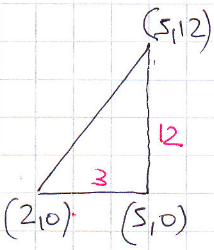
C2, IYGB, PAPER Z

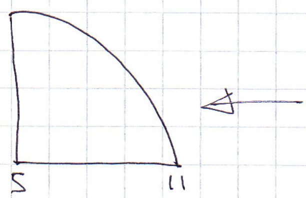
1.

$y = 4x - 8$
$0 = 4x - 8$
$x = 2$
$\boxed{A(2,0)}$

$y = -x^2 + 14x - 33$
$0 = -x^2 + 14x - 33$
$x^2 - 14x + 33 = 0$
$(x-11)(x-3) = 0$
$x = 3, 11$ $\quad \boxed{B(11,0)}$



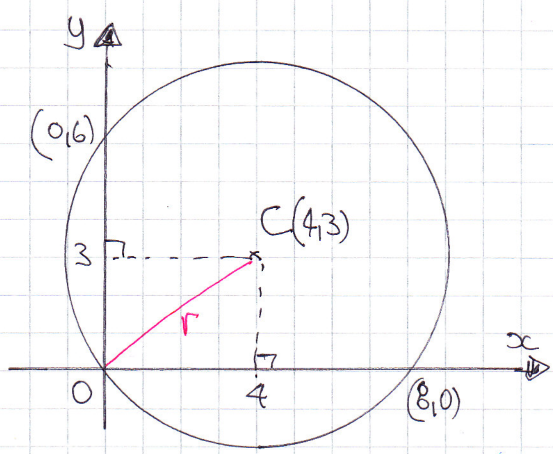
$\frac{1}{2} \times 3 \times 12 = 18$



$$\int_5^{11} -x^2 + 14x - 33 \, dx$$
$$= \left[-\tfrac{1}{3}x^3 + 7x^2 - 33x\right]_5^{11}$$
$$= \left(-\tfrac{1331}{3} + 847 - 363\right) - \left(-\tfrac{125}{3} + 175 - 165\right)$$
$$= \frac{121}{3} - \left(-\frac{95}{3}\right) = 72$$

∴ REQUIRED AREA $= 18 + 72 = 90$

2.



- BY INSPECTION CENTRE IS AT $(4,3)$
- RADIUS $= 5$ $\quad$ (3, 4, 5 TRIANGLE)

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3. a) $(1+2x)^5 = 1 + 5(2x)^1 + 10(2x)^2 + 10(2x)^3 + 5(2x)^4 + 1(2x)^5$
$= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

b) $f(-x) = (1+2(-x))^5 = (1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

c) $f(x) - f(-x) = 64x$

$$\begin{array}{r} \cancel{1} + 10x + \cancel{40x^2} + 80x^3 + \cancel{80x^4} + 32x^5 \\ \cancel{-1} + 10x - \cancel{40x^2} + 80x^3 - \cancel{80x^4} + 32x^5 \\ \hline 20x \qquad + 160x^3 \qquad + 64x^5 \end{array} = 64x$$

$\Rightarrow 64x^5 + 160x^3 - 44x = 0$
$\Rightarrow 64x^4 + 160x^2 - 44 = 0 \quad (x \neq 0)$
$\Rightarrow 16x^4 + 40x^2 - 11 = 0$

FACTORIZE OR QUADRATIC FORMULA

$\Rightarrow (4x^2 - 1)(4x^2 + 11) = 0$

$\Rightarrow x^2 = \begin{cases} 1/4 \\ \cancel{-11/4} \end{cases}$

$\Rightarrow x = \pm \frac{1}{2}$

4. $\log_4 x - 2\log_x 4 = 1$

$\Rightarrow \log_4 x - 2 \times \frac{1}{\log_4 x} = 1$

$\boxed{\log_a b \equiv \frac{1}{\log_b a}}$

$\Rightarrow y - \frac{2}{y} = 1 \quad$ WITH $y = \log_4 x$

$\Rightarrow y^2 - 2 = y$

C2, IYGB, PAPER Z

$\Rightarrow y^2 - y - 2 = 0$

$\Rightarrow (y+1)(y-2)$

$\Rightarrow y = \begin{cases} -1 \\ 2 \end{cases}$

$\log_4 x = \begin{cases} -1 \\ 2 \end{cases}$

$x = \begin{cases} 1/4 \\ 16 \end{cases}$

EITHER $\log_b x = a \Rightarrow x = b^a$

OR
$\log_4 x = -1$
$\log_4 x = -1 \times \log_4 4$
$\log_4 x = \log_4 4^{-1}$
$\log_4 x = \log_4 \frac{1}{4}$
$x = \frac{1}{4}$

