IYGB GCE

Core Mathematics C3

Advanced

Practice Paper V

Difficulty Rating: 3.88/1.8868

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1 **(***)**

$$f(x) = x\ln\left(1+x^2\right), \ x \in \mathbb{R}.$$

Show that an equation of the tangent to the curve with equation $y = f(x)$, at the point where $x = 1$, is given by

$$y = x(1+\ln 2) - 1. \qquad \textbf{(7)}$$

Question 2 **(***+)**

$$\sin x = \frac{12}{13} \text{ and } \cos y = \frac{15}{17}.$$

If x is obtuse and y is acute, show clearly that

$$\sin(x-y) = \frac{220}{221}. \qquad \textbf{(5)}$$

Question 3 **(****)**

The functions f and g are defined by

$$f(x) = x^2 + 3x - 7, \ x \in \mathbb{R}$$

$$g(x) = ax + b, \ x \in \mathbb{R},$$

where a and b are positive constants.

When the composition $fg(x)$ is divided by $(x+2)$ the remainder is 21, while $(x-1)$ is a factor of the composition $gf(x)$.

Determine the value of a and the value of b. **(8)**

Question 4 (****)

The function f is defined as

$$f(x) = a\ln(bx),\ x \in \mathbb{R},\ x > 0,$$

where a and b are positive constants.

a) Given that the graph of $f(x)$ passes through the points $\left(\frac{1}{3},0\right)$ and $(3,4)$, find the exact value of a and the value of b. **(5)**

b) Sketch the graph of

$$y = |f(x)|.$$ **(2)**

c) Find, in exact form where appropriate, the solutions of the equation

$$|f(x)| = 8.$$ **(5)**

Question 5 (****)

It is given that

$$\cos 3x \equiv 4\cos^3 x - 3\cos x.$$

a) Prove the validity of the above trigonometric identity by writing $\cos 3x$ as $\cos(2x + x)$. **(5)**

b) Hence, or otherwise, solve the trigonometric equation

$$8\cos^3 x - 6\cos x + 1 = 0,\ 0 \le x < 2\pi,$$

giving the answers in terms of π. **(5)**

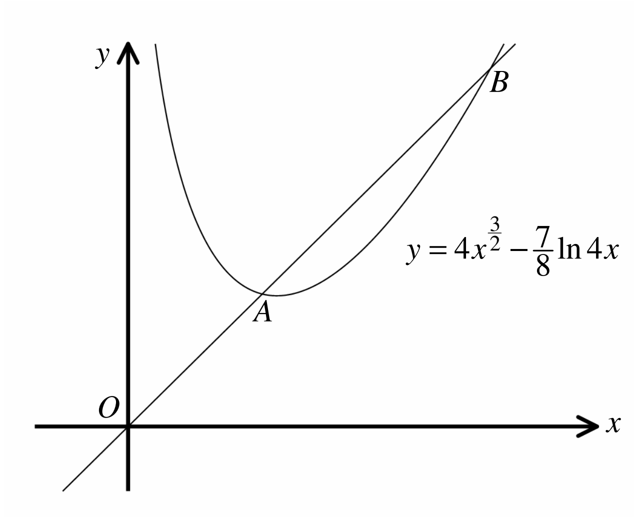
Question 6 (***+)

A curve C has equation

$$y = 4x^{\frac{3}{2}} - \frac{7}{8}\ln 4x, \quad x \in \mathbb{R}, \quad x > 0.$$

The point A is on C, where $x = \frac{1}{4}$.

a) Find an equation of the normal to the curve at A. **(4)**



This normal meets the curve again at the point B, as shown in the figure above.

b) Show that the x coordinate of B satisfies the equation **(3)**

$$x = \left(\frac{16x + 7\ln 4x}{32}\right)^{\frac{2}{3}}.$$

The recurrence relation

$$x_{n+1} = \left(\frac{16x_n + 7\ln 4x_n}{32}\right)^{\frac{2}{3}}, \; x_0 = 0.7$$

is to be used to find the x coordinate of B.

c) Find, to 3 decimal places, the value of x_1, x_2, x_3 and x_4. **(2)**

d) Show that the x coordinate of B is 0.6755, correct to 4 decimal places. **(3)**

Question 7 **(****)**

The curve C has equation

$$y = \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}(2x+1), \quad -1 < x \le 1.$$

By taking logarithms on both sides of this equation, or otherwise, show that at the point on C where $x = \frac{1}{2}$, the gradient is $-\frac{2}{9}\sqrt{3}$. **(8)**

Question 8 **(****+)**

The populations P_1 and P_2 of two bacterial cultures, t hours after a certain instant, are modelled by the following equations

$$P_1 = 1600\mathrm{e}^{\frac{1}{4}t}, \; P_2 = 100\mathrm{e}^{\frac{1}{2}t}, \; t \in \mathbb{R}, \; t \ge 0.$$

When $t = T$, P_1 contains 4800 more bacteria than P_2.

a) Find, in terms of natural logarithms, the possible values of T. **(6)**

At a certain time there are P extra bacteria in P_1 compared with P_2.

b) Determine the greatest value of P. **(7)**
