C3 IYGB PAPER J

1. $y = \dfrac{x}{1+\ln x}$

$\dfrac{dy}{dx} = \dfrac{(1+\ln x)\times 1 - x \times \frac{1}{x}}{(1+\ln x)^2} = \dfrac{1+\ln x - 1}{(1+\ln x)^2} = \dfrac{\ln x}{(1+\ln x)^2}$

FOR STATIONARY POINTS $\dfrac{dy}{dx} = 0$

$\dfrac{\ln x}{(1+\ln x)^2} = 0$

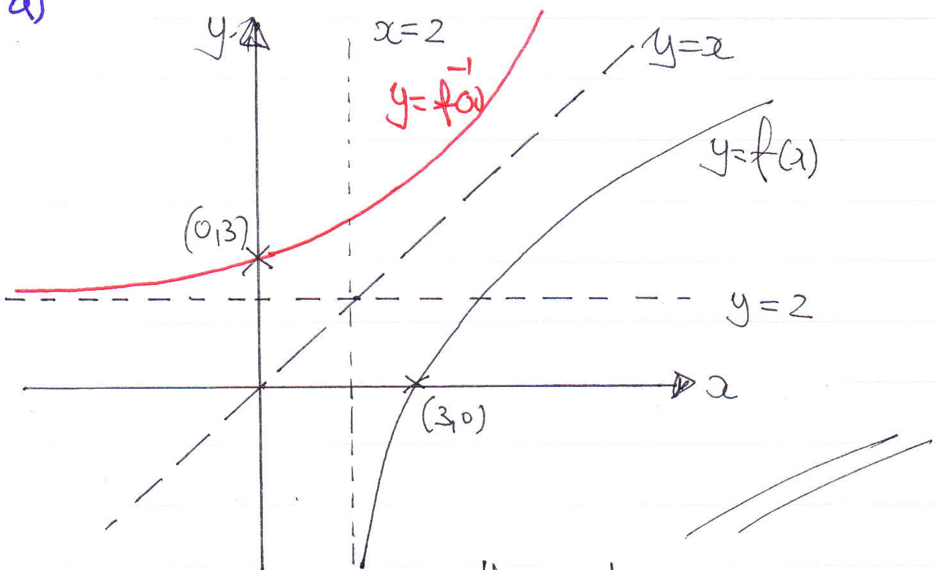
$\ln x = 0$

$x = 1$

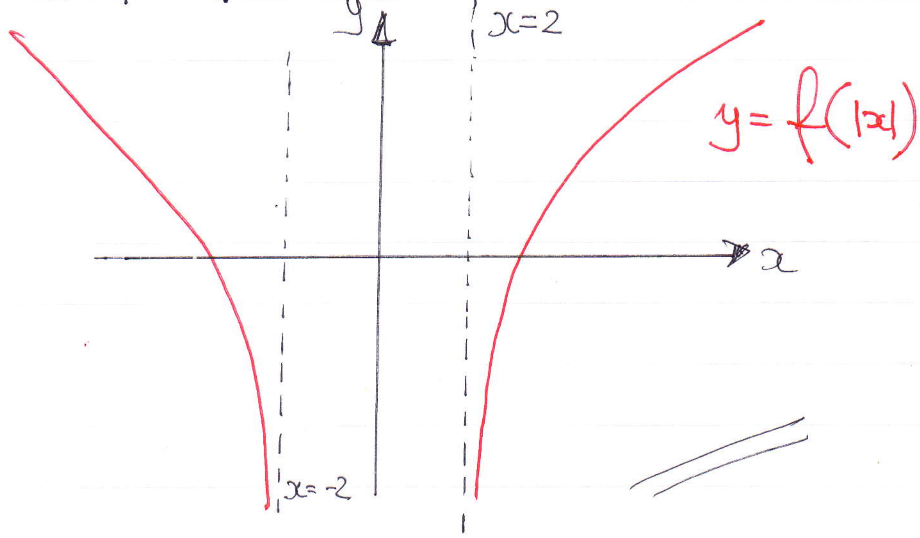
$y = \dfrac{1}{1+\cancel{\ln 1}} = 1$

$y = 1$ $\quad$ $\therefore$ JUST A SINGLE STATIONARY POINT AT $(1,1)$

2. a)



b)



