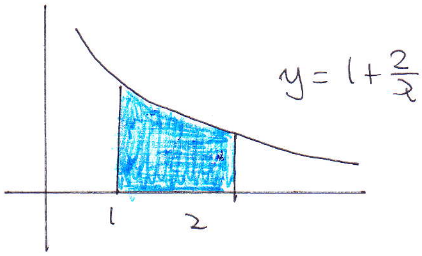
C4, IYGB, PAPER K

1.



$y = 1 + \frac{2}{x}$

$y^2 = \left(1 + \frac{2}{x}\right)^2 = 1 + \frac{4}{x} + \frac{4}{x^2}$

$$V = \pi \int_{x_1}^{x_2} (y(x))^2 \, dx = \pi \int_1^2 1 + \frac{4}{x} + 4x^{-2} \, dx$$

$$= \pi \left[x + 4\ln|x| - 4x^{-1} \right]_1^2 = \pi \left[x + 4\ln|x| - \frac{4}{x} \right]_1^2$$

$$= \pi \left[(\cancel{2} + 4\ln 2 - \cancel{2}) - (1 + \cancel{4\ln 1} - 4) \right] = \pi \left[4\ln 2 - 1 + 4 \right]$$

$$= \pi \left[3 + 4\ln 2 \right]$$

AS REQUIRED

2. a)

$$\frac{(1+2x)^2}{1-2x} = (1+2x)^2(1-2x)^{-1} = (1 + 4x + 4x^2)(1-2x)^{-1}$$

$$= (1 + 4x + 4x^2)\left[1 + \frac{-1}{1}(-2x)^1 + \frac{-1(-2)}{1\times 2}(-2x)^2 + \frac{-1(-2)(-3)}{1\times 2\times 3}(-2x)^3 + O(x^4)\right]$$

$$= (1 + 4x + 4x^2)\left[1 + 2x + 4x^2 + 8x^3 + O(x^4)\right]$$

$$\begin{aligned} = \; & 1 + 2x + 4x^2 + 8x^3 + O(x^4) \\ & \quad + 4x + 8x^2 + 16x^3 + O(x^4) \\ & \qquad\qquad 4x^2 + 8x^3 + O(x^4) \\ \hline & 1 + 6x + 16x^2 + 32x^3 + O(x^4) \end{aligned}$$

b) VALID FOR $|2x| < 1$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

C4, IYGB, PAPER K

3.
$$x^2\frac{dy}{dx} = y^2 - 3x^4y^2$$
$$\Rightarrow x^2\frac{dy}{dx} = y^2(1-3x^4)$$
$$\Rightarrow \frac{1}{y^2}\,dy = \frac{1-3x^4}{x^2}\,dx$$
$$\Rightarrow \int \frac{1}{y^2}\,dy = \int \frac{1}{x^2} - \frac{3x^4}{x^2}\,dx$$
$$\Rightarrow \int y^{-2}\,dy = \int x^{-2} - 3x^2\,dx$$
$$\Rightarrow -y^{-1} = -x^{-1} - x^3 + C$$
$$\Rightarrow -\frac{1}{y} = -\frac{1}{x} - x^3 + C$$
$$\Rightarrow \frac{1}{y} = \frac{1}{x} + x^3 + C$$

APPLY CONDITIONS

$x=1 \quad y=\frac{1}{2}$

$2 = 1 + 1 + C$

$\boxed{C=0}$

$$\Rightarrow \frac{1}{y} = \frac{1}{x} + x^3$$
$$\Rightarrow \frac{1}{y} = \frac{1+x^4}{x}$$
$$\Rightarrow y = \frac{x}{1+x^4}$$

4. a)
$$f(x) = \frac{5}{3x^2-5x} = \frac{5}{x(3x-5)} \equiv \frac{A}{x} + \frac{B}{3x-5}$$

$$\boxed{5 \equiv A(3x-5) + Bx}$$

If $x=0 \Rightarrow 5 = -5A \Rightarrow A = -1$

If $x=\frac{5}{3} \Rightarrow 5 = \frac{5}{3}B \Rightarrow B = 3$

$$\therefore f(x) = \frac{3}{3x-5} - \frac{1}{x}$$

b)
$$\int_3^5 f(x)\,dx = \int_3^5 \frac{3}{3x-5} - \frac{1}{x}\,dx = \Big[\ln|3x-5| - \ln|x|\Big]_3^5$$
$$= (\ln 10 - \ln 5) - (\ln 4 - \ln 3) = \ln 2 - \ln\frac{4}{3}$$
$$= \ln\frac{2}{\frac{4}{3}} = \ln\frac{3}{2}$$

