C3, IYGB, PAPER M

1.

$$1-\frac{1}{x-2}+\frac{3}{x^2-x-2}=1-\frac{1}{x-2}+\frac{3}{(x-2)(x+1)}$$

$$\frac{(x-2)(x+1)-(x+1)+3}{(x-2)(x+1)}=\frac{x^2-x-2-x-1+3}{(x-2)(x+1)}=\frac{x^2-2x}{(x-2)(x+1)}$$

$$=\frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}=\frac{x}{x+1}$$

2. a)

$$x^3+1=4x$$

$$x^3-4x+1=0$$

$$\boxed{f(x)=x^3-4x+1}$$

$$f(0)=1>0$$

$$f(1)=-2<0$$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN IN THE INTERVAL $[0,1]$ THERE MUST BE A ROOT IN THE INTERVAL

b)

$$x^3+1=4x$$

$$\Longrightarrow\ 1=4x-x^3$$

$$\Longrightarrow\ 1=x(4-x^2)$$

$$\Longrightarrow\ x=\frac{1}{4-x^2}$$

c)

$$x_{n+1}=\frac{1}{4-x_n^2}$$

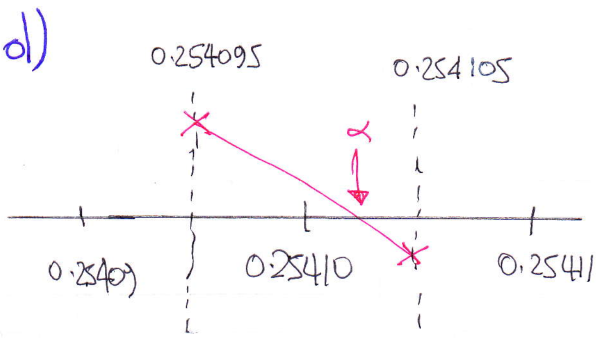
$$x_1=0.1$$

$$x_2=0.2506$$

$$x_3=0.2540$$

$$x_4=0.2541$$

d)



$$f(x)=x^3-4x+1$$

$$f(0.254095)=0.000025>0$$

$$f(0.254105)=-0.000013<0$$

CHANGE OF SIGN & CONTINUITY IMPLIES THAT

$$0.254095<\alpha<0.254105$$

$$\therefore\ \alpha=0.25410$$

5 d.p.

C3, IYGB, PAPER M

3. $y = \ln\left(\frac{x}{4}\right) = \ln\left(\frac{1}{4}x\right)$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{4}x} \times \frac{1}{4} = \frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{4}$$

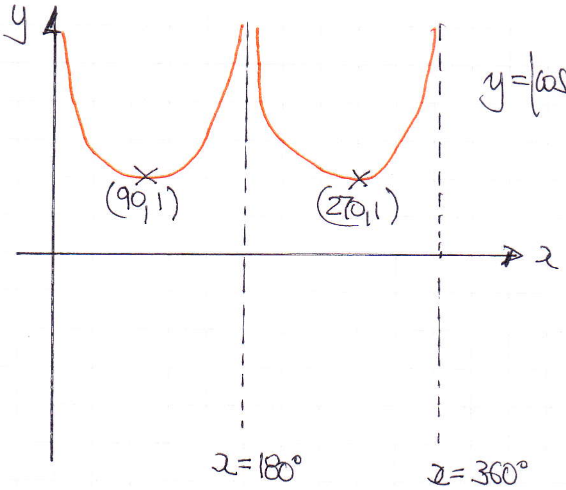
$\therefore$ NORMAL GRADIENT IS -4

WITH $x = 4$ $\quad y = \ln\left(\frac{4}{4}\right) = \cancel{\ln 1} = 0$

$\therefore$ $(4,0)$ WITH GRAD -4

$$y - y_0 = m(x - x_0)$$
$$y - 0 = -4(x - 4)$$
$$y = 16 - 4x$$

4. a)



b) $\operatorname{cosec} x = 2$

$\sin x = \frac{1}{2}$

$\arcsin\left(\frac{1}{2}\right) = 30°$

$$\begin{cases} x = 30 \pm 360n \\ x = 150 \pm 360n \end{cases} \quad n = 0, 1, 2, 3 \ldots$$