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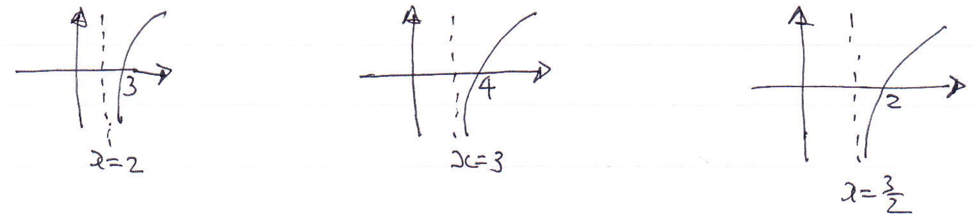
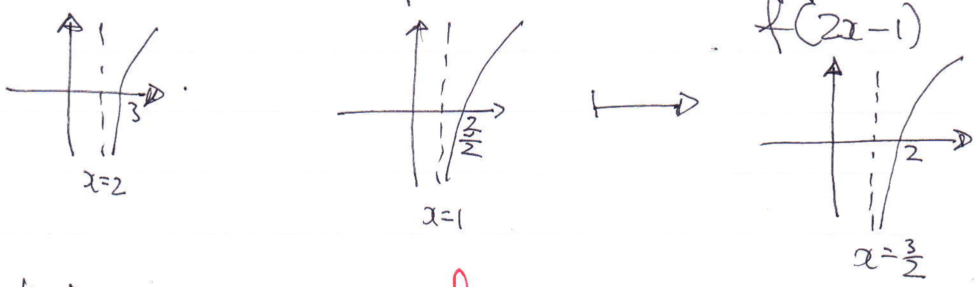
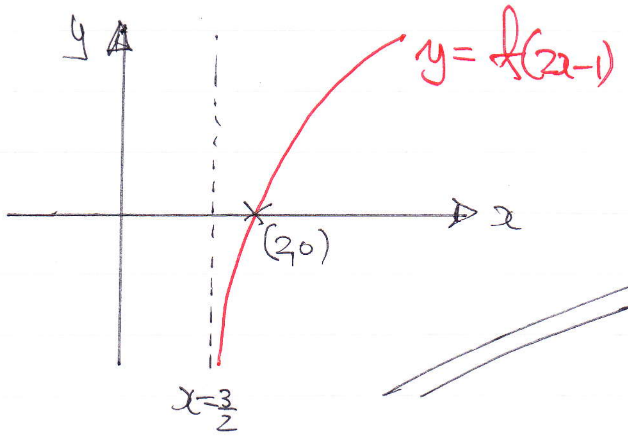
c) $f(x) \longmapsto f(x-1) \longmapsto f(2x-1)$



OR $f(x) \longmapsto f(2x) \longmapsto f\left(2\left(x-\tfrac{1}{2}\right)\right)$

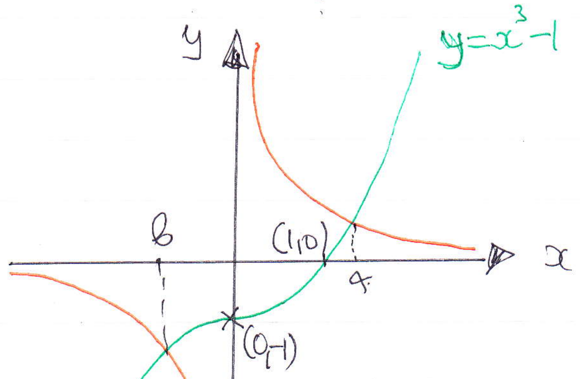
$f(2x-1)$





3. a) $x^3 - 1 - \frac{1}{x} = 0$

$x^3 - 1 = \frac{1}{x}$



TWO ROOTS, ONE POSITIVE & ONE NEGATIVE INDEED

b) BECAUSE THE INTERSECTION WHICH PRODUCES α MUST BE LARGER THAN THE x INTERCEPT OF $y = x^3 - 1$

(SEE DIAGRAM)

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c) $x_{n+1} = \sqrt[3]{\frac{1}{x_n} + 1}$

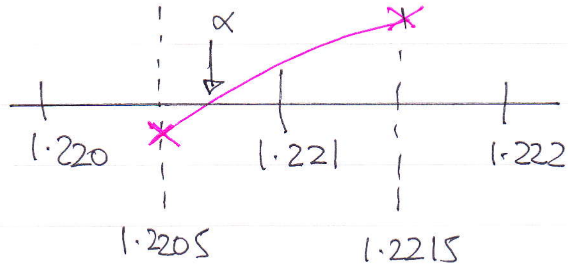
$x_0 = 1.5$

$x_1 = 1.19$

$x_2 = 1.23$

$x_3 = 1.22$

d)



- LET $f(x) = x^3 - 1 - \frac{1}{x}$

$f(1.2205) = -0.0013$

$f(1.2215) = 0.0039$

- CHANGE OF SIGN & CONTINUITY IMPLIES

$1.2205 < \alpha < 1.2215$

$\therefore \quad \alpha = 1.221$ 3 d.p

4. $f(x) = e^x \sin 2x$ (BY PRODUCT RULE)

