IYGB GCE

Core Mathematics C4

Advanced

Practice Paper Y

Difficulty Rating: 3.9333/1.9395

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet “Mathematical Formulae and Statistical Tables” may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

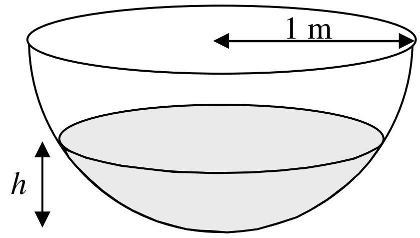
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Use trigonometric identities to integrate

$$\int \frac{\cos 2x}{1-\cos^2 2x}\, dx. \qquad (4)$$

Question 2



A tank for storing water is in the shape of a hollow inverted hemisphere with a radius of one metre.

It can be shown by calculus that when the depth of the water in the tank is h m, its volume, V m^3, is given by the formula

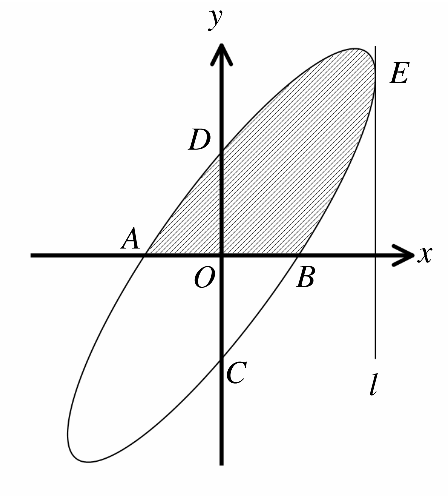
$$V = \frac{1}{3}\pi h^2 (3-h).$$

a) Determine the volume of the water in the tank when $h = 0.5$. **(1)**

The tank is initially empty and water then begins to pour in at the constant rate of $\frac{\pi}{24}$ m^3 per hour.

b) Find the rate at which the height of the water is increasing 5 hours later. **(5)**

Question 3



The figure above shows an ellipse with parametric equations

$$x = 2\cos\theta, \quad y = 6\sin\left(\theta + \frac{\pi}{3}\right), \quad -\pi \le \theta < \pi .$$

The curve meets the coordinate axes at the points A, B, C and D.

a) Find the coordinates of the points A, B, C and D. **(4)**

The straight line l is the tangent to the ellipse at the point E.

b) State the equation of l, given it parallel to the y axis. **(1)**

c) Find the value of θ at the point E. **(1)**

The finite region bounded by the ellipse and the x axis for which $y \ge 0$ is shown shaded in the figure above.

d) Show that the area of this region is given by the integral

$$\int_{-\frac{\pi}{3}}^{\frac{2\pi}{3}} 3 - 3\cos 2\theta + 3\sqrt{3}\sin 2\theta \ d\theta .$$ **(4)**

e) Hence find the area of the shaded region. **(3)**

Question 4

The straight line L_1 passes through the point $A(5,-2,1)$ and is parallel to the vector

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$

a) Find a vector equation for L_1, in terms of a scalar parameter λ. **(2)**

The straight line L_2 has a vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

where μ is scalar parameter.

b) Show that the lines intersect at some point P, and find its coordinates . **(4)**

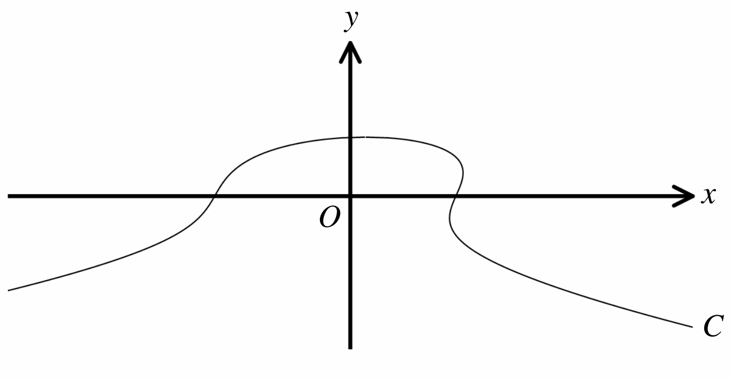
The point B lies on L_2 where $\mu = -2$.

The point C lies on a straight line which is parallel to L_1 and passes through B.

The points A, B, C and P are vertices of a parallelogram.

c) Show that one of the possible positions for C is the origin O and find the coordinates of the other possible position for C. **(4)**

Question 5



The figure above shows part of the curve C with equation

$$x^2 + 2x + y^3 = 63 + xy.$$

a) Show that

$$\frac{dy}{dx} = \frac{y - 2x - 2}{3y^2 - x}$$ **(4)**

b) Show further that C has only one stationary point at $(1,4)$. **(7)**

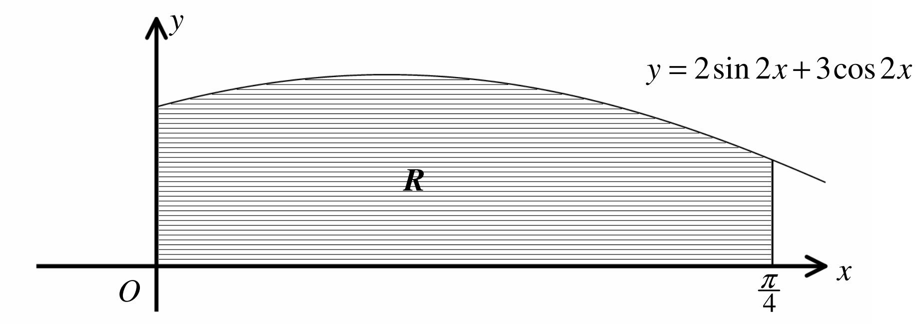
Question 6

$$f(x) = \sqrt{\frac{1+ax}{4-x}}, \ -1 < x < 1.$$

The value of the constant a is such so that the coefficient of x^2 in the convergent binomial expansion of $f(x)$ is $\frac{1}{64}$.

Find the value of a. **(7)**

Question 7



The figure above shows part of the curve C, with equation

$$y = 2\sin 2x + 3\cos 2x .$$

a) Show that

$$y^2 = A + B\cos 4x + C\sin 4x ,$$

where A, B and C are constants. **(4)**

The shaded region R is bounded by the curve, the line $x = \frac{\pi}{4}$ and the coordinate axes.

b) Find the area of R. **(3)**

The region R is rotated by 2π radians in the x axis forming a solid of revolution S.

c) Show that the volume of S is

$$\frac{\pi}{8}(13\pi + 24) .$$ **(3)**

Question 8

Water is pouring into a container at a constant rate of $200 \ \text{cm}^3\text{s}^{-1}$ and is leaking from a hole at the base of the container at a rate proportional the volume V of the water already in the container.

a) Form a differential equation connecting the volume V cm^3, the time t in seconds and a proportionality constant k. **(2)**

b) Show that a general solution of the differential equation is given by

$$V = \frac{200}{k} + A\text{e}^{-kt},$$

where A is a constant. **(6)**

The container was initially empty and after 10 seconds the volume of the water V is increasing at the rate of $100 \ \text{cm}^3\text{s}^{-1}$.

c) Show further that

$$V = \frac{2000}{\ln 2}\left(1 - 2^{-\frac{1}{10}t}\right).$$ **(6)**