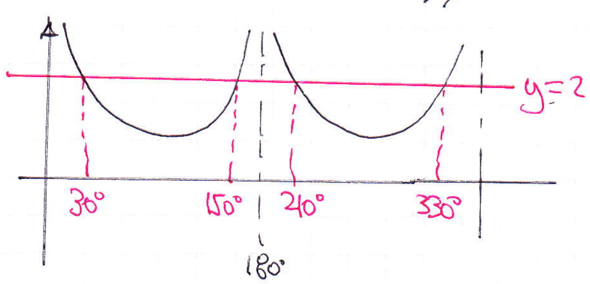
$x_1 = 30°$

$x_2 = 150°$

c) USING PART (a) & (b)

$x_1 = 30°$ $\quad x_3 = 210°$

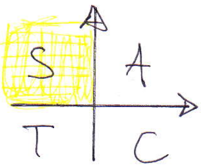
$x_2 = 150°$ $\quad x_4 = 330°$

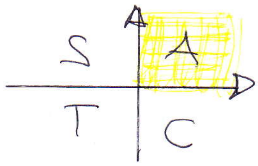


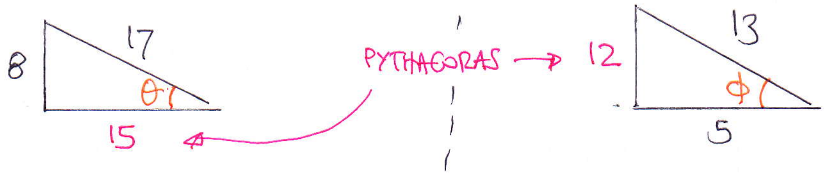
5.

$\sin\theta = \frac{8}{17}$ $\cos\phi = \frac{5}{13}$

OBTUSE   ACUTE







$\cos\theta = -\frac{15}{17}$ $\sin\phi = \frac{12}{13}$

Thus

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$= -\frac{15}{17}\times\frac{5}{13} - \frac{8}{17}\times\frac{12}{13} = -\frac{75}{221} - \frac{96}{221} = -\frac{171}{221}$$

AS REQUIRED

6. a)

$$x = (2y+1)^{\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{2}(2y+1)^{-\frac{1}{2}}\times 2$$

$$\frac{dx}{dy} = (2y+1)^{-\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{\sqrt{2y+1}}$$

$$\frac{dy}{dx} = \sqrt{2y+1}$$

$$\frac{dy}{dx} = x$$

b)

$$x = \sqrt{2y+1}$$

$$\Rightarrow x^2 = 2y+1$$

$$\Rightarrow x^2 - 1 = 2y$$

$$\Rightarrow y = \frac{1}{2}x^2 - \frac{1}{2}$$

$$\therefore \frac{dy}{dx} = x$$

IE. AS IN PART (a)

7. a) $T = 20 + 480e^{-0.1t}$

I) when $t=0$

$T = 20 + 480e^{0}$

$T = 500°C$

II) As $t \to \infty$, $e^{-0.1t} \to 0$

, $480e^{-0.1t} \to 0$

, $T \to 20$

$\therefore$ WATER TEMPERATURE IS 20°C

b) $\boxed{260} = 20 + 480e^{-0.1t}$

$\Rightarrow 240 = 480e^{-0.1t}$

$\Rightarrow \frac{1}{2} = e^{-0.1t}$

$\Rightarrow 2 = e^{0.1t}$

$\Rightarrow \ln 2 = 0.1t$

$\Rightarrow \frac{1}{10}t = \ln 2$

$\Rightarrow t = 10\ln 2$

$\Rightarrow t \simeq 6.93$

c) "RATE" = "DIFFERENTIATION"

$\Rightarrow \frac{dT}{dt} = 480e^{-0.1t} \times (-0.1)$

$\Rightarrow \frac{dT}{dt} = -48e^{-0.1t}$

$\Rightarrow -0.533 = -48e^{-0.1t}$

COOLING

$\Rightarrow 0.011104... = e^{-0.1t}$

$\Rightarrow 90.056... = e^{0.1t}$

$\Rightarrow \ln(90.056...) = 0.1t$

$\Rightarrow t = 45.0$

d) FROM PART (c)