$f'(x) = e^x \sin 2x + e^x(2\cos 2x)$

$f'(x) = e^x(\sin 2x + 2\cos 2x)$

NOW

$f'(\pi) = e^\pi(\cancel{\sin 2\pi} + 2\cos 2\pi) = 2e^\pi$ ← GRAD OF TANGENT

$f(\pi) = e^\pi(\cancel{\sin 2\pi}) = 0 \quad \therefore P(\pi, 0)$

$\therefore$ EQUATION OF NORMAL

$y - y_0 = m(x - x_0)$

$y - 0 = -\frac{1}{2e^\pi}(x - \pi)$

$2e^\pi y = -x + \pi$

$x + 2ye^\pi = \pi$

AS REQUIRED

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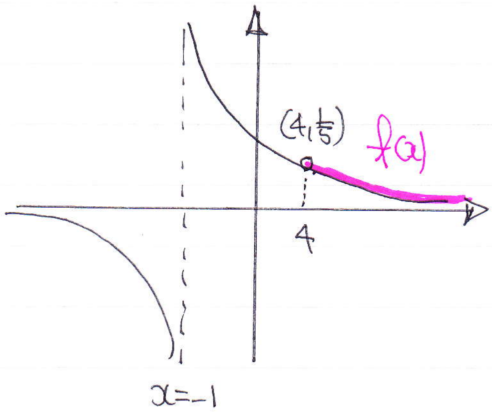
5\. a) $f(x) = \dfrac{2x-1}{x^2-x-2} - \dfrac{1}{x-2} = \dfrac{2x-1}{(x-2)(x+1)} - \dfrac{1}{x-2}$

$= \dfrac{2x-1-(x+1)}{(x-2)(x+1)} = \dfrac{2x-1-x-1}{(x-2)(x+1)} = \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x+1)}$

$\therefore f(x) = \dfrac{1}{x+1}$ AS REQUIRED

b) 

$\therefore$ Range

$0 < f(x) < \frac{1}{5}$

c) $y = \dfrac{1}{x+1}$

$yx + y = 1$

$yx = 1 - y$

$x = \dfrac{1-y}{y}$ $\left(\text{OR } \dfrac{1}{y} - 1\right)$

$\therefore f^{-1}(x) = \dfrac{1}{x} - 1$

d)

	$f(x)$	$f^{-1}(x)$
D	$x > 4$	$0 < x < \frac{1}{5}$
R	$0 < f(x) < \frac{1}{5}$	$f^{-1}(x) > 4$

$\therefore$ DOMAIN : $0 < x < \frac{1}{5}$

RANGE : $f^{-1}(x) > 4$

d) $f(g(x)) = \frac{1}{11}$

$\Rightarrow f(3x^2-2) = \frac{1}{11}$

$\Rightarrow \dfrac{1}{(3x^2-2)+1} = \dfrac{1}{11}$

$\Rightarrow \dfrac{1}{3x^2-1} = \dfrac{1}{11}$

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$\Rightarrow \; 3x^2 - 1 = 11$

$\Rightarrow \; 3x^2 = 12$

$\Rightarrow \; x^2 = 4$

$\Rightarrow \; x = \begin{cases} 2 \\ -2 \end{cases}$ // (BOTH O.K!)

6. a) $\tan 25 = \tan(45 - 20) = \dfrac{\tan 45 - \tan 20}{1 + \tan 45 \tan 20}$

$= \dfrac{1 - t}{1 + t}$ // (SINCE $\tan 45 = 1$)

b) $\tan 65 = \tan(45 + 20) = \dfrac{\tan 45 + \tan 20}{1 - \tan 45 \tan 20} = \dfrac{1 + t}{1 - t}$

$\therefore \; \tan 25 \tan 65 = \dfrac{\cancel{1-t}}{\cancel{1+t}} \times \dfrac{\cancel{1+t}}{\cancel{1-t}} = 1$ // AS REQUIRED

c) $2\cos(\theta + 20) = 5\sin(\theta - 20)$

$2\cos\theta\cos 20 - 2\sin\theta\sin 20 = 5\sin\theta\cos 20 - 5\cos\theta\sin 20$

DIVIDE BY $\cos 20 \cos\theta$

$\dfrac{2\cancel{\cos\theta}\cancel{\cos 20}}{\cancel{\cos 20\cos\theta}} - \dfrac{2\sin\theta\sin 20}{\cos 20\cos\theta} = \dfrac{5\sin\theta\cancel{\cos 20}}{\cancel{\cos 20}\cos\theta} - \dfrac{5\cancel{\cos\theta}\sin 20}{\cos 20\cancel{\cos\theta}}$

