C4, IYGB, PAPER L

1. a) $\int_0^4 e^{\frac{1}{2}x}\,dx = \left[2e^{\frac{1}{2}x}\right]_0^4 = 2e^2 - 2e^0 = 2(e^2-1)$

b) $\int_0^{\frac{\pi}{3}} \tan x\,dx = \left[\ln|\sec x|\right]_0^{\frac{\pi}{3}} = \ln(\sec\tfrac{\pi}{3}) - \ln(\sec 0)$

$= \ln 2 - \cancel{\ln 1} = \ln 2$

2. a) $f(x) = (1+3x)(1-\tfrac{2}{3}x)^{-2}$

$= (1+3x)\left[1 + \frac{-2}{1}(-\tfrac{2}{3}x)^1 + \frac{-2(-3)}{1\times 2}(-\tfrac{2}{3}x)^2 + \frac{-2(-3)(-4)}{1\times2\times3}(-\tfrac{2}{3}x)^3 + O(x^4)\right]$

$= (1+3x)(1 + \tfrac{4}{3}x + \tfrac{4}{3}x^2 + \tfrac{32}{27}x^3 + O(x^4))$

$= 1 + \tfrac{4}{3}x + \tfrac{4}{3}x^2 + \tfrac{32}{27}x^3 + O(x^4)$

$\quad\;\; 3x + 4x^2 + 4x^3 + O(x^4)$

$= 1 + \tfrac{13}{3}x + \tfrac{16}{3}x^2 + \tfrac{140}{27}x^3 + O(x^4)$

b) VALID FOR $|\tfrac{2}{3}x| < 1$

$|x| < \tfrac{3}{2}$ $\quad\therefore\; -\tfrac{3}{2} < x < \tfrac{3}{2}$

3. $\boxed{x = \frac{1-t^2}{1+t^2} \quad y = \frac{2t}{1+t^2} \quad \& \quad 3y = 4x}$

SOLVING SIMULTANEOUSLY

$\Rightarrow 3\left(\frac{2t}{\cancel{1+t^2}}\right) = 4\left(\frac{1-t^2}{\cancel{1+t^2}}\right)$

$\Rightarrow 6t = 4 - 4t^2$

$\Rightarrow 4t^2 + 6t - 4 = 0$

$\Rightarrow 2t^2 + 3t - 2 = 0$

$\Rightarrow (2t-1)(t+2)$

$t = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$

$x = \begin{cases} -\frac{3}{5} \\ 3/5 \end{cases}$

$y = \begin{cases} -\frac{4}{5} \\ 4/5 \end{cases}$

$(\tfrac{3}{5}, \tfrac{4}{5})$ & $(-\tfrac{3}{5}, -\tfrac{4}{5})$

4. a) $4y + y^2 e^{3x} = x^3 + C$

Diff w.r.t x

$\Rightarrow 4\frac{dy}{dx} + 2y\frac{dy}{dx}e^{3x} + 3y^2 e^{3x} = 3x^2$

$\Rightarrow (4 + 2ye^{3x})\frac{dy}{dx} = 3x^2 - 3y^2 e^{3x}$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2 e^{3x}}{4 + 2ye^{3x}}$

b) $\left.\frac{dy}{dx}\right|_{(1,k)} = 0$

$3 - 3k^2 e^3 = 0$

$1 - k^2 e^3 = 0$

$\boxed{k^2 e^3 = 1}$

$k^2 = \frac{1}{e^3} = e^{-3}$

$\boxed{k = e^{-\frac{3}{2}}}$

& $4 \times k + k^2 e^3 = 1^3 + C$

$4k + k^2 e^3 = 1 + C$

$4(e^{-\frac{3}{2}}) \cancel{+1} = \cancel{1} + C$

$C = 4e^{-\frac{3}{2}}$

5. a) $\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_0^{4\pi} 6\sin\frac{x}{4}\,dx$

$= \left[-24\cos\frac{x}{4}\right]_0^{4\pi} = 24\left[\cos\frac{x}{4}\right]_{4\pi}^{0}$

$= 24\left[\cos 0 - \cos\pi\right] = 24\left[1 - (-1)\right] = 48$

b) $\text{VOLUME} = \pi\int_{x_1}^{x_2}\left[y(x)\right]^2 dx = \pi\int_0^{4\pi}\left[6\sin\frac{x}{4}\right]^2 dx$

