IYGB GCE

Core Mathematics C4

Advanced

Practice Paper L

Difficulty Rating: 3.6333/1.6901

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Evaluate each of the following integrals, giving the answers in exact form.

a) $\displaystyle\int_0^4 e^{\frac{1}{2}x} \, dx$. **(2)**

b) $\displaystyle\int_0^{\frac{\pi}{3}} \tan x \, dx$. **(3)**

Question 2

$$f(x) = (1+3x)\left(1-\tfrac{2}{3}x\right)^{-2}.$$

a) Show that if x is numerically small

$$f(x) \approx 1 + \frac{13}{3}x + \frac{16}{3}x^2 + \frac{140}{27}x^3. \qquad \textbf{(6)}$$

b) State the range of values of x for which the expansion of $f(x)$ is valid. **(1)**

Question 3

A curve C is given by the parametric equations

$$x = \frac{1-t^2}{1+t^2}, \quad y = \frac{2t}{1+t^2}, \quad t \in \mathbb{R}.$$

Determine the coordinates of the points of intersection between C and the straight line with equation

$$3y = 4x. \qquad \textbf{(7)}$$

Question 4

The equation of a curve is given implicitly by

$$4y + y^2 e^{3x} = x^3 + C,$$

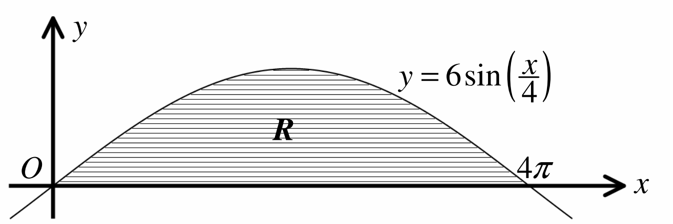
where C is a non zero constant.

a) Find a simplified expression for $\frac{dy}{dx}$. **(5)**

The point $P(1, k)$, where $k > 0$, is a stationary point of the curve.

b) Find an exact value for C. **(6)**

Question 5



The figure below shows the graph of the curve with equation

$$y = 6\sin\left(\frac{x}{4}\right),\ 0 \le x \le 4\pi.$$

The shaded region R, is bounded by the curve and the x axis.

a) Determine the area of R. **(3)**

This region R is rotated through $360°$ about the x axis to form a solid of revolution.

b) Show that the volume of the solid generated is $72\pi^2$. **(6)**

Question 6

The straight line l passes through the points A and C whose respective coordinates are $(-2,7,9)$ and $(8,-3,-1)$.

a) Find a vector equation for l. (2)

The point $E(2,p,q)$ lies on l and the point B has coordinates $(-4,1,1)$.

b) Determine the value of p and the value of q. (3)

c) Show that BE is perpendicular to l. (2)

The point D is such, so that $ABCD$ is a kite with $\measuredangle ABC = \measuredangle ADC$.

Determine …

d) … the coordinates of D. (2)

e) … the area of the kite $ABCD$. (3)

Question 7

$$\frac{4}{(1-u)^2(1+u)} \equiv \frac{A}{(1-u)^2} + \frac{B}{1-u} + \frac{C}{1+u}.$$

a) Find the value of A, B and C in the above identity. (3)

b) Hence by using a suitable substitution find the exact value of

$$\int_0^{\frac{\pi}{6}} \frac{4}{\cos x(1-\sin x)}\, dx.$$ (8)

Question 8

A cylindrical tank has constant radius of 0.9 m.

The volume, V m^3, of the water in the tank has height h m.

Water can be poured into the tank from a tap at the top of the tank and can be drained out of a tap at the base of the tank, which are initially both turned off.

Water then starts pouring in at the constant rate of 0.36π m^3 per minute and at the same time water begins to drain out at the rate of $0.45\pi h$ m^3 per minute.

a) Given further that t is measured from the instant when both taps were turned on, show that

$$9\frac{dh}{dt} = 4 - 5h. \qquad \textbf{(5)}$$

Initially the water in the tank has a height of 4 m.

b) Solve the above differential equation to show that

$$h = \frac{4}{5}\left(1 + 4\mathrm{e}^{-\frac{5}{9}t}\right). \qquad \textbf{(6)}$$

c) Find the value of t when $h = 1.6$. **(2)**
