IYGB GCE

Core Mathematics C3

Advanced

Practice Paper S

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Show that the expression

$$\left[\mathrm{e}-\left(\frac{\mathrm{e}^{x+\frac{1}{2}}}{\mathrm{e}^{-2x}}\right)^2\right]\times\frac{1}{\mathrm{e}^{3x}+\ln\mathrm{e}}$$

simplifies to $\mathrm{e}-\mathrm{e}^{3x+1}$. **(8)**

Question 2

$$f(x)\equiv\frac{\left(3-2\cos^2 x\right)\left(1+6\sin^2 x\right)^{\frac{1}{2}}}{\left(1+\tan x\right)^2},\ \tan x\neq -1.$$

Determine the value of $f'\left(\frac{\pi}{4}\right)$. **(10)**

Question 3

Solve the trigonometric equation

$$\sin x\cos x+\frac{1}{2}=\cos^2\left(\frac{x}{2}-\frac{\pi}{6}\right),\ 0\leq x<\pi,$$

giving the answers in terms of π. **(12)**

Question 4

Sketch the graph of

$$y=\left|x^2-16\right|+2x,\ x\in\mathbb{R}.$$

The sketch must include the coordinates of any cusps or any stationary points **(7)**

Question 5

The functions f and g are defined by

$$f(x)=\begin{cases} x^2, & x\in\mathbb{R},\ 0<x\leq 1 \\ 2-x, & x\in\mathbb{R},\ 1<x\leq 2 \end{cases}$$

$$g(x)=\frac{1}{x},\ x\in\mathbb{R},\ x\neq 0.$$

a) Find expressions for the function compositions $fg(x)$ and $gf(x)$, giving full descriptions of their domains. **(6)**

b) Sketch the graphs of the function compositions $fg(x)$ and $gf(x)$, and hence state the ranges of $fg(x)$ and $gf(x)$. **(8)**

Question 6

It is given that θ and φ satisfy the simultaneous equations

$$\frac{\sin 2\theta}{1+\sin\theta}=1-\sin\varphi,$$

$$\theta+\varphi=\pi$$

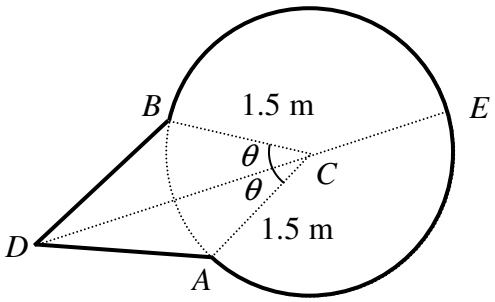
where $0<\theta<\pi$, $\theta\neq\frac{\pi}{2}$, $0<\varphi<\pi$.

a) Determine the value of $\tan\theta$. **(6)**

b) Show clearly that

$$\tan(3\theta+5\varphi)=-\frac{4}{3}.$$ **(6)**

Question 7



The figure above shows a circle with centre at C and radius 1.5 metres.

The points A and B lie on the circle so that $\measuredangle BCA = 2\theta$, $0 < \theta < \pi$.

The point D lies outside the circle so that the line segments BD and AD are equal in length and the length of DC is 3 metres. The point E lies on the circle so that DCE is a straight line segment of length 4.5 metres.

The **total length** of the line segment BD, the line segment AD and the circular arc $\overset{\frown}{AEB}$ denoted by L.

Given that θ varies, show that L has a stationary value when $\theta = \dfrac{\pi}{3}$ and determine further the value L and the nature of this stationary value.

(12)