& SIMILARLY THE OTHER

5. a)

u_1	u_2	u_3
$2x+4$	$3x+2$	x^2-11

$2x+4 \xrightarrow{\times r} 3x+2 \xrightarrow{\times r} x^2-11$

$\frac{3x+2}{2x+4} = \frac{x^2-11}{3x+2}$

$(3x+2)^2 = (2x+4)(x^2-11)$

$9x^2+12x+4 = 2x^3-22x+4x^2-44$

$0 = 2x^3-5x^2-34x-48$

AS REQUIRED

b) BY LONG DIVISION OR INSPECTION

$2x^3-5x^2-34x-48=0$

$(x-6)(2x^2+7x+8)=0$

$b^2-4ac = 7^2-4\times2\times8$
$= 49-64 = -15 < 0$
$\therefore$ IRREDUCIBLE

$\therefore$ ONLY SOLUTION $x=6$

c) If $x=6$,

u_1	u_2	u_3
16	20	25

$a=16$
$r=1.25=\frac{5}{4}$
$n=8$

$$S_n=\frac{a(r^n-1)}{r-1}$$

$$S_8=\frac{16(1.25^8-1)}{1.25-1}=317.469\ldots \simeq 317 \quad (3\text{ s.f.})$$

6.
$$\sin\theta\tan^2\theta(2\sin\theta+3)+\tan^2\theta=0$$
$$\Rightarrow \tan^2\theta\left[\sin\theta(2\sin\theta+3)+1\right]=0$$
$$\Rightarrow \tan^2\theta\left[2\sin^2\theta+3\sin\theta+1\right]=0$$
$$\Rightarrow \tan^2\theta(2\sin\theta+1)(\sin\theta+1)=0$$

- $\tan\theta=0$

$\theta=0\pm180n$

$n=0,1,2,3,\ldots$

- $\sin\theta=-1$

$\theta=-90\pm360n$
$\theta=270\pm360n$

$n=0,1,2,\ldots$

- $\sin\theta=-\frac{1}{2}$

$\theta=-30^\circ\pm360n$
$\theta=210^\circ\pm360n$

$n=0,1,2,3\ldots$

$\theta=0,180$

~~$\theta=270$~~ BECAUSE OF $\tan\theta$

$\theta=330^\circ, 210^\circ$

$$\theta=0^\circ, 180^\circ, 210^\circ, 330^\circ$$

ALTERNATIVE

$$\Rightarrow \sin\theta\tan^2\theta(2\sin\theta+3)+\tan^2\theta=0$$
$$\Rightarrow \sin\theta\times\frac{\sin^2\theta}{\cos^2\theta}\times(2\sin\theta+3)+\frac{\sin^2\theta}{\cos^2\theta}=0$$
$$\Rightarrow \frac{\sin^3\theta(2\sin\theta+3)+\sin^2\theta}{\cos^2\theta}=0$$
$$\Rightarrow 2\sin^4\theta+3\sin^3\theta+\sin^2\theta=0$$
$$\Rightarrow \sin^2\theta\left[2\sin^2\theta+3\sin\theta+1\right]=0$$

EITHER $\sin\theta=0$

OR $\sin\theta=\begin{cases}-1\\-\frac{1}{2}\end{cases}$

AS BEFORE

THIS TIME $\theta=270^\circ$ IS REJECTED BECAUSE OF DIVISION BY $\cos^2\theta$

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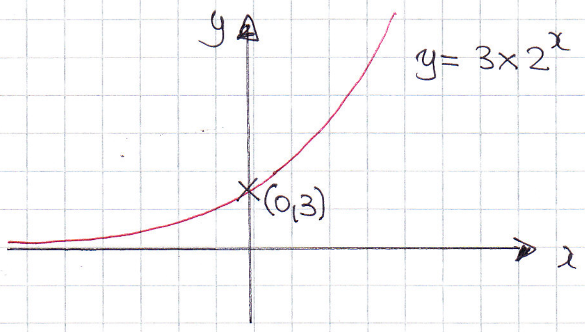
7. a) $y = \left(\frac{1}{2}\right)^x = \left(2^{-1}\right)^x = 2^{-x}$ I.E $f(-x)$

$\therefore$ REFLECTION, IN THE y AXIS

b)



c) $\left.\begin{array}{l} y = \left(\frac{1}{2}\right)^x \\ y = 3 \times 2^x \end{array}\right\} \Rightarrow \frac{1}{2^x} = 3 \times 2^x$

$\Rightarrow 1 = 3 \times 2^x \times 2^x$

$\frac{1}{3} = 2^{2x}$

$\frac{1}{3} = \left(2^x\right)^2$

$\pm\sqrt{\frac{1}{3}} = 2^x$

$+\frac{1}{\sqrt{3}} = 2^x \quad (2^x > 0)$

$\therefore 3 \times 2^x = \frac{3}{\sqrt{3}}$

$y = \sqrt{3}$

$\log \frac{1}{3} = \log 2^{2x}$

$\log \frac{1}{3} = 2x \log 2$

$x = \frac{\log \frac{1}{3}}{2 \log 2} = \frac{\log \frac{1}{3}}{\log 4}$

$x = -\frac{\log 3}{\log 4}$

PROBLEM GETTING EXACT ANSWERS!

