IYGB GCE

Core Mathematics C4

Advanced

Practice Paper O

Difficulty Rating: 3.4400/1.5625

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$\frac{30}{(x+3)(9-2x)} \equiv \frac{A}{x+3} + \frac{B}{9-2x}.$$

a) Determine the value of each of the constants A and B. **(3)**

b) Evaluate

$$\int_{1}^{4} \frac{30}{(x+3)(9-2x)} \, dx,$$

giving the answer as a single simplified natural logarithm. **(5)**

Question 2

$$f(x) = \frac{20}{\sqrt{4+2x}}, \quad |x| < 2.$$

a) Expand $f(x)$ as an infinite series, up and including the term in x^3. **(5)**

b) By substituting $x = \frac{1}{12}$ in the above expansion, show that **(3)**

$$\sqrt{6} \approx 2.45.$$

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Question 3

A curve C has implicit equation

$$x^2 + 4xy + 2y^2 = 7 .$$

a) Show that …

i. … $\dfrac{dy}{dx} = -\dfrac{x+2y}{2x+2y}$. **(5)**

ii. … the equation of the tangent to the curve at $P(1,1)$ is **(2)**

$$3x + 4y = 7 .$$

The tangent to the curve at the point Q is parallel to the tangent to the curve at P.

b) Find the coordinates of Q. **(6)**

Question 4

Liquid is pouring into a container at the constant rate of $30 \text{ cm}^3\text{s}^{-1}$.

The container is initially empty and when the height of the liquid in the container is h cm the volume of the liquid, V cm^3, is given by

$$V = 36h^2 .$$

a) Find the rate at which the height of the liquid in the container is rising when the height of the liquid reaches 3 cm. **(4)**

b) Determine the rate at which the height of the liquid in the container is rising 12.5 minutes after the liquid started pouring in. **(6)**

Question 5

The points with coordinates $A(3,0,3)$ and $B(4,-1,5)$ are given.

a) Find a vector equation of the straight line l_1 that passes through A and B. **(3)**

The straight line l_2 has equation

$$\mathbf{r} = 5\mathbf{i} + 10\mathbf{j} + 4\mathbf{k} + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 are perpendicular. **(2)**

c) Show further that l_1 and l_2 intersect at some point P and find its coordinates. **(5)**

The point E is on the l_1.

A circle with centre at E is drawn so that it cuts l_2 at the points C and D.

d) Given that the coordinates of C are $(0,-5,-1)$, find the coordinates of D. **(3)**

Question 6

A machine is used to produce waves in the swimming pool of a water theme park.

Let x cm be the height of the wave produced above a certain level in the pool, and suppose it can be modelled by the differential equation

$$\frac{dx}{dt} = 2x \sin 2t, \quad t \geq 0,$$

where t is the time in seconds.

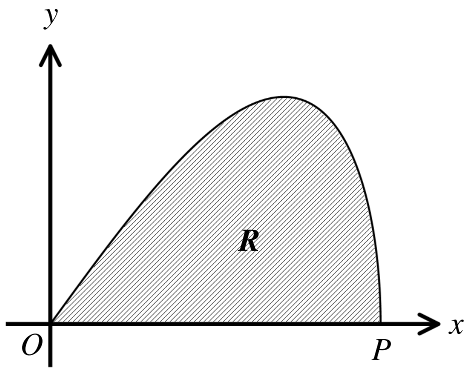
When $t = 0$, $x = 6$.

a) Solve the differential equation to show

$$x = 6e^{1-\cos 2t}. \qquad \textbf{(8)}$$

b) Find the maximum height of the wave. **(2)**

Question 7



The figure above shows the curve C with parametric equations

$$x = 5\cos t, \quad y = 3\sin 2t \;, \quad 0 \le t \le \frac{\pi}{2}.$$

The curve meets the x axis at the origin O and at the point P.

a) Find the value of t at O and at P. **(3)**

The finite region R bounded by C and the x axis is revolved by 2π radians in the x axis forming a solid of revolution S.

b) Show that the volume of S is given by the integral

$$\pi \int_0^{\frac{\pi}{2}} 180 \sin^3 t \cos^2 t \;\, dt \,.$$ **(4)**

c) By using the substitution $u = \cos t$, or otherwise, find the volume of S. **(6)**