C3, IYGB, PAPER R

1. a) $y=0$ IF $x^3-3x^2-3=0$

LET $\boxed{f(x)=x^3-3x^2-3}$

$f(3) = 27-27-3 = -3 < 0$
$f(4) = 64-48-3 = 15 > 0$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN, THERE IS AT LEAST ONE SOLUTION IN THE INTERVAL

b) $x^3-3x^2-3=0$
$x^3 = 3x^2+3$
$x = \frac{3x^2+3}{x^2}$
$x = 3+\frac{3}{x^2}$

c) $x_{n+1} = 3+\frac{3}{x_n^2}$

$x_1 = 4$
$x_2 = 3.1875$
$x_3 = 3.2953$
$x_4 = 3.2763$
$x_5 = 3.2795$

2.

$$\frac{\frac{1}{12}-\frac{1}{3x^2}}{\frac{1}{12}+\frac{1}{4x}+\frac{1}{6x^2}} = \frac{\frac{1}{12}(12x^2)-\frac{1}{3x^2}(12x^2)}{\frac{1}{12}(12x^2)+\frac{1}{4x}(12x^2)+\frac{1}{6x^2}(12x^2)}$$

$$= \frac{x^2-4}{x^2+3x+2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+1)} = \frac{x-2}{x+1}$$

IF $a=-2$
$b=1$

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3. $4\sin x\cos x = 1$

$\Rightarrow 2\sin x\cos x = \frac{1}{2}$

$\Rightarrow \sin 2x = \frac{1}{2}$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\left(\begin{array}{l} 2x = \frac{\pi}{6} \pm 2n\pi \\ 2x = \frac{5\pi}{6} \pm 2n\pi \end{array} \right.$ $n = 0, 1, 2, 3, \ldots$

$\left(\begin{array}{l} x = \pi/12 \pm n\pi \\ x = 5\pi/12 \pm n\pi \end{array} \right.$

$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}$

4. a) $y = e^{2x}(\cos x + \sin x)$ (BY PRODUCT RULE)

$\frac{dy}{dx} = 2e^{2x}(\cos x + \sin x) + e^{2x}(-\sin x + \cos x)$

b) $\frac{dy}{dx} = \underline{2e^{2x}\cos x} + 2e^{2x}\sin x - e^{2x}\sin x + \underline{e^{2x}\cos x}$

$\frac{dy}{dx} = 3e^{2x}\cos x + e^{2x}\sin x$

$\frac{dy}{dx} = e^{2x}(3\cos x + \sin x)$ AS REQUIRED

c) $\frac{dy}{dx} = 0$

$e^{2x}(3\cos x + \sin x) = 0$

$3\cos x + \sin x = 0$ $\quad e^{2x} \neq 0$

$\frac{3\cos x}{\cos x} + \frac{\sin x}{\cos x} = 0$

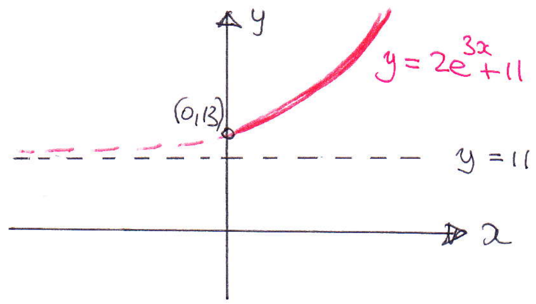
$3 + \tan x = 0$

$\therefore \tan x = -3$

5. a) $g(f(x)) = g(e^x) = 2(e^x)^3 + 11 = 2e^{3x} + 11$

b) $\xrightarrow{x>0} \boxed{f} \rightarrow \quad \xrightarrow{x \in \mathbb{R}} \boxed{g}$

DOMAIN, $x > 0$



RANGE, $g(f(x)) > 13$

c) $g(f(x)) = 27$

$2e^{3x} + 11 = 27$

$2e^{3x} = 16$

$e^{3x} = 8$

$3x = \ln 8$

$3x = 3\ln 2$

$x = \ln 2$

$\simeq 0.693$

d) FROM GRAPH ABOUT $k > 13$

(ANY HORIZONTAL LINE GREATER THAN $y = 13$ WILL CROSS THE GRAPH OF $gf(x)$)

6. a)

$$y = \frac{x}{x^2+9}$$

$$\frac{dy}{dx} = \frac{(x^2+9)\times 1 - x(2x)}{x^2+9} = \frac{x^2+9-2x^2}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2}$$