$\Rightarrow \; 2 - 2\tan\theta\tan 20 = 5\tan\theta - 5\tan 20$

$\Rightarrow \; 2 - 2t\tan\theta = 5\tan\theta - 5t$

$\Rightarrow \; 2 + 5t = 5\tan\theta + 2t\tan\theta$

$\Rightarrow \; 2 + 5t = \tan\theta(5 + 2t)$

$\Rightarrow \; \tan\theta = \dfrac{2 + 5t}{5 + 2t}$ // AS REQUIRED

C3, IYGB, PAPER J

7. a) $LHS = \cos(A+B) + \cos(A-B)$

$= \cos A\cos B - \cancel{\sin A\sin B} + \cos A\cos B + \sin A\sin B$

$= 2\cos A\cos B$

$= RHS$

b) $2\cos\left(x+\frac{\pi}{6}\right) = \sec\left(x+\frac{\pi}{2}\right)$

$\Rightarrow 2\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{\cos\left(x+\frac{\pi}{2}\right)}$

$\Rightarrow 2\cos\left(\overset{A}{x+\frac{\pi}{6}}\right)\cos\left(\overset{B}{x+\frac{\pi}{2}}\right) = 1$

LOOKING AT THE IDENTITY FROM RHS TO LHS

$\Rightarrow \cos\left[\left(x+\frac{\pi}{6}\right)\overset{A+B}{+}\left(x+\frac{\pi}{2}\right)\right] + \cos\left[\left(x+\frac{\pi}{6}\right)\overset{A-B}{-}\left(x+\frac{\pi}{2}\right)\right] = 1$

$\Rightarrow \cos\left(2x+\frac{2\pi}{3}\right) + \cos\left(-\frac{\pi}{3}\right) = 1$

$\Rightarrow \cos\left(2x+\frac{2\pi}{3}\right) + \frac{1}{2} = 1$

$\Rightarrow \cos\left(2x+\frac{2\pi}{3}\right) = \frac{1}{2}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\left(\begin{array}{l} 2x+\frac{2\pi}{3} = \frac{\pi}{3} \pm 2n\pi \\ 2x+\frac{2\pi}{3} = \frac{5\pi}{3} \pm 2n\pi \end{array}\right. \qquad n = 0,1,2,3,\ldots$

$\left(\begin{array}{l} 2x = -\frac{\pi}{3} \pm 2n\pi \\ 2x = \pi \pm 2n\pi \end{array}\right.$

$\left(\begin{array}{l} x = -\frac{\pi}{6} \pm n\pi \\ x = \frac{\pi}{2} \pm n\pi \end{array}\right.$

$\therefore\ x_1 = \frac{5\pi}{6}$

$x_2 = \frac{\pi}{2}$

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8. a) $\theta = 300 - 280e^{-0.05t}$

when $t=0$, $\theta = 300 - 280e^{0}$

$\theta = 300 - 280$

$\theta = 20°C$

b) I) $160 = 300 - 280e^{-0.05t}$

$280e^{-0.05t} = 140$

$e^{-0.05t} = \frac{1}{2}$

$e^{0.05t} = 2$

$0.05t = \ln 2$

$t = 20\ln 2$

$t \simeq 13.86$ min

II) $\frac{d\theta}{dt} = -0.05(-280e^{-0.05t})$

$\frac{d\theta}{dt} = 14e^{-0.05t}$

$4 = 14e^{-0.05t}$

$\frac{2}{7} = e^{-0.05t}$

$e^{0.05t} = \frac{7}{2}$

$0.05t = \ln\frac{7}{2}$

$t \simeq 25.06$

c) As $t \to \infty$, $e^{-0.05t} \to 0$

$-280e^{-0.05t} \to 0$

$\theta \to 300$

$\therefore \theta_{MAX} = 300°C$

d) $300 - 280e^{-0.05t} = 250 - 230e^{-0.1t}$

MULTIPLY THROUGH BY $e^{0.1t}$ AFTER GROUPING CONSTANTS

$\Rightarrow 50 - 280e^{-0.05t} = -230e^{-0.1t}$

$\Rightarrow 50e^{0.1t} - 280e^{0.05t} = -230$.

$\Rightarrow 50(e^{0.05t})^2 - 280(e^{0.05t}) + 230 = 0$

$\Rightarrow 50X^2 - 280X + 230 = 0$

$\Rightarrow 5X^2 - 28X + 23 = 0$

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$\Rightarrow (5x - 23)(x - 1) = 0$

$\Rightarrow x = \begin{cases} 1 \\ \frac{23}{5} \end{cases}$

$\Rightarrow e^{0.05t} = \begin{cases} 1 \\ \frac{23}{5} \end{cases}$

$\Rightarrow t = \begin{cases} 0 & \text{(ALREADY KNOWN)} \\ 20 \ln \frac{23}{5} \simeq 30.52 \text{ min} \end{cases}$