$= \pi\int_0^{4\pi} 36\sin^2\frac{x}{4}\,dx = \pi\int_0^{4\pi} 36\left(\frac{1}{2} - \frac{1}{2}\cos\frac{x}{2}\right) dx$

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$$= \pi \int_0^{4\pi} 18 - 18\cos\frac{x}{2}\,dx = \pi\left[18x - 36\sin\frac{x}{2}\right]_0^{4\pi}$$

$$= \pi\left[(72\pi - 0) - (0 - 0)\right] = 72\pi^2$$

6. a) $\vec{AC} = \underline{c} - \underline{a} = (8,-3,-1) - (-2,7,9) = (10,-10,-10)$

SCALE IT TO $(-1,1,1)$

$\underline{r} = (-2,7,9) + \lambda(-1,1,1)$

$(x,y,z) = (-2-\lambda,\ \lambda+7,\ \lambda+9)$

b) $E(2,p,q)$

(i): $-2-\lambda = 2$, $-4 = \lambda$, $\boxed{\lambda = -4}$

(j): $p = \lambda + 7$, $p = -4+7$, $p = 3$

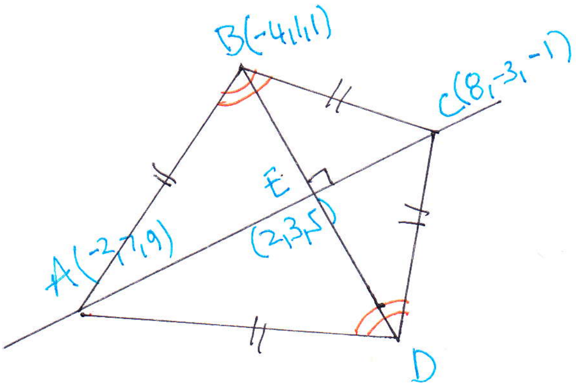
(k): $q = \lambda + 9$, $q = -4+9$, $q = 5$

c) $\vec{BE} = \underline{e} - \underline{b} = (2,3,5) - (-4,1,1) = (6,2,4)$

$$\underbrace{(6,2,4)}_{\vec{BE}} \cdot \underbrace{(-1,1,1)}_{\text{DIRECTION OF } \ell} = -6+2+4 = 0$$

INDEED PERPENDICULAR

d)



BY INSPECTION E IS THE MIDPOINT OF BD

$\therefore D(8,5,9)$

e) AREA OF KITE $= 2 \times$ AREA OF $\overset{\Delta}{ABC}$

$= \cancel{2 \times \frac{1}{2}} |\vec{AC}| \, |\vec{BE}|$

$= |10, -10, -10| \, |6, 2, 4|$

$= \sqrt{100+100+100} \sqrt{36+4+16}$

$= \sqrt{300} \sqrt{56}$

$= 10\sqrt{3} \; 2\sqrt{14}$

$= 20\sqrt{42}$

7. a) $\dfrac{4}{(1-u)^2(1+u)} \equiv \dfrac{A}{(1-u)^2} + \dfrac{B}{1-u} + \dfrac{C}{1+u}$

$4 \equiv A(1+u) + B(1-u)(1+u) + C(1-u)^2$

If $u=1$, $4 = 2A \Rightarrow \boxed{A=2}$

If $u=-1$, $4 = 4C \Rightarrow \boxed{C=1}$

If $u=0$, $4 = A+B+C \Rightarrow \boxed{B=1}$

b) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{4}{\cos x(1-\sin x)}\, dx = \dots$ SUBSTITUTION

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$x=0 \quad u=0$

$x=\frac{\pi}{6} \quad u=\frac{1}{2}$

$= \displaystyle\int_0^{\frac{1}{2}} \frac{4}{\cos x(1-u)} \frac{du}{\cos x} = \int_0^{\frac{1}{2}} \frac{4}{\cos^2 x(1-u)}\, du$

$= \displaystyle\int_0^{\frac{1}{2}} \frac{4}{(1-\sin^2 x)(1-u)}\, du = \int_0^{\frac{1}{2}} \frac{4}{(1-u^2)(1-u)}\, du$

