda\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}x\\y\end{pmatrix} - \frac{1}{2}\begin{pmatrix}14\\0\end{pmatrix}$$

$$2\lambda\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}14\\0\end{pmatrix}$$

$$\begin{pmatrix}2\lambda\\2\lambda\end{pmatrix} = \begin{pmatrix}x-14\\y\end{pmatrix}$$

I.E. $x = 2\lambda + 14$
$y = 2\lambda$

BUT MAGNITUDE OF $\underline{v}$ IS 34

$\Rightarrow \sqrt{x^2+y^2} = 34$

$\Rightarrow x^2+y^2 = 1156$

$\Rightarrow (2\lambda+14)^2 + 4\lambda^2 = 1156$

$\Rightarrow 4\lambda^2 + 56\lambda + 196 + 4\lambda^2 = 1156$

$\Rightarrow 8\lambda^2 + 56\lambda - 960 = 0$

$\Rightarrow \lambda^2 + 7\lambda - 120 = 0$

$\Rightarrow (\lambda - 8)(\lambda + 15) = 0$

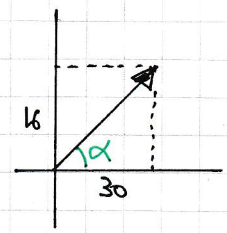
$\Rightarrow \lambda = 8$ // $\lambda > 0$

DRAWING A DIAGRAM FOR $\underline{v}$

$x = 2(8) + 14 = 30$

$y = 16$

I.E $\underline{v} = \begin{pmatrix}30\\16\end{pmatrix} = 30\underline{i} + 16\underline{j}$



$\tan\alpha = \frac{16}{30} = \frac{8}{15}$

$\alpha \simeq 28.0724\ldots$

$\alpha \simeq 28°$ //

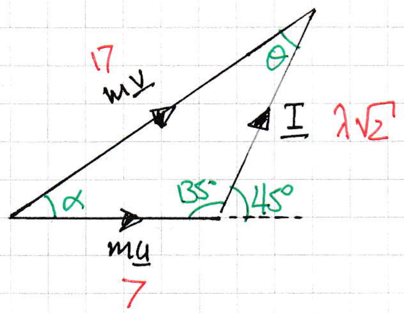
IYGB - FM1 PAPER Q - QUESTION 8

A GEOMETRIC APPROACH

$$\underline{I} = m\underline{v} - m\underline{u}$$

$$m\underline{v} = m\underline{u} + \underline{I}$$

DRAWING A VECTOR TRIANGLE



- $\underline{I} = \lambda(\underline{i}+\underline{j})$
  $|\underline{I}| = \lambda\sqrt{2}$
- $|m\underline{u}| = |\frac{1}{2} \times 14\underline{i}| = 7$
- $|m\underline{v}| = \frac{1}{2} \times 34 = 17$

BY THE SINE RULE

$$\frac{\sin 135^\circ}{17} = \frac{\sin\theta}{7}$$

$$\sin\theta = \frac{7\sin 135^\circ}{17}$$

$$\sin\theta = \frac{7}{34}\sqrt{2}$$

$$\theta \simeq 16.93^\circ$$

$$\therefore \alpha \simeq 180 - 135 - 16.93$$

$$\alpha \simeq 28^\circ$$

BY THE SINE RULE AGAIN OR COSINE RULE

$$17^2 = 7^2 + 2\lambda^2 - 2\times 7\times\lambda\sqrt{2}\cos 135$$

$$289 = 49 + 2\lambda^2 + 14\lambda$$

$$0 = 2\lambda^2 + 14\lambda - 240$$

$$0 = \lambda^2 + 7\lambda - 120$$

$$(\lambda+15)(\lambda-8) = 0$$

$$\lambda = 15 \qquad (\lambda > 0)$$