IYGB GCE

Core Mathematics C3

Advanced

Practice Paper R

Difficulty Rating: 3.52/1.6129

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve C has equation

$$y = x^3 - 3x^2 - 3,$$

and crosses the x axis at the point $A(\alpha, 0)$.

a) Show that α lies between 3 and 4. **(2)**

b) Show further that the equation $x^3 - 3x^2 - 3 = 0$ can be rearranged to

$$x = 3 + \frac{3}{x^2}, \quad x \neq 0.$$ **(2)**

The equation rearrangement of part **(b)** is written as the following recurrence relation

$$x_{n+1} = 3 + \frac{3}{x_n^{\ 2}}, \quad x_1 = 4.$$

c) Use the above iterative formula to find, to 4 decimal places, the value of x_2, x_3, x_4 and x_5. **(2)**

Question 2

Show clearly that

$$\frac{\frac{1}{12} - \frac{1}{3x^2}}{\frac{1}{12} + \frac{1}{4x} + \frac{1}{6x^2}} \equiv \frac{x+a}{x+b}.$$

where a and b are integers to be found **(5)**

Question 3

Solve the following trigonometric equation

$$4\sin x\cos x = 1, \quad 0 \le x < \pi,$$

giving the answers in terms of π. **(5)**

Question 4

The equation of the curve C is given by

$$y = e^{2x}\left(\cos x + \sin x\right).$$

a) Find an expression for $\dfrac{dy}{dx}$. **(3)**

b) Show further that $\dfrac{dy}{dx}$ can be simplified to

$$\frac{dy}{dx} = e^{2x}\left(\sin x + 3\cos x\right).$$ **(2)**

c) Hence show that the x coordinates of the turning points of C satisfy

$$\tan x = -3$$ **(2)**

Question 5

$$f(x) = \mathrm{e}^x, \ x \in \mathbb{R}, \ x > 0.$$

$$g(x) = 2x^3 + 11, \ x \in \mathbb{R}.$$

a) Find and simplify an expression for the composite function $gf(x)$. **(2)**

b) State the domain and range of $gf(x)$. **(2)**

c) Solve the equation

$$gf(x) = 27.$$

(3)

The equation $gf(x) = k$, where k is a constant, has solutions.

d) State the range of the possible values of k. **(1)**

Question 6

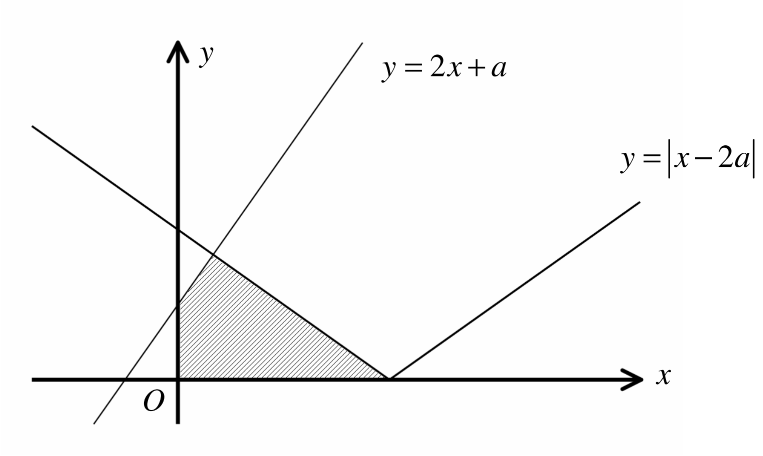
The equation of a curve C is

$$y = \frac{x}{x^2 + 9}, \ x \in \mathbb{R}.$$

a) Find the coordinates of the stationary points of C. **(6)**

b) Evaluate the exact value of $\frac{d^2y}{dx^2}$ at each of these stationary points. **(4)**

Question 7



The figure above shows the graphs of

$$C_1: \ y = |x - 2a|, \ x \in \mathbb{R},$$

$$C_2: \ y = 2x + a, \ x \in \mathbb{R}.$$

The finite region bounded by C_1, C_2 and the coordinate axes is shown shaded in the above diagram.

Find, in terms of a, the exact area of the shaded region. **(8)**

Question 8

By considering the compound angle identity for $\tan(A+B)$, with suitable values for A and B, show that

$$\cot 75° = 2 - \sqrt{3}.$$ **(5)**

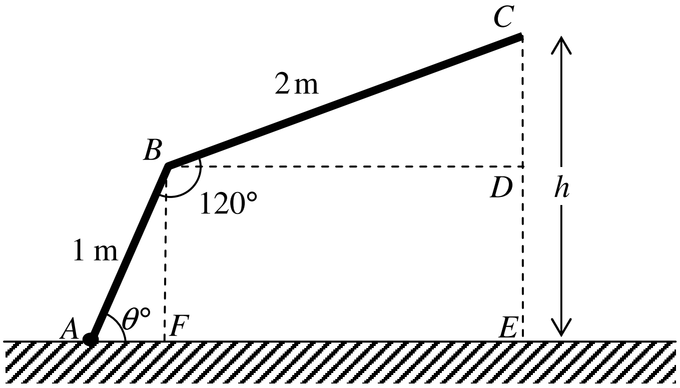
Question 9

Solve the equation

$$\ln\left(\mathrm{e}x^2\right)\ln x = 1,\ \ x > 0,$$

giving the answers in exact form. **(7)**

Question 10



The figure above shows a rigid rod ABC where AB is 1 metre, BC is 2 metres and the angle ABC is $120°$. The rod is hinged at A so it can be rotated in a vertical plane forming an angle $\theta°$ with the horizontal ground.

Let h metres be the height of the point C from the horizontal ground.

a) Show that …

i. … $\measuredangle DBC = \theta° - 60°$. **(2)**

ii. … $h = 2\sin\theta - \sqrt{3}\cos\theta$. **(4)**

iii. … when AC is horizontal, $\tan\theta = \frac{1}{2}\sqrt{3}$. **(2)**

b) Express h in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, and hence find the maximum value of h and the value of θ when h takes this maximum value. **(6)**