SOLVE $\frac{dy}{dx} = 0$

$$\frac{9-x^2}{(x^2+9)^2} = 0$$

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$9 - x^2 = 0$

$x^2 = 9$

$x = \begin{cases} 3 \\ -3 \end{cases}$ $\quad y = \begin{cases} \frac{1}{6} \\ -\frac{1}{6} \end{cases}$ $\quad$ IE $\quad (3, \frac{1}{6})$ $\quad (-3, -\frac{1}{6})$

b) $\frac{dy}{dx} = \frac{9 - x^2}{(x^2+9)^2}$

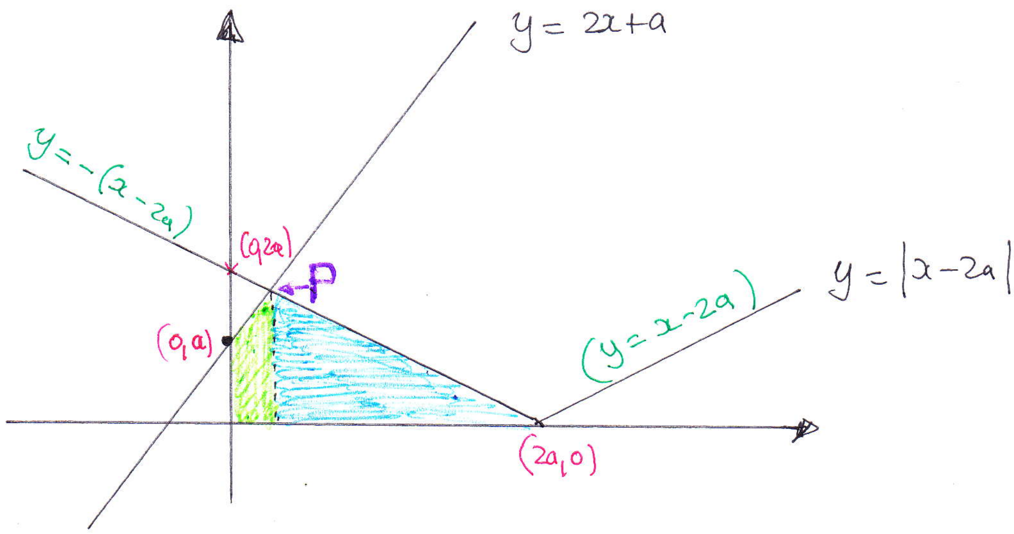
$\frac{d^2y}{dx^2} = \frac{(x^2+9)^2(-2x) - (9-x^2) \times 2(x^2+9)(2x)}{(x^2+9)^4}$

$\frac{d^2y}{dx^2} = \frac{-2x(x^2+9)^{\cancel{2}} - 4x\cancel{(x^2+9)}(9-x^2)}{(x^2+9)^{\cancel{4}\,3}}$

$\frac{d^2y}{dx^2} = \frac{-2x(x^2+9) - 4x(9-x^2)}{(x^2+9)^3}$

$\left.\frac{d^2y}{dx}\right|_{x=\pm 3} = \frac{-2(\pm 3) \times 18 - 0}{18^3} = \mp \frac{1}{54}$

7.



- FIND x & y INTERCEPTS BY INSPECTION
- FIND INTERSECTION

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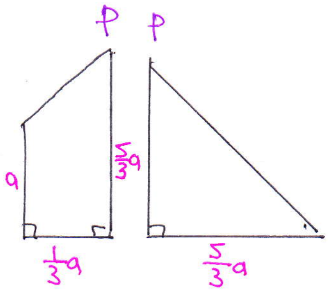
Hmce $\left.\begin{array}{l} y=-(x-2a) \\ y=2x+a \end{array}\right\} \Longrightarrow$

$$-x+2a = 2x+a$$
$$a = 3x$$
$$\boxed{x = \tfrac{1}{3}a}$$

$$y = 2\left(\tfrac{1}{3}a\right)+a$$
$$\boxed{y = \tfrac{5}{3}a}$$

$$\therefore P\left(\tfrac{1}{3}a, \tfrac{5}{3}a\right)$$



$$= \left[\frac{a+\frac{5}{3}a}{2} \times \frac{1}{3}a\right] + \left[\frac{1}{2}\times\frac{5}{3}a\times\frac{5}{3}a\right]$$

$$= \frac{4}{9}a^2 + \frac{25}{18}a^2 = \frac{11}{6}a^2$$

8. LET $A = 45°$
$B = 30°$

$$\cot(45+30) = \frac{1}{\tan(45+30)} = \frac{1}{\frac{\tan 45 + \tan 30}{1-\tan 45 \tan 30}}$$

