IYGB GCE

Core Mathematics C4

Advanced

Practice Paper M

Difficulty Rating: 3.4800/1.5873

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Use an appropriate integration method to find

$$\int (x+1)e^{x+1}\, dx. \qquad \textbf{(4)}$$

Question 2

$$y = \sqrt{4-12x}, \quad -\frac{1}{3} < x < \frac{1}{3}.$$

a) Find the binomial expansion of y in ascending powers of x up and including the term in x^3, writing all coefficients in their simplest form. **(4)**

b) Hence find the coefficient of x^2 in the expansion of

$$(12x-4)(4-12x)^{\frac{1}{2}}. \qquad \textbf{(2)}$$

Question 3

Fine sand is dropping on a horizontal floor at the constant rate of $4 \text{ cm}^3\text{s}^{-1}$ and forms a pile whose volume, V cm^3, and height, h cm, are connected by the formula

$$V = -8 + \sqrt{h^4 + 64}.$$

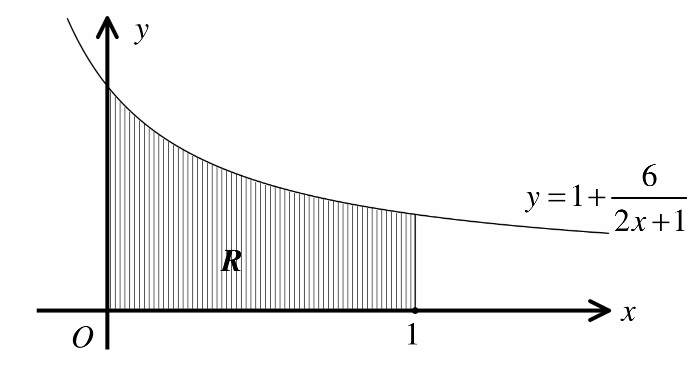
Find the rate at which the height of the pile is increasing, when the height of the pile has reached 2 cm. **(6)**

Question 4

By using the substitution $u = \sin x$, or otherwise, find

$$\int \cos^3 x\, dx. \qquad \textbf{(7)}$$

Question 5



The figure above shows the graph of the curve with equation

$$y = 1 + \frac{6}{2x+1}, \quad x \neq -\frac{1}{2}.$$

a) Show that

$$\left(1 + \frac{6}{2x+1}\right)^2 \equiv 1 + \frac{A}{2x+1} + \frac{B}{(2x+1)^2},$$

where A and B are constants to be found. **(2)**

The shaded region, labelled as R, bounded by the curve, the coordinate axes and the line $x = 1$ is rotated by 2π radians in the x axis to form a solid of revolution.

b) Show further that the volume generated is

$$\pi(13 + 6\ln 3).$$

(6)

Question 6

Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{x(2-x)}, \quad y > 0$$

subject to the condition $y = 1$ at $x = 1$, giving the answer in the form $y^2 = f(x)$. **(9)**

Question 7

The straight line l passes through the points with coordinates $(4,-1,1)$ and $(-1,4,6)$.

a) Determine a vector equation of l. **(3)**

The points C and D have coordinates $(4,-2,-3)$ and $(p,q,-1)$, respectively.

The midpoint of CD is the point M, where M lies on l.

Find in any order …

b) … the coordinates of M. **(3)**

c) … the value of p and the value of q. **(2)**

d) … the size of the acute angle θ, between CD and l. **(3)**

Question 8

The equation of a curve is given by

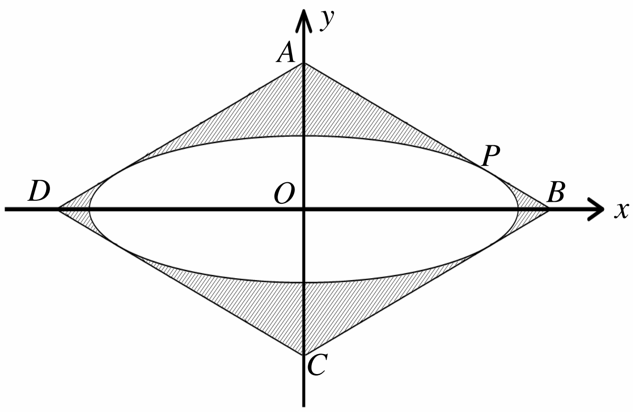
$$e^y = \frac{x^2+3}{x-1}.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{(x-3)(x+1)}{(x^2+3)(x-1)}. \qquad \textbf{(6)}$$

b) Find the exact coordinates of the turning point of the curve. **(3)**

Question 9



The figure above shows the design of a pendant $ABCD$ in the shape of a rhombus, made up of two different types of metal.

The innermost part of the design is enclosed by a curve and is made of silver. The rest of the design is made of gold.

The design is symmetrical about both the x and the y axis.

The innermost part of the design is modelled by an ellipse, given parametrically by

$$x = 12\cos\theta, \ y = 6\sin\theta, \ 0 \le \theta < 2\pi .$$

a) Use integration to show that the area enclosed by the ellipse is exactly 72π. **(8)**

The point P lies on the ellipse where $\theta = \frac{\pi}{6}$.

The straight line AB is the tangent to the ellipse at P.

b) Show that the equation of the tangent AB can be written as

$$2y + \sqrt{3}x = 24 .$$ **(5)**

c) Hence find an exact value for the area of the pendant that is made up of gold. **(2)**