C4, IYGB, PAPER K

5. $y = 15\left[4 - \frac{27}{(x+3)^3}\right]$ & $\ln(x+3) = \frac{1}{3}t$

$y = 15\left[4 - 27(x+3)^{-3}\right]$ $\qquad$ $t = 3\ln(x+3)$

$\frac{dy}{dx} = 15\left[81(x+3)^{-4}\right]$ $\qquad$ $\frac{dt}{dx} = \frac{3}{x+3}$

$\frac{dy}{dx} = \frac{1215}{(x+3)^4}$ $\qquad$ $\frac{dx}{dt} = \frac{x+3}{3}$

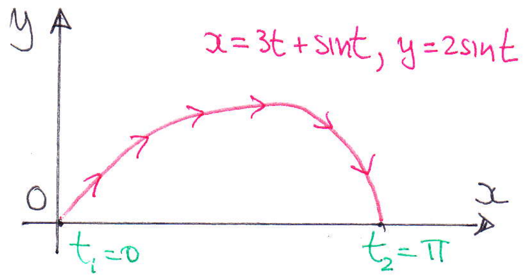
$\therefore \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} =$

$\frac{dy}{dt} = \frac{1215}{(x+3)^4} \times \frac{x+3}{3}$

$\frac{dy}{dt} = \frac{405}{(x+3)^3}$

$\left.\frac{dy}{dt}\right|_{x=9} = \frac{405}{1728} = \frac{15}{64}$ //

6.



$y = 0$
$0 = 2\sin t$
$\sin t = 0$
$t = 0, \pi$

AREA $= \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt = \int_0^{\pi} (2\sin t)(3+\cos t)\,dt$

$= \int_0^{\pi} 6\sin t + 2\sin t\cos t\,dt = \int_0^{\pi} 6\sin t + \sin 2t\,dt$

C4, IYGB, PAPER K

$$= \left[-6\cos t - \tfrac{1}{2}\cos 2t\right]_0^{\pi} = \left[6\cos t + \tfrac{1}{2}\cos 2t\right]_{\pi}^{0}$$

$$= \left(6+\tfrac{1}{2}\right) - \left(-6+\tfrac{1}{2}\right) = 6+\tfrac{1}{2}+6-\tfrac{1}{2} = 12$$

7. $\displaystyle\int_4^8 \frac{6x}{\sqrt{2x-7}}\,dx = \ldots$ BY SUBSTITUTION

$$u = \sqrt{2x-7}$$
$$u^2 = 2x-7$$
$$\cancel{2}u\frac{du}{dx} = \cancel{2}$$
$$u\,du = dx$$

$x=4 \mapsto u=1$

$x=8 \mapsto u=3$

$$2x = u^2+7$$
$$6x = 3u^2+21$$

$$= \int_1^3 \frac{6x}{\cancel{u}}(\cancel{u}\,du) = \int_1^3 6x\,du$$

$$= \int_1^3 3u^2+21\,du = \left[u^3+21u\right]_1^3$$

$$= (27+63)-(1+21) = 90-22$$

$$= 68$$

8. a) $\underline{a} = (7,2,3)$

$\underline{c} = (3,-2,1)$

$\overrightarrow{AC} = \underline{c}-\underline{a} = (3,-2,1)-(7,2,3)$

$= (-4,-4,-2)$

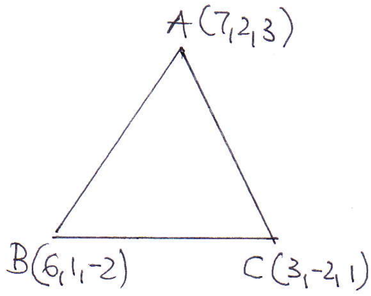
b) MIDPOINT $= \left(\frac{7+3}{2}, \frac{2-2}{2}, \frac{3+1}{2}\right)$ IE $(5,0,2)$

c) $(-4,-4,-2)\cdot(1,1,-4) = -4-4+8 = 0$

DIRECTION VECTOR OF LINE

INDEED PERPENDICULAR

C4, IYGB, PAPER K



d) $\underline{r} = (5,0,2) + \lambda(1,1,-4) = (\lambda+5, \lambda, -4\lambda+2)$