$= \displaystyle\int_0^{\frac{1}{2}} \frac{4}{(1-u)(1+u)(1-u)}\, du = \int_0^{\frac{1}{2}} \frac{1}{(1-u)^2(1+u)}\, du$

$$= \int_0^{\frac{1}{2}} \frac{2}{(1-u)^2} + \frac{1}{1-u} + \frac{1}{1+u}\,du$$

$$\left[2(1-u)^{-2}\right]$$

$$= \left[2(1-u)^{-1} - \ln|1-u| + \ln|1+u|\right]_0^{\frac{1}{2}}$$

$$= \left[\ln|1+u| - \ln|1-u| + \frac{2}{1-u}\right]_0^{\frac{1}{2}}$$

$$= \left(\ln\tfrac{3}{2} - \ln\tfrac{1}{2} + 4\right) - \left(\cancel{\ln 1} - \cancel{\ln 1} + 2\right)$$

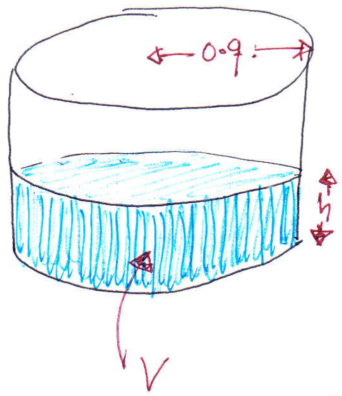
$$= \ln 3 + 2$$

8. a)

IN : $\frac{dV}{dt} = 0.36\pi$

OUT : $\frac{dV}{dt} = -0.45\pi h$

NET : $\frac{dV}{dt} = 0.36\pi - 0.45\pi h$



THUS

$$\frac{dV}{dt} = 0.36\pi - 0.45\pi h$$

$$\frac{dV}{dh} \times \frac{dh}{dt} = 0.36\pi - 0.45\pi h$$

$V = $ "πr^2" h

$V = \pi(0.9)^2 h$

$V = 0.81\pi h$

$\frac{dV}{dh} = 0.81\pi$

$$0.81\cancel{\pi}\frac{dh}{dt} = 0.36\cancel{\pi} - 0.45\cancel{\pi}h$$

$$0.81\frac{dh}{dt} = 0.36 - 0.45h$$

$$81\frac{dh}{dt} = 36 - 45h$$

$$9\frac{dh}{dt} = 4 - 5h$$

AS REQUIRED

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b) SOLVING BY SEPARATING VARIABLES

$$\Rightarrow \frac{9}{4-5h}\,dh = 1\,dt$$

$$\Rightarrow \int \frac{9}{4-5h}\,dh = \int 1\,dt$$

$$\Rightarrow -\frac{9}{5}\ln|4-5h| = t+C$$

$$\Rightarrow \ln|4-5h| = -\frac{5}{9}t + C$$

$$\Rightarrow 4-5h = e^{-\frac{5}{9}t+C} = e^{-\frac{5}{9}t} \times e^{C} = Ae^{-\frac{5}{9}t}$$

$$\Rightarrow 4-5h = Ae^{-\frac{5}{9}t}$$

$$\Rightarrow 4 + Ae^{-\frac{5}{9}t} = 5h$$

$$\Rightarrow h = \frac{4}{5} + Ae^{-\frac{5}{9}t}$$

WHEN $t=0$ $h=4$

$$4 = \frac{4}{5} + Ae^{0}$$

$$A = \frac{16}{5}$$

$$\therefore h = \frac{4}{5} + \frac{16}{5}e^{-\frac{5}{9}t}$$

$$h = \frac{4}{5}\left[1+4e^{-\frac{5}{9}t}\right]$$ // AS REQUIRED

c) $h = 1.6 = \frac{8}{5} \Rightarrow \frac{8}{5} = \frac{4}{5}\left[1+4e^{-\frac{5}{9}t}\right]$

$$2 = 1 + 4e^{-\frac{5}{9}t}$$

$$\frac{1}{4} = e^{-\frac{5}{9}t}$$

$$4 = e^{\frac{5}{9}t}$$

$$\frac{5}{9}t = \ln 4$$

$$\therefore t = \frac{9}{5}\ln 4 \approx 2.50$$ //