C3, IYGB, PAPER Q

1. a) $y = \sqrt{x^2-1} = (x^2-1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \times 2x$

$\frac{dy}{dx} = x(x^2-1)^{-\frac{1}{2}}$

$= \frac{x}{\sqrt{x^2-1}}$

b) $y = x^4 \ln x$

$\frac{dy}{dx} = 4x^3 \ln x + x^4 \times \frac{1}{x}$

$\frac{dy}{dx} = 4x^3 \ln x + x^3$

$\frac{dy}{dx} = x^3(4\ln x + 1)$

c) $y = \frac{e^x - 1}{e^x + 1}$

$\frac{dy}{dx} = \frac{(e^x+1)e^x - (e^x-1)e^x}{(e^x+1)^2} = \frac{\cancel{e^{2x}} + e^x - \cancel{e^{2x}} + e^x}{(e^x+1)^2}$

$= \frac{2e^x}{(e^x+1)^2}$

2. $\frac{[(3x-1)(2x+3) - 2(4x-1)](3x-1)}{(3x+1)} = \frac{(6x^2 + 9x - 2x - 3 - 8x + 2)(3x-1)}{3x+1}$

$= \frac{(6x^2 - x - 1)(3x-1)}{3x+1} = \frac{\cancel{(3x+1)}(2x-1)(3x-1)}{\cancel{(3x+1)}}$

$= 6x^2 - 2x - 3x + 1 = 6x^2 - 5x + 1$

$(a = 6,\ b = -5,\ c = 1)$

3. I) $2\sec\theta - 1 = 2\sec\theta \sin^2\theta$

$\Rightarrow 2\sec\theta - 1 = 2\sec\theta(1 - \cos^2\theta)$

$\Rightarrow \cancel{2\sec\theta} - 1 = \cancel{2\sec\theta} - 2\sec\theta\cos^2\theta$

$\Rightarrow 2\sec\theta\cos^2\theta = 1$

$\Rightarrow \frac{2}{\cos\theta}\cos^2\theta = 1$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\arccos\frac{1}{2} = 60$

$\theta = 60 \pm 360n$

$\theta = 300 \pm 360n$ $\quad n = 0, 1, 2, 3, \ldots$

$\therefore \theta = 60$

II) $4\cot^2 x - 9\operatorname{cosec} x + 6 = 0$

$\Rightarrow 4(\operatorname{cosec}^2 x - 1) - 9\operatorname{cosec} x + 6 = 0$

$\Rightarrow 4\operatorname{cosec}^2 x - 4 - 9\operatorname{cosec} x + 6 = 0$

$\Rightarrow 4\operatorname{cosec}^2 x - 9\operatorname{cosec} x + 2 = 0$

$\Rightarrow (4\operatorname{cosec} x - 1)(\operatorname{cosec} x - 2) = 0$

$\Rightarrow \operatorname{cosec} x = \begin{cases} 2 \\ \cancel{\frac{1}{4}} \end{cases}$

$\Rightarrow \sin\theta = \begin{cases} \frac{1}{2} \\ \cancel{4} \end{cases}$

$\arcsin\left(\frac{1}{2}\right) = 30°$

$\begin{cases} \theta = 30 \pm 360n \\ \theta = 150 \pm 360n \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$\therefore \theta = 30, 150$

4. a) $y = x^2 e^x$ BY PRODUCT RULE

$\Rightarrow \frac{dy}{dx} = 2xe^x + x^2e^x = xe^x(2+x)$

FOR MIN/MAX $\frac{dy}{dx} = 0$

$xe^x(x+2) = 0 \quad e^x \neq 0$

$x = \begin{cases} 0 \\ -2 \end{cases} \quad y = \begin{cases} 0 \\ 4e^{-2} \end{cases} \quad \begin{matrix} (0,0) \\ (-2, 4e^{-2}) \end{matrix}$

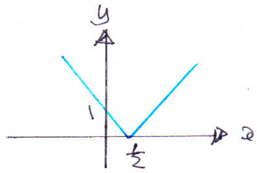
b) $\frac{dy}{dx} = xe^x(x+2) = e^x(x^2+2x)$