ALTERNATIVE (BETTER APPROACH)

$y = \left(\frac{1}{2}\right)^x = \frac{1}{2^x}$

$\therefore \boxed{\frac{1}{y} = 2^x}$

THUS $y = 3 \times 2^x$

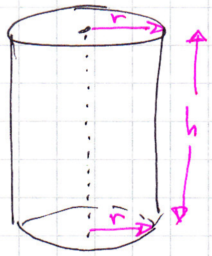
$y = 3 \times \frac{1}{y}$

$y^2 = 3$

$y = \pm\sqrt{3}$

$y = \sqrt{3} \quad (y > 0)$

8.



- $V = 16\pi$

$\pi r^2 h = 16\pi$

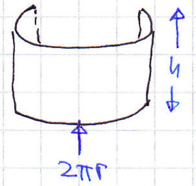
$\boxed{r^2 h = 16}$

$$A = 2\pi r^2 + 2\pi r h$$

πr^2

$+$

πr^2



- $A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$A = 2\pi r^2 + \frac{32\pi}{r}$

$A = 2\pi r^2 + 32\pi r^{-1}$

$\frac{dA}{dr} = 4\pi r - 32\pi r^{-2}$

$\frac{dA}{dr} = 4\pi r - \frac{32\pi}{r^2}$

- SOLVE FOR ZERO

$4\pi r - \frac{32\pi}{r^2} = 0$

$4\cancel{\pi} r = \frac{32\cancel{\pi}}{r^2}$

$4r^3 = 32$

$r^3 = 8$

$r = 2$

NO NEED TO SHOW THAT THIS VALUE OF r MINIMIZES A, AS IT IS <u>GIVEN</u> IN THE QUESTION

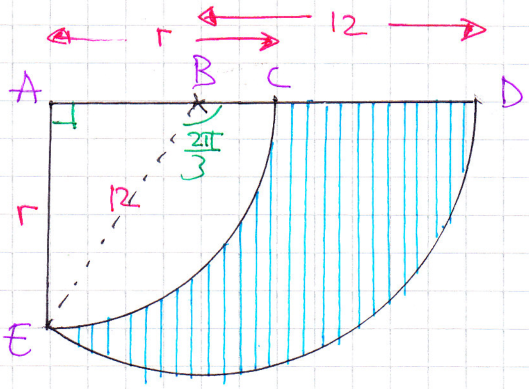
$r^2 h = 16$

$4h = 16$

$h = 4$

C2, IYGB, PAPER Z

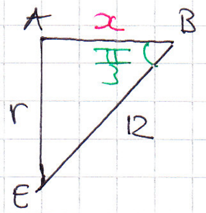
9.



- AREA OF SECTOR EBD

$$"\tfrac{1}{2}r^2\theta" = \tfrac{1}{2}\times 12^2 \times \tfrac{2\pi}{3}$$
$$= 48\pi$$

- $\hat{ABE} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

- 

$$\sin\frac{\pi}{3} = \frac{r}{12}$$
$$\frac{\sqrt{3}}{2} = \frac{r}{12}$$
$$\boxed{r = 6\sqrt{3}}$$

- AREA OF $\frac{1}{4}$ CIRCLE

$$"\tfrac{1}{4}\pi r^2" = \tfrac{1}{4}\pi \times (6\sqrt{3})^2 = 27\pi$$

- $x^2 + r^2 = 12^2$

$$x^2 + 108 = 144$$
$$\boxed{x = 6}$$

- AREA OF TRIANGLE $\overset{\triangle}{ABE}$

$$\tfrac{1}{2}xr = \tfrac{1}{2}\times 6 \times 6\sqrt{3} = 18\sqrt{3}$$

REQUIRED AREA = TRIANGLE ABE + SECTOR EBD − QUARTER CIRCLE EAC

$$= 18\sqrt{3} + 48\pi - 27\pi$$
$$= 21\pi + 18\sqrt{3}$$
$$= 3(7\pi + 6\sqrt{3})$$

// AS REQUIRED