$$= \frac{1-\tan 45\tan 30}{\tan 45 + \tan 30} = \frac{1 - 1\times\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}} = \frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{(3-\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$= \frac{9-3\sqrt{3}-3\sqrt{3}+3}{9\cancel{-3\sqrt{3}+3\sqrt{3}}-3} = \frac{12-6\sqrt{3}}{6}$$

$$= 2-\sqrt{3}$$

C3, IYGB, PAPER R

9. $\ln(ex^2)\ln x = 1$

$\Rightarrow [\ln e + \ln x^2]\ln x = 1$

$\Rightarrow [1 + 2\ln x]\ln x = 1$

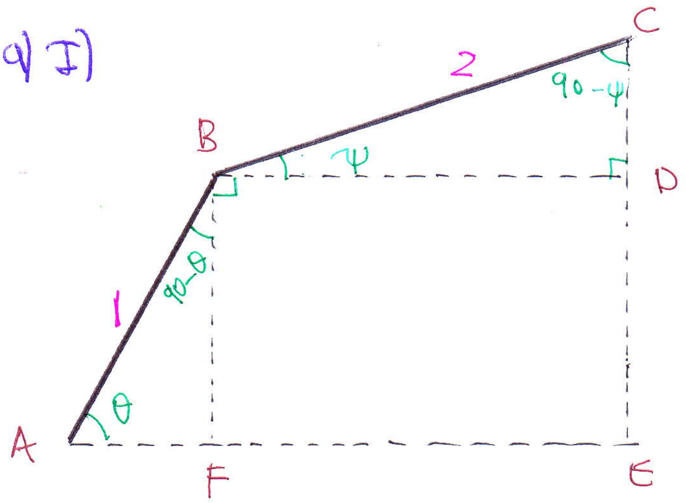
$\Rightarrow \ln x + 2(\ln x)^2 = 1$

$\Rightarrow 2(\ln x)^2 + \ln x - 1 = 0$

$\Rightarrow (2\ln x - 1)(\ln x + 1) = 0$

$\Rightarrow \ln x = \begin{cases} -1 \\ \frac{1}{2} \end{cases} \Rightarrow x = \begin{cases} e^{-1} = \frac{1}{e} \\ e^{\frac{1}{2}} = \sqrt{e} \end{cases}$

10. a I)



LET $\psi = D\hat{B}C$

$\Rightarrow (90-\theta) + 90 + \psi = 120$

$\Rightarrow 180 + \psi - \theta = 120$

$\Rightarrow \psi = \theta - 60$

II) $h = |CD| + |DE|$

$h = |CD| + |BF|$

$h = 2\sin\psi + 1\sin\theta$

$h = 2\sin(\theta - 60) + \sin\theta$

$h = 2\sin\theta \overset{\frac{1}{2}}{\cos 60} - 2\cos\theta \overset{\frac{\sqrt{3}}{2}}{\sin 60} + \sin\theta$

$h = \sin\theta - \sqrt{3}\cos\theta + \sin\theta$

$h = 2\sin\theta - \sqrt{3}\cos\theta$

AS REQUIRED

II) WHEN AC IS HORIZONTAL, IT C TOUCHES THE GROUND, THEN $h=0$

$$2\sin\theta - \sqrt{3}\cos\theta = 0$$

$$\frac{2\sin\theta}{\cos\theta} - \frac{\sqrt{3}\cos\theta}{\cos\theta} = \frac{0}{\cos\theta}$$

$$2\tan\theta - \sqrt{3} = 0$$

$$\tan\theta = \frac{\sqrt{3}}{2}$$

AS REQUIRED

b) $$2\sin\theta - \sqrt{3}\cos\theta \equiv R\sin(\theta-\alpha)$$

$$\equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

$$\equiv (R\cos\alpha)\sin\theta - (R\sin\alpha)\cos\theta$$

$$\therefore \left.\begin{array}{l} R\cos\alpha = 2 \\ R\sin\alpha = \sqrt{3} \end{array}\right\} \Rightarrow R = \sqrt{2^2+\sqrt{3}^2} = \sqrt{7}$$

$$\Rightarrow \tan\alpha = \frac{\sqrt{3}}{2} \qquad \alpha \simeq 40.9°$$

$$h = \sqrt{7}\sin(\theta - 40.9°)$$

- $h_{MAX} = \sqrt{7} \simeq 2.65\text{ m}$

- IT OCCURS WHEN $\theta - 40.9 = 90$ ← $\sin(\theta-40.9) = +1$

$$\theta = 130.9°$$

$$\theta \simeq 131°$$