$\frac{d^2y}{dx^2} = e^x(x^2+2x) + e^x(2x+2) = e^x\left[x^2+2x+2x+2\right] = e^x(x^2+4x+2)$

THUS $\left.\frac{d^2y}{dx^2}\right|_{x=0} = 2 > 0 \quad \therefore (0,0)$ IS A LOCAL MIN

$\left.\frac{d^2y}{dx^2}\right|_{x=-2} = -2e^{-2} < 0 \quad \therefore (-2, 4e^{-2})$ IS A LOCAL MAX

5. a) $f(x) \geq 0$



b) $gf(x) = 2$

$\Rightarrow g(f(x)) = 2$

$\Rightarrow g(|2x-1|) = 2$

$\Rightarrow \ln(|2x-1| + 2) = 2$

$\ln(2x+1) = 2$ OR $\ln(-2x+1+2) = 2$

$2x+1 = e^2$ $\qquad$ $3-2x = e^2$

$2x = e^2 - 1$ $\qquad$ $3 - e^2 = 2x$

$x = \frac{1}{2}(e^2-1)$ $\qquad$ $x = \frac{1}{2}(3-e^2)$

BOTH CHECKED O.K.

c) $f(x) = g(x)$

$\Rightarrow |2x-1| = \ln(x+2)$

$\boxed{\text{SINCE } 1 \leq x \leq 2 \quad |2x-1| = 2x-1 \text{ OR "LEAVE IT IN"}}$

$\Rightarrow |2x-1| = \ln(x+2)$

$\Rightarrow |2x-1| - \ln(x+2) = 0$

LET $h(x) = |2x-1| - \ln(x+2)$

$h(1) = -0.099 < 0$

$h(2) = 1.614 > 0$

AS $h(x)$ IS CONTINUOUS AND CHANGES SIGN, THERE MUST BE A ROOT BETWEEN 1 & 2

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d) $\boxed{x_{n+1} = \frac{1}{2}\left[1 + \ln(x_n+2)\right]}$

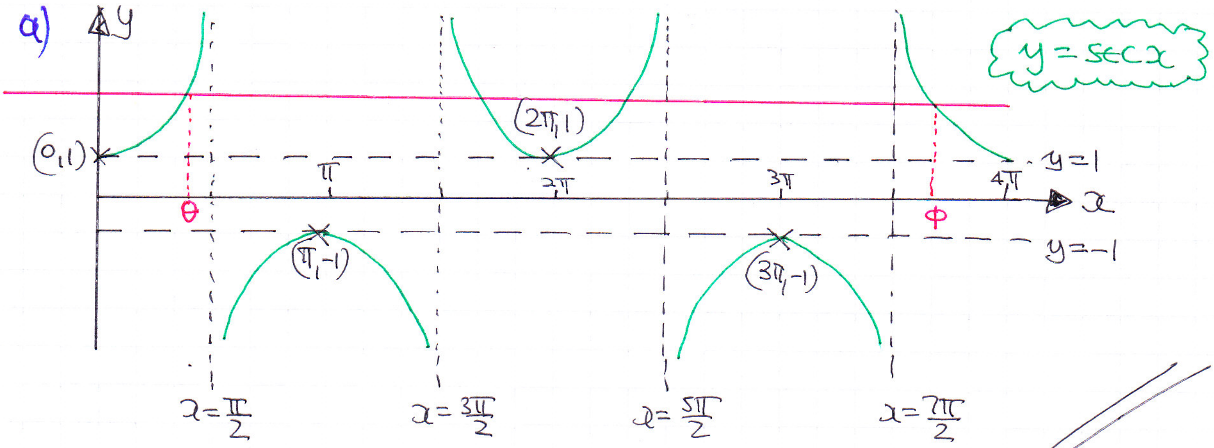
$x_1 = 1$

$x_2 = 1.049$

$x_3 = 1.057$

$x_4 = 1.059$

6. a)



b) LOOKING AT THE "RED LINE"

$\phi = 4\pi - \theta$

7.

$$\left.\begin{array}{l} x + e^y = 5 \\ \ln(x+1)^2 = 2y \end{array}\right\} \Rightarrow \left.\begin{array}{l} e^y = 5 - x \\ \cancel{2}\ln(x+1) = \cancel{2}y \end{array}\right\} \Rightarrow \begin{array}{l} y = \ln(5-x) \\ y = \ln(x+1) \end{array}$$

