IYGB GCE

Core Mathematics C3

Advanced

Practice Paper N

Difficulty Rating: 3.4267/1.5544

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Given that

$$\frac{2x^3+x^2-4x+1}{x^2+x-2} \equiv Ax+B+\frac{C}{x+D},$$

use polynomial division, or another appropriate method, to find the value of each of the constants A, B, C and D. **(5)**

Question 2

A curve C has equation

$$y = x\mathrm{e}^{2x}, \; x \in \mathbb{R}.$$

Show that an equation of the tangent to C at the point where $x=\frac{1}{2}$ is **(6)**

$$2y = \mathrm{e}(4x-1).$$

Question 3

$$x^3 = 5x+1, \; x \in \mathbb{R}.$$

a) Show that the above equation has a root α between 2 and 3. **(3)**

The iterative formula

$$x_{n+1} = \sqrt[3]{5x_n+1}, \; x_1 = 2,$$

is to be used to find α

b) Find, to 2 decimal places, the value of x_2, x_3 and x_4. **(2)**

c) By considering the graph of a suitable function in an appropriate interval, show that $\alpha = 2.330$, correct to 3 decimal places. **(2)**

Question 4

It is given that

$$\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta, \quad \theta \neq 90k°, \ k \in \mathbb{Z}.$$

a) Prove the validity of the above trigonometric identity. **(4)**

b) Hence show that

$$\tan 15° = 2-\sqrt{3}.$$ **(3)**

Question 5

The functions f and g are defined by

$$f(x) = e^{2x}-1, \quad x \in \mathbb{R}$$

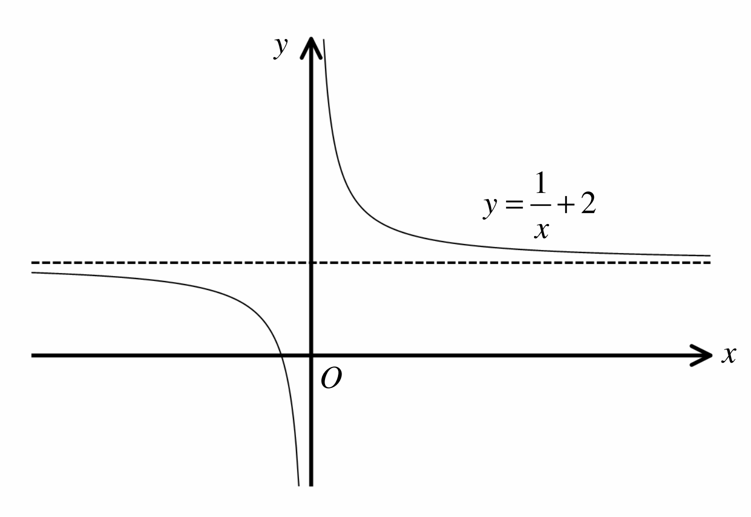
$$g(x) = |x|, \quad x \in \mathbb{R}.$$

a) Find the composite function $gf(x)$, and sketch its graph. **(5)**

b) Solve the equation

$$gf(x) = 1.$$ **(3)**

Question 6



The figure above shows the graph of

$$y=\frac{1}{x}+2,\ x\neq 0.$$

a) State the equation of the horizontal asymptote to the curve, marked as a dotted line in the figure. **(1)**

The function f is defined

$$f(x)=\frac{1}{x}+2,\ x\in\mathbb{R},\ x>1.$$

b) State the range of $f(x)$. **(1)**

c) Obtain an expression for $f^{-1}(x)$. **(3)**

d) State the domain and range of $f^{-1}(x)$. **(2)**

Question 7

Determine, in exact form where appropriate, the solutions of each of the following equations.

a) $2\ln 56 - \left[\ln 168 - \ln\left(\frac{3}{7}\right)\right] = x\ln 2$. **(3)**

b) $e^{y}3^{e} = 3$. **(5)**

c) $e^{\cos(\ln w)} = 1$, $1 \le w < 5$. **(5)**

Question 8

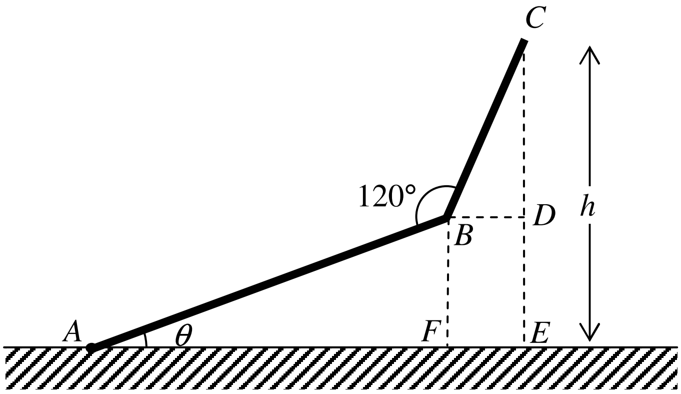
The curve C has equation

$$y = \frac{x}{x^2+1}, \quad x \in \mathbb{R} .$$

a) Show that there is no point on C where the gradient is -1. **(5)**

b) Find the coordinates of the points on C where the gradient is $\frac{12}{25}$. **(5)**

Question 9



The figure above shows a rigid rod ABC where AB is 6 metres, BC is 4 metres and the angle ABC is $120°$. The rod is hinged at A so it can be rotated in a vertical plane forming an angle $\theta°$ with the horizontal ground.

Let h metres be the height of the point C from the horizontal ground.

a) Show clearly that …

i. … $\measuredangle DBC = \theta° + 60°$. **(2)**

ii. … $h = 8\sin\theta + 2\sqrt{3}\cos\theta$. **(4)**

b) By expressing h in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, find to the nearest degree, the values of θ when $h = 6$. **(6)**