$\frac{dT}{dt} = -48e^{-0.1t}$

$\Rightarrow -10\frac{dT}{dt} = 480e^{-0.1t}$

$\Rightarrow -10\frac{dT}{dt} + 20 = 20 + 480e^{-0.1t}$

$\Rightarrow -10\frac{dT}{dt} + 20 = T$

$\Rightarrow -10\frac{dT}{dt} = T - 20$

$\Rightarrow \frac{dT}{dt} = -\frac{1}{10}(T - 20)$

AS REQUIRED

8. a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

ADDING

$$\sin(\underbrace{A+B}_{P}) + \sin(\underbrace{A-B}_{Q}) = 2\sin A\cos B$$

LET $\left.\begin{array}{l} A+B = P \\ A-B = Q \end{array}\right\} \Rightarrow$ ADD: $2A = P+Q$

$$\boxed{A = \frac{P+Q}{2}}$$

$\Downarrow$

SUBTRACT: $2B = P-Q$

$$\boxed{B = \frac{P-Q}{2}}$$

HENCE: $\sin(\underbrace{A+B}_{P}) + \sin(\underbrace{A-B}_{Q}) = 2\sin \underbrace{A}_{\frac{P+Q}{2}} \cos \underbrace{B}_{\frac{P-Q}{2}}$

ie $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

AS REQUIRED

b) $\sin\underbrace{4\theta}_{P} + \sin\underbrace{2\theta}_{Q} = \cos\theta$

$\Rightarrow 2\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right) = \cos\theta$

$\Rightarrow 2\sin 3\theta\cos\theta = \cos\theta$

$\Rightarrow 2\sin 3\theta\cos\theta - \cos\theta = 0$

$\Rightarrow \cos\theta(2\sin 3\theta - 1) = 0$

- $\cos\theta = 0$

  $\arccos 0 = \frac{\pi}{2}$

  $\left(\begin{array}{l} \theta = \frac{\pi}{2} \pm 2n\pi \\ \theta = \frac{3\pi}{2} \pm 2n\pi \end{array}\right.$ $n = 0, 1, 3, 3 \ldots$

- $\sin 3\theta = \frac{1}{2}$

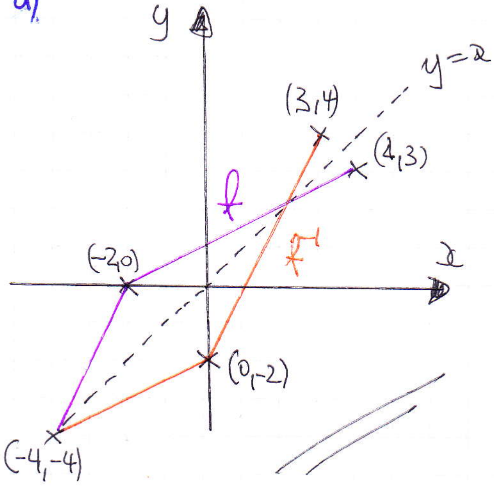
  $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

  $\left(\begin{array}{l} 3\theta = \frac{\pi}{6} \pm 2n\pi \\ 3\theta = \frac{5\pi}{6} \pm 2n\pi \end{array}\right.$ $n = 0, 1, 2, 3, \ldots$

  $\left(\begin{array}{l} \theta = \frac{\pi}{18} \pm \frac{2n\pi}{3} \\ \theta = \frac{5\pi}{18} \pm \frac{2n\pi}{3} \end{array}\right.$

$$\theta = \frac{\pi}{2}, \frac{\pi}{18}, \frac{13\pi}{18}, \frac{5\pi}{18}, \frac{17\pi}{18}$$

9. a)

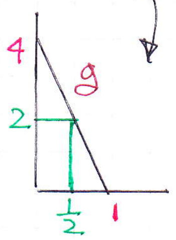


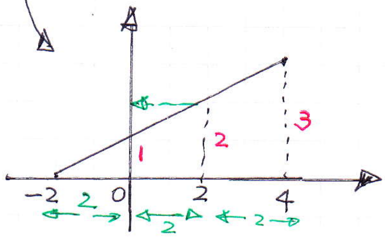
● FOR $f^{-1}(x)$

DOMAIN: $-4 \leq x \leq 3$

RANGE: $-4 \leq f^{-1}(x) \leq 4$

b) I) $fg(\tfrac{1}{2}) = f(g(\tfrac{1}{2})) = f(2) = 2$





II) $f(g(f^{-1}(1))) = f(g(0)) = f(4) = 3$