$\therefore \ln(5-x) = \ln(x+1)$

$5 - x = x + 1$

$4 = 2x$

$x = 2$

$\therefore y = \ln 3$

C3, IYGB, PAPER Q

8. a) $\cos x + \sqrt{3}\sin x \equiv R\cos(x-\alpha)$

$\equiv R\cos x\cos\alpha + R\sin x\sin\alpha$

$\equiv (R\cos\alpha)\cos x + (R\sin\alpha)\sin x$

$\left.\begin{array}{l} R\cos\alpha = 1 \\ R\sin\alpha = \sqrt{3} \end{array}\right\} \Rightarrow$ SQUARE & ADD $R = \sqrt{1^2 + (\sqrt{3})^2}$

$R = 2$

DIVIDE $\tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

$\therefore f(x) = 2\cos(x - \frac{\pi}{3})$

b) $\cos 2\theta + \sqrt{3}\sin 2\theta = 2\cos\theta$

$\cancel{2}\cos(2\theta - \frac{\pi}{3}) = \cancel{2}\cos\theta$

$\cos(2\theta - \frac{\pi}{3}) = \cos\theta$

$\left.\begin{array}{l} 2\theta - \frac{\pi}{3} = \theta \pm 2n\pi \\ 2\theta - \frac{\pi}{3} = (2\pi - \theta) \pm 2n\pi \end{array}\right\} \quad n = 0, 1, 2, 3, \ldots$

$\theta = \frac{\pi}{3} \pm 2n\pi$

$3\theta = \frac{7\pi}{3} \pm 2n\pi$

$\theta = \frac{\pi}{3} \pm 2n\pi$

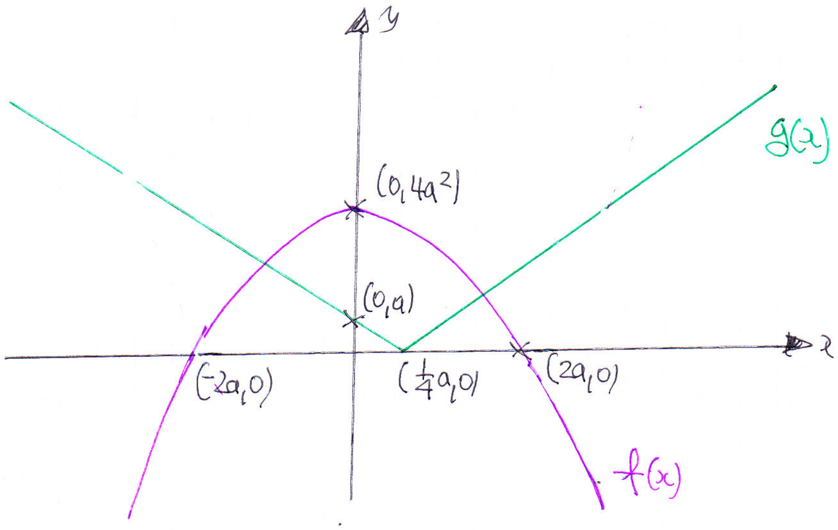
$\theta = \frac{7\pi}{9} \pm \frac{2n\pi}{3}$

$\theta_1 = \frac{\pi}{3}$

$\theta_2 = \frac{7\pi}{9}$

$\theta_3 = \frac{13\pi}{9}$

9. a)



b) $4 - x^2 = |4x - 1|$ USING ABOVE GRAPH WITH $a = 1$

- $4 - x^2 = 4x - 1$

$0 = x^2 + 4x - 5$

$0 = (x+5)(x-1)$

$x = \begin{cases} \cancel{-5} \\ 1 \end{cases}$

- $4 - x^2 = 1 - 4x$

$0 = x^2 - 4x - 3$

$0 = (x-2)^2 - 7$

$(x-2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

$x = \begin{cases} \cancel{2+\sqrt{7}} \\ 2 - \sqrt{7} \end{cases}$

$\therefore x = \begin{cases} 1 \\ 2 - \sqrt{7} \end{cases}$