IF $\lambda = 1$ $\boxed{B(6,1,-2)}$

$\underline{a} = (7,2,3)$
$\underline{b} = (6,1,-2)$
$\underline{c} = (3,-2,1)$

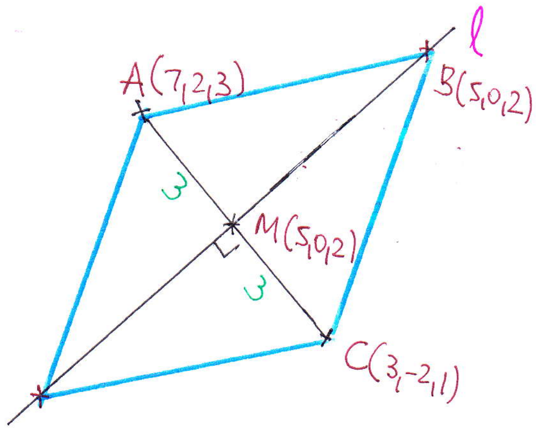
- $|\vec{AB}| = |\underline{b} - \underline{a}| = |(6,1,-2) - (7,2,3)| = |-1,-1,-5|$
$= \sqrt{1+1+25} = \sqrt{27}$

- $|\vec{BC}| = |\underline{c} - \underline{b}| = |(3,-2,1) - (6,1,-2)| = |-3,-3,3|$
$= \sqrt{\quad +9+9} = \sqrt{27}$

- $|\vec{AC}| = |-4,-4,-2| = \sqrt{16+16+4} = \sqrt{36} = 6$

$\therefore |\vec{AB}| = |\vec{BC}| \neq |\vec{AC}|$ ISOSCELES BUT NOT EQUILATERAL

e)



- $|\vec{MB}| = |\underline{b} - \underline{m}| = |(6,1,-2) - (5,0,2)|$
$= |1,1,-4| = \sqrt{1+1+16} = \sqrt{18}$

- AREA $= 2 \times$ AREA OF $\hat{ABC}$

$= \cancel{2} \times \frac{1}{\cancel{2}} |AC||MB|$

$= 6 \times \sqrt{18}$

$= 6 \times 3\sqrt{2}$

$= 18\sqrt{2}$

AS REQUIRED.

9. a) $4y - 2xy + 6 = y^2 + 3x^2$

Diff w.r.t x

$$4\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} + 0 = 2y\frac{dy}{dx} + 6x$$

$$(4 - 2x - 2y)\frac{dy}{dx} = 6x + 2y$$

$$\frac{dy}{dx} = \frac{6x + 2y}{4 - 2x - 2y}$$

$$\frac{dy}{dx} = \frac{3x + y}{2 - x - y}$$ // AS REQUIRED

b) GRADIENT L_1 = GRADIENT $L_2 = 1$

$$1 = \frac{3x + y}{2 - x - y}$$

$$2 - x - y = 3x + y$$

$$2 = 2y + 4x$$

$$1 = y + 2x$$

$$\boxed{y = 1 - 2x}$$

SOLVING SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

$\Rightarrow 4(1-2x) - 2x(1-2x) + 6 = (1-2x)^2 + 3x^2$

$\Rightarrow 4 - 8x - 2x + \cancel{4x^2} + 6 = 1 - 4x + \cancel{4x^2} + 3x^2$

$\Rightarrow 0 = 3x^2 + 6x - 9$

$\Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1) = 0$

$\Rightarrow x = \begin{cases} 1 \\ -3 \end{cases} \qquad y = \begin{cases} -1 \\ 7 \end{cases} \qquad \begin{matrix} (1,-1) \\ (-3,7) \end{matrix}$

$L_1: \quad y+1 = 1(x-1)$

$y+1 = x-1$

$\boxed{y = x-2}$

$L_2: \quad y-7 = 1(x+3)$

$y-7 = x+3$

$